4.8 Analyzing Graphs of Polynomial Functions

What have we learned so far in this chapter?

Concept Summary

Zeros, Factors, Solutions, and Intercepts

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial function. The following statements are equivalent.

Zero: k is a zero of the polynomial function f.

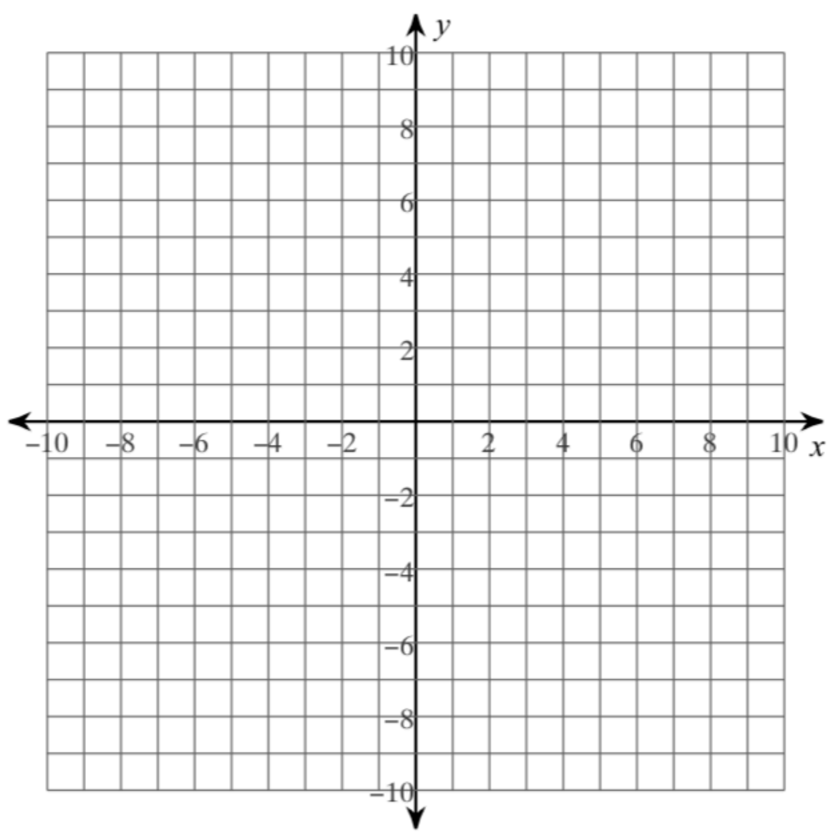
Factor: $x - k$ is a factor of the polynomial $f(x)$.

Solution: k is a solution (or root) of the polynomial equation $f(x) = 0$.

***x*-Intercept:** If k is a real number, then k is an x-intercept of the graph of the polynomial function f. The graph of f passes through $(k, 0)$.

Example 1: Using x-intercepts to graph a polynomial function

Graph the function: $f(x) = \frac{1}{6}(x+3)(x-2)^2$



The Location Principle

Back in my day…

Core Concept

The Location Principle

If f is a polynomial function, and a and b are two real numbers such that $f(a) < 0$ and $f(b) > 0$, then f has at least one real zero between a and b.

Example 2: Locating real zeros of a polynomial function.

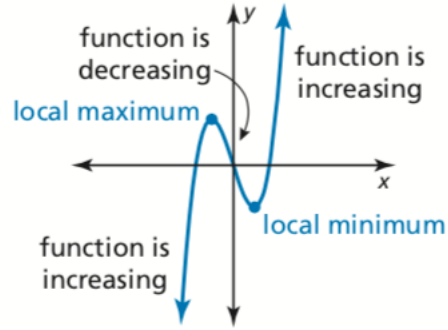
Find all real zeros of: $f(x) = 6x^3 + 5x^2 - 17x - 6$

Turning points

Core Concept

Turning Points of Polynomial Functions

1. The graph of every polynomial function of degree n has *at most* $n - 1$ turning points.
2. If a polynomial function of degree n has n distinct real zeros, then its graph has *exactly* $n - 1$ turning points.



