4.4 Factoring Polynomials

Do Now: Determine the factors of 72

Example 1: Finding a Common Monomial Factor

Factor each polynomial completely.

a) $y^5 - 48y^3$

b) $x^3 - 4x^2 - 5x$

c) $5z^5 + 30z^3 + 45z^2$

Key Notes!

Core Concept

Special Factoring Patterns

Sum of Two Cubes	Example
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$	$64x^3 + 1 = (4x)^3 + 1^3$ $= (4x + 1)(16x^2 - 4x + 1)$

Difference of Two Cubes	Example
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$	$27x^3 - 8 = (3x)^3 - 2^3$ $= (3x - 2)(9x^2 + 6x + 4)$

Example 2: Factoring the Sum of Difference of Two Cubes

Factor $x^3 - 125$

Factor $16m^5 + 54m^2$

Example 3: Factoring by Grouping

Factor $k^3 + 5k^2 - 4k - 20$

Example 4: Factoring Polynomials in Quadratic Form

Factor $16x^4 - 81$

Example 5: Determining Whether a Linear Binomial is a Factor.

Determine whether $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$

Try on your own.

Determine whether $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - 2x^2 + 25$

Example 6: Factoring a Polynomial

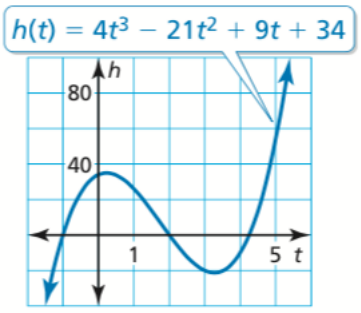
Show that $x + 3$ is a factor of $f(x) = x^4 - 3x^3 - x - 3$. Then factor $f(x)$ completely.

Example 7:

EXAMPLE 7 **Real-Life Application**

During the first 5 seconds of a roller coaster ride, the function $h(t) = 4t^3 - 21t^2 + 9t + 34$ represents the height h (in feet) of the roller coaster after t seconds. How long is the roller coaster at or below ground level in the first 5 seconds?

SOLUTION



Classwork/Homework (I know this is a lot, but it is one of the most important sections!)
5, 7, 9, 13, 16, 17, 23, 26, 27, 30, 31, 35, 36, 39, 40, 42, 43, 45, 46, 48, 51-54, 56, 72, 73

4.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The expression $9x^4 - 49$ is in _____ form because it can be written as $u^2 - 49$ where $u =$ _____.
2. **VOCABULARY** Explain when you should try factoring a polynomial by grouping.
3. **WRITING** How do you know when a polynomial is factored completely?
4. **WRITING** Explain the Factor Theorem and why it is useful.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, factor the polynomial completely. *(See Example 1.)*

5. $x^3 - 2x^2 - 24x$
6. $4k^5 - 100k^3$
7. $3p^5 - 192p^3$
8. $2m^6 - 24m^5 + 64m^4$
9. $2q^4 + 9q^3 - 18q^2$
10. $3r^6 - 11r^5 - 20r^4$
11. $10w^{10} - 19w^9 + 6w^8$
12. $18v^9 + 33v^8 + 14v^7$

In Exercises 13–20, factor the polynomial completely. *(See Example 2.)*

13. $x^3 + 64$
14. $y^3 + 512$
15. $g^3 - 343$
16. $c^3 - 27$
17. $3h^9 - 192h^6$
18. $9n^6 - 6561n^3$
19. $16t^7 + 250t^4$
20. $135z^{11} - 1080z^8$

ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in factoring the polynomial.**

21. ✗
$$3x^3 + 27x = 3x(x^2 + 9)$$
$$= 3x(x + 3)(x - 3)$$

22. ✗
$$x^9 + 8x^3 = (x^3)^3 + (2x)^3$$
$$= (x^3 + 2x)[(x^3)^2 - (x^3)(2x) + (2x)^2]$$
$$= (x^3 + 2x)(x^6 - 2x^4 + 4x^2)$$

In Exercises 23–30, factor the polynomial completely. *(See Example 3.)*

23. $y^3 - 5y^2 + 6y - 30$
24. $m^3 - m^2 + 7m - 7$
25. $3a^3 + 18a^2 + 8a + 48$
26. $2k^3 - 20k^2 + 5k - 50$
27. $x^3 - 8x^2 - 4x + 32$
28. $z^3 - 5z^2 - 9z + 45$
29. $4q^3 - 16q^2 - 9q + 36$
30. $16n^3 + 32n^2 - n - 2$

