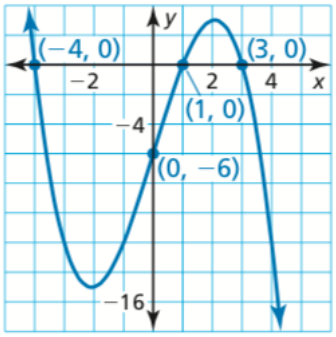
4.9 Modeling with Polynomial Functions

Example 1: Writing a Cubic Function

Write a cubic function whose graph is shown.



#FBF
Finite Differences.

Lets think back to Chapter 2. Recall how we could determine if a function was quadratic.

x	−3	−2	−1	0	1	2	3
y	9	4	1	0	1	4	9

Example 2: Writing a Function Using Finite Differences

Use finite differences to determine the degree of the polynomial function that fits the data.

x	1	2	3	4	5	6	7
$f(x)$	1	4	10	20	35	56	84

Try on your own:

x	-3	-2	-1	0	1	2
$f(x)$	6	15	22	21	6	-29

Lets Think of the Big Picture

Do you think that in the real world the data sets will fit exactly like the data points?

How do you think we could model something that is not in a perfect cubic function?

Example 3: Real World Applications:

The table shows the total U.S. biomass energy consumptions y (in trillions of British thermal units, or Btus) in the year t, where $t = 1$ corresponds to 2001. Find a polynomial model for the data. Use the model to estimate the total U.S. biomass energy consumption in 2013.

t	1	2	3	4	5	6
y	2622	2701	2807	3010	3117	3267

t	7	8	9	10	11	12
y	3493	3866	3951	4286	4421	4316

Homework
3, 4, 7, 9, 12, 14, **16**, 19, 20, 23

4.9 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

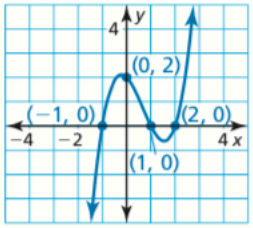
1. **COMPLETE THE SENTENCE** When the x-values in a set of data are equally spaced, the differences of consecutive y-values are called _____.

2. **WRITING** Explain how you know when a set of data could be modeled by a cubic function.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, write a cubic function whose graph is shown. *(See Example 1.)*

3.



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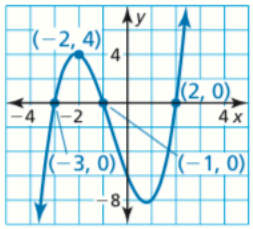

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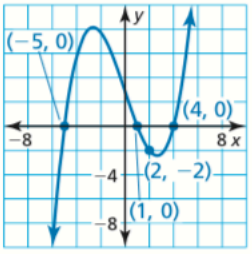

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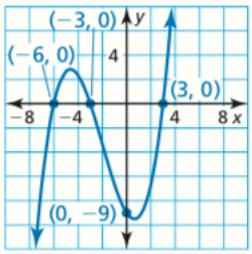


In Exercises 7–12, use finite differences to determine the degree of the polynomial function that fits the data. Then use technology to find the polynomial function. *(See Example 2.)*

7.

x	−6	−3	0	3	6	9
$f(x)$	−2	15	−4	49	282	803

8.

x	−1	0	1	2	3	4
$f(x)$	−14	−5	−2	7	34	91

9. $(-4, -317), (-3, -37), (-2, 21), (-1, 7), (0, -1), (1, 3), (2, -47), (3, -289), (4, -933)$

10. $(-6, 744), (-4, 154), (-2, 4), (0, -6), (2, 16), (4, 154), (6, 684), (8, 2074), (10, 4984)$

11. $(-2, 968), (-1, 422), (0, 142), (1, 26), (2, -4), (3, -2), (4, 2), (5, 2), (6, 16)$

12. $(1, 0), (2, 6), (3, 2), (4, 6), (5, 12), (6, -10), (7, -114), (8, -378), (9, -904)$

13. **ERROR ANALYSIS** Describe and correct the error in writing a cubic function whose graph passes through the given points.