Example 3: Finding Turning Points

Graph each function. Identify the x-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which each function is increasing or decreasing.

a. $f(x) = x^3 - 3x^2 + 6$

b. $g(x) = x^4 - 6x^3 + 3x^2 + 10x - 3$

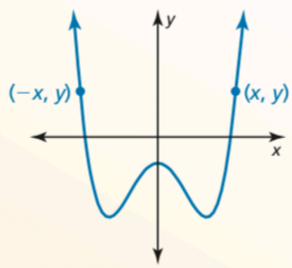
Core Concept

Even and Odd Functions

A function f is an **even function** when $f(-x) = f(x)$ for all x in its domain. The graph of an even function is *symmetric about the y-axis*.

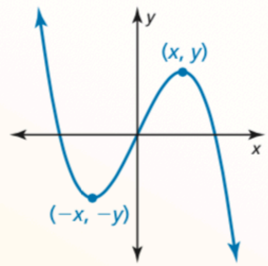
A function f is an **odd function** when $f(-x) = -f(x)$ for all x in its domain. The graph of an odd function is *symmetric about the origin*. One way to recognize a graph that is symmetric about the origin is that it looks the same after a 180° rotation about the origin.

Even Function



For an even function, if (x, y) is on the graph, then $(-x, y)$ is also on the graph.

Odd Function



For an odd function, if (x, y) is on the graph, then $(-x, -y)$ is also on the graph.

Example 4: Identifying even and odd functions

Determine whether each function is *even*, *odd*, or *neither*.

a. $f(x) = x^3 - 7x$ **b.** $g(x) = x^4 + x^2 - 1$ **c.** $h(x) = x^3 + 2$

Homework

3-6, 7, 9, 12, 14, 17, 18, 21, 23, 26, 30, 32, 40, 42, 44

4.8 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A local maximum or local minimum of a polynomial function occurs at a ______________ point of the graph of the function.

2. **WRITING** Explain what a local maximum of a function is and how it may be different from the maximum value of the function.

Monitoring Progress and Modeling with Mathematics

ANALYZING RELATIONSHIPS In Exercises 3–6, match the function with its graph.

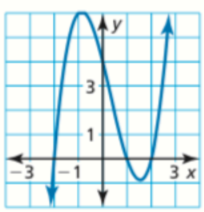
3. $f(x) = (x - 1)(x - 2)(x + 2)$

4. $h(x) = (x + 2)^2(x + 1)$

5. $g(x) = (x + 1)(x - 1)(x + 2)$

6. $f(x) = (x - 1)^2(x + 2)$

A.



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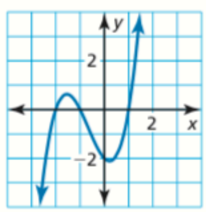

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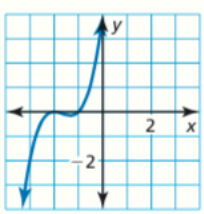

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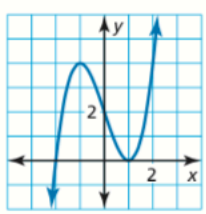


In Exercises 7–14, graph the function. *(See Example 1.)*

7. $f(x) = (x - 2)^2(x + 1)$

8. $f(x) = (x + 2)^2(x + 4)^2$

9. $h(x) = (x + 1)^2(x - 1)(x - 3)$

10. $g(x) = 4(x + 1)(x + 2)(x - 1)$

11. $h(x) = \frac{1}{3}(x - 5)(x + 2)(x - 3)$

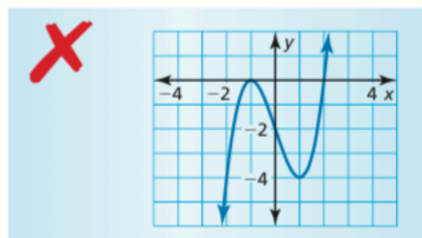
12. $g(x) = \frac{1}{12}(x + 4)(x + 8)(x - 1)$

13. $h(x) = (x - 3)(x^2 + x + 1)$

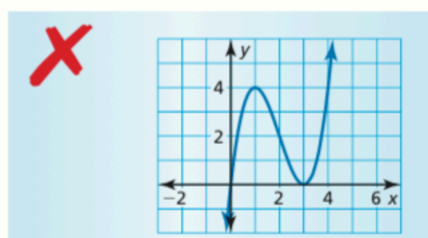
14. $f(x) = (x - 4)(2x^2 - 2x + 1)$

ERROR ANALYSIS In Exercises 15 and 16, describe and correct the error in using factors to graph *f*.