In Exercises 31–38, factor the polynomial completely. *(See Example 4.)*

31. $49k^4 - 9$
32. $4m^4 - 25$
33. $c^4 + 9c^2 + 20$
34. $y^4 - 3y^2 - 28$
35. $16z^4 - 81$
36. $81a^4 - 256$
37. $3r^8 + 3r^5 - 60r^2$
38. $4n^{12} - 32n^7 + 48n^2$

In Exercises 39–44, determine whether the binomial is a factor of the polynomial. *(See Example 5.)*

39. $f(x) = 2x^3 + 5x^2 - 37x - 60;\ x - 4$
40. $g(x) = 3x^3 - 28x^2 + 29x + 140;\ x + 7$
41. $h(x) = 6x^5 - 15x^4 - 9x^3;\ x + 3$
42. $g(x) = 8x^5 - 58x^4 + 60x^3 + 140;\ x - 6$
43. $h(x) = 6x^4 - 6x^3 - 84x^2 + 144x;\ x + 4$
44. $t(x) = 48x^4 + 36x^3 - 138x^2 - 36x;\ x + 2$

In Exercises 45–50, show that the binomial is a factor of the polynomial. Then factor the polynomial completely. *(See Example 6.)*

45. $g(x) = x^3 - x^2 - 20x; x + 4$

46. $t(x) = x^3 - 5x^2 - 9x + 45; x - 5$

47. $f(x) = x^4 - 6x^3 - 8x + 48; x - 6$

48. $s(x) = x^4 + 4x^3 - 64x - 256; x + 4$

49. $r(x) = x^3 - 37x + 84; x + 7$

50. $h(x) = x^3 - x^2 - 24x - 36; x + 2$

ANALYZING RELATIONSHIPS In Exercises 51–54, match the function with the correct graph. Explain your reasoning.

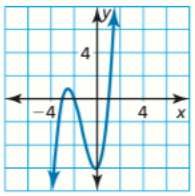
51. $f(x) = (x - 2)(x - 3)(x + 1)$

52. $g(x) = x(x + 2)(x + 1)(x - 2)$

53. $h(x) = (x + 2)(x + 3)(x - 1)$

54. $k(x) = x(x - 2)(x - 1)(x + 2)$

A.



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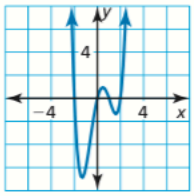

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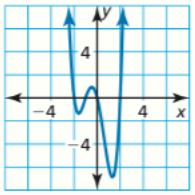

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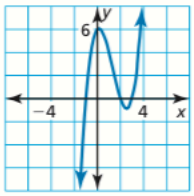
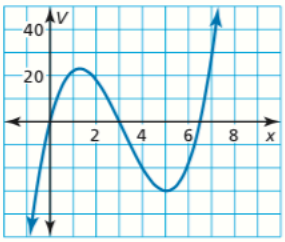
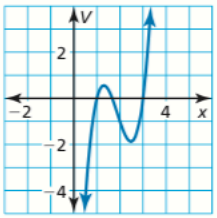


55. MODELING WITH MATHEMATICS The volume (in cubic inches) of a shipping box is modeled by $V = 2x^3 - 19x^2 + 39x$, where x is the length (in inches). Determine the values of x for which the model makes sense. Explain your reasoning. *(See Example 7.)*



56. MODELING WITH MATHEMATICS The volume (in cubic inches) of a rectangular birdcage can be modeled by $V = 3x^3 - 17x^2 + 29x - 15$, where x is the length (in inches). Determine the values of x for which the model makes sense. Explain your reasoning.



USING STRUCTURE In Exercises 57–64, use the method of your choice to factor the polynomial completely. Explain your reasoning.

57. $a^6 + a^5 - 30a^4$

58. $8m^3 - 343$

59. $z^3 - 7z^2 - 9z + 63$

60. $2p^8 - 12p^5 + 16p^2$

61. $64r^3 + 729$

62. $5x^5 - 10x^4 - 40x^3$

63. $16n^4 - 1$

64. $9k^3 - 24k^2 + 3k - 8$

65. REASONING Determine whether each polynomial is factored completely. If not, factor completely.

a. $7z^4(2z^2 - z - 6)$

b. $(2 - n)(n^2 + 6n)(3n - 11)$

c. $3(4y - 5)(9y^2 - 6y - 4)$

66. PROBLEM SOLVING The profit P (in millions of dollars) for a T-shirt manufacturer can be modeled by $P = -x^3 + 4x^2 + x$, where x is the number (in millions) of T-shirts produced. Currently the company produces 4 million T-shirts and makes a profit of \$4 million. What lesser number of T-shirts could the company produce and still make the same profit?

