5.4 Equilateral and Isosceles Triangles

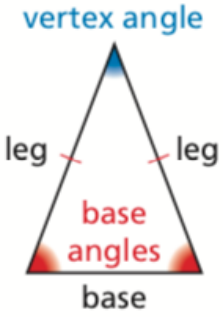
Spiral Review: Sketch and correctly label the following.

Sketch an Equilateral Triangle:

Sketch an Isosceles Triangle:

Using the Base Angles Theorem:

A triangle is isosceles when it has at least two congruent sides. If there are only two congruent sides, then these are called **legs**. The angle formed by the legs is the **vertex angle**. The third side Is known as the **base.** The two angles adjacent to the base are Known as the **base angles**.



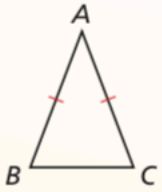
Theorems

Theorem 5.6 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.

Proof p. 252

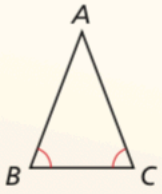


Theorem 5.7 Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

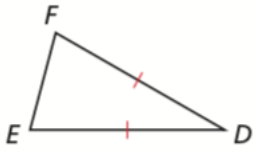
If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

Proof Ex. 27, p. 275



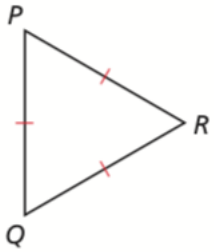
Example 1: Using the base angles theorem.

In $\triangle DEF, \overline{DE} \cong \overline{DF}$. Name two congruent angles.



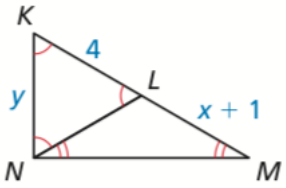
Example 2: Measures of a triangle

Determine the measure of angle P, R, and Q.



Example 3: Using Isosceles and Equilateral Triangles

Find the values of x and y in the diagram.



Example 4: Multistep problem

In the lifeguard tower, $\overline{PS} \cong \overline{QR}$ and $\angle QPS \cong \angle PQR$.

a. Explain how to prove that $\triangle QPS \cong \triangle PQR$.

b. Explain why $\triangle PQT$ is isosceles.

Classwork / Homework
3-10, 13-16, 21, 29, 30, 31, 35

5.4 Exercises

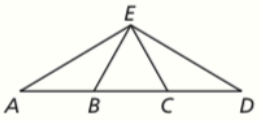
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Describe how to identify the *vertex angle* of an isosceles triangle.
2. **WRITING** What is