✗

$(-6, 0), (1, 0), (3, 0), (0, 54)$

$54 = a(0 - 6)(0 + 1)(0 + 3)$

$54 = -18a$

$a = -3$

$f(x) = -3(x - 6)(x + 1)(x + 3)$

14. **MODELING WITH MATHEMATICS** The dot patterns show pentagonal numbers. The number of dots in the nth pentagonal number is given by $f(n) = \frac{1}{2}n(3n - 1)$. Show that this function has constant second-order differences.

15. **OPEN-ENDED** Write three different cubic functions that pass through the points (3, 0), (4, 0), and (2, 6). Justify your answers.

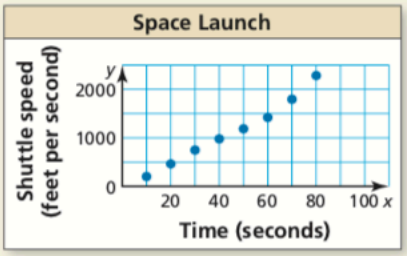
16. **MODELING WITH MATHEMATICS** The table shows the ages of cats and their corresponding ages in human years. Find a polynomial model for the data for the first 8 years of a cat's life. Use the model to estimate the age (in human years) of a cat that is 3 years old. *(See Example 3.)*

Age of cat, x	1	2	4	6	7	8
Human years, y	15	24	32	40	44	48

17. MODELING WITH MATHEMATICS The data in the table show the average speeds y (in miles per hour) of a pontoon boat for several different engine speeds x (in hundreds of revolutions per minute, or RPMs). Find a polynomial model for the data. Estimate the average speed of the pontoon boat when the engine speed is 2800 RPMs.

x	10	20	25	30	45	55
y	4.5	8.9	13.8	18.9	29.9	37.7

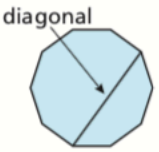
18. HOW DO YOU SEE IT? The graph shows typical speeds y (in feet per second) of a space shuttle x seconds after it is launched.



a. What type of polynomial function models the data? Explain.

b. Which nth-order finite difference should be constant for the function in part (a)? Explain.

19. MATHEMATICAL CONNECTIONS The table shows the number of diagonals for polygons with n sides. Find a polynomial function that fits the data. Determine the total number of diagonals in the decagon shown.



Number of sides, n	3	4	5	6	7	8
Number of diagonals, d	0	2	5	9	14	20

20. MAKING AN ARGUMENT Your friend states that it is not possible to determine the degree of a function given the first-order differences. Is your friend correct? Explain your reasoning.

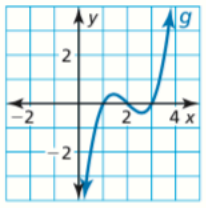
21. WRITING Explain why you cannot always use finite differences to find a model for real-life data sets.

22. THOUGHT PROVOKING A, B, and C are zeros of a cubic polynomial function. Choose values for A, B, and C such that the distance from A to B is less than or equal to the distance from A to C. Then write the function using the A, B, and C values you chose.

23. MULTIPLE REPRESENTATIONS Order the polynomial functions according to their degree, from least to greatest.

A. $f(x) = -3x + 2x^2 + 1$

B.



C.

x	−2	−1	0	1	2	3
$h(x)$	8	6	4	2	0	−2

D.

x	−2	−1	0	1	2	3
$k(x)$	25	6	7	4	−3	10

24. ABSTRACT REASONING Substitute the expressions $z, z + 1, z + 2, \ldots, z + 5$ for x in the function $f(x) = ax^3 + bx^2 + cx + d$ to generate six equally-spaced ordered pairs. Then show that the third-order differences are constant.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation using square roots. *(Section 3.1)*

25. $x^2 - 6 = 30$

26. $5x^2 - 38 = 187$

27. $2(x - 3)^2 = 24$

28. $\frac{4}{3}(x + 5)^2 = 4$

Solve the equation using the Quadratic Formula. *(Section 3.4)*

29. $2x^2 + 3x = 5$

30. $2x^2 + \frac{1}{2} = 2x$

31. $2x^2 + 3x = -3x^2 + 1$

32. $4x - 20 = x^2$