67. PROBLEM SOLVING The profit P (in millions of dollars) for a shoe manufacturer can be modeled by $P = -21x^3 + 46x$, where x is the number (in millions) of shoes produced. The company now produces 1 million shoes and makes a profit of \$25 million, but it would like to cut back production. What lesser number of shoes could the company produce and still make the same profit?

68. THOUGHT PROVOKING Find a value of k such that $\frac{f(x)}{x-k}$ has a remainder of 0. Justify your answer.

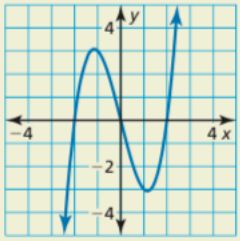
$$f(x) = x^3 - 3x^2 - 4x$$

69. COMPARING METHODS You are taking a test where calculators are not permitted. One question asks you to evaluate $g(7)$ for the function $g(x) = x^3 - 7x^2 - 4x + 28$. You use the Factor Theorem and synthetic division and your friend uses direct substitution. Whose method do you prefer? Explain your reasoning.

70. MAKING AN ARGUMENT You divide $f(x)$ by $(x - a)$ and find that the remainder does not equal 0. Your friend concludes that $f(x)$ cannot be factored. Is your friend correct? Explain your reasoning.

71. CRITICAL THINKING What is the value of k such that $x - 7$ is a factor of $h(x) = 2x^3 - 13x^2 - kx + 105$? Justify your answer.

72. HOW DO YOU SEE IT? Use the graph to write an equation of the cubic function in factored form. Explain your reasoning.



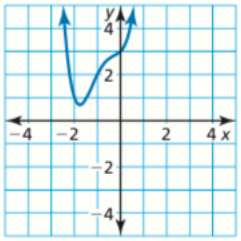
73. ABSTRACT REASONING Factor each polynomial completely.

a. $7ac^2 + bc^2 - 7ad^2 - bd^2$

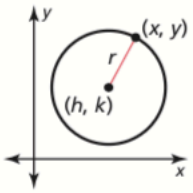
b. $x^{2n} - 2x^n + 1$

c. $a^5b^2 - a^2b^4 + 2a^4b - 2ab^3 + a^3 - b^2$

74. REASONING The graph of the function $f(x) = x^4 + 3x^3 + 2x^2 + x + 3$ is shown. Can you use the Factor Theorem to factor $f(x)$? Explain.



75. MATHEMATICAL CONNECTIONS The standard equation of a circle with radius r and center (h, k) is $(x - h)^2 + (y - k)^2 = r^2$. Rewrite each equation of a circle in standard form. Identify the center and radius of the circle. Then graph the circle.



a. $x^2 + 6x + 9 + y^2 = 25$

b. $x^2 - 4x + 4 + y^2 = 9$

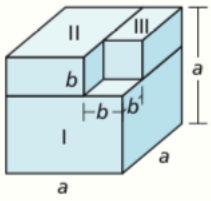
c. $x^2 - 8x + 16 + y^2 + 2y + 1 = 36$

76. CRITICAL THINKING Use the diagram to complete parts (a)–(c).

a. Explain why $a^3 - b^3$ is equal to the sum of the volumes of the solids I, II, and III.

b. Write an algebraic expression for the volume of each of the three solids. Leave your expressions in factored form.

c. Use the results from part (a) and part (b) to derive the factoring pattern $a^3 - b^3$.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the quadratic equation by factoring. *(Section 3.1)*

77. $x^2 - x - 30 = 0$

78. $2x^2 - 10x - 72 = 0$

79. $3x^2 - 11x + 10 = 0$

80. $9x^2 - 28x + 3 = 0$

Solve the quadratic equation by completing the square. *(Section 3.3)*

81. $x^2 - 12x + 36 = 144$

82. $x^2 - 8x - 11 = 0$

83. $3x^2 + 30x + 63 = 0$

84. $4x^2 + 36x - 4 = 0$