15. $f(x) = (x + 2)(x - 1)^2$



16. $f(x) = x^2(x - 3)^3$



In Exercises 17–22, find all real zeros of the function. *(See Example 2.)*

17. $f(x) = x^3 - 4x^2 - x + 4$

18. $f(x) = x^3 - 3x^2 - 4x + 12$

19. $h(x) = 2x^3 + 7x^2 - 5x - 4$

20. $h(x) = 4x^3 - 2x^2 - 24x - 18$

21. $g(x) = 4x^3 + x^2 - 51x + 36$

22. $f(x) = 2x^3 - 3x^2 - 32x - 15$

In Exercises 23–30, graph the function. Identify the x-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing. *(See Example 3.)*

23. $g(x) = 2x^3 + 8x^2 - 3$

24. $g(x) = -x^4 + 3x$

25. $h(x) = x^4 - 3x^2 + x$

26. $f(x) = x^5 - 4x^3 + x^2 + 2$

27. $f(x) = 0.5x^3 - 2x + 2.5$

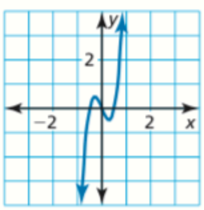
28. $f(x) = 0.7x^4 - 3x^3 + 5x$

29. $h(x) = x^5 + 2x^2 - 17x - 4$

30. $g(x) = x^4 - 5x^3 + 2x^2 + x - 3$

In Exercises 31–36, estimate the coordinates of each turning point. State whether each corresponds to a local maximum or a local minimum. Then estimate the real zeros and find the least possible degree of the function.

31.



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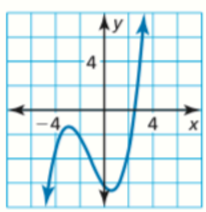

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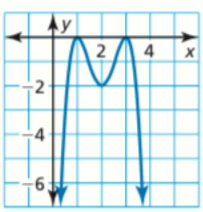

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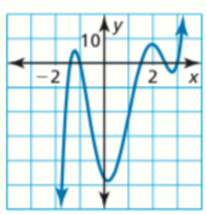

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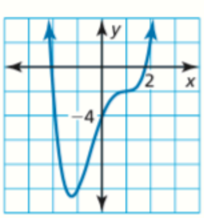

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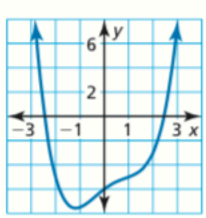

OPEN-ENDED In Exercises 37 and 38, sketch a graph of a polynomial function f having the given characteristics.

37.
- The graph of f has x-intercepts at $x = -4$, $x = 0$, and $x = 2$.
- f has a local maximum value when $x = 1$.
- f has a local minimum value when $x = -2$.

38.
- The graph of f has x-intercepts at $x = -3$, $x = 1$, and $x = 5$.
- f has a local maximum value when $x = 1$.
- f has a local minimum value when $x = -2$ and when $x = 4$.

In Exercises 39–46, determine whether the function is *even, odd,* or *neither*. *(See Example 4.)*

39. $h(x) = 4x^7$

40. $g(x) = -2x^6 + x^2$

41. $f(x) = x^4 + 3x^2 - 2$

42. $f(x) = x^5 + 3x^3 - x$

43. $g(x) = x^2 + 5x + 1$

44. $f(x) = -x^3 + 2x - 9$

45. $f(x) = x^4 - 12x^2$

46. $h(x) = x^5 + 3x^4$

47. USING TOOLS When a swimmer does the breaststroke, the function

$$S = -241t^7 + 1060t^6 - 1870t^5 + 1650t^4 - 737t^3 + 144t^2 - 2.43t$$

models the speed S (in meters per second) of the swimmer during one complete stroke, where t is the number of seconds since the start of the stroke and $0 \leq t \leq 1.22$. Use a graphing calculator to graph the function. At what time during the stroke is the swimmer traveling the fastest?

48. USING TOOLS During a recent period of time, the number S (in thousands) of students enrolled in public schools in a certain country can be modeled by $S = 1.64x^3 - 102x^2 + 1710x + 36{,}300$, where x is time (in years). Use a graphing calculator to graph the function for the interval $0 \leq x \leq 41$. Then describe how the public school enrollment changes over this period of time.

49. WRITING Why is the adjective *local*, used to describe the maximums and minimums of cubic functions, sometimes not required for quadratic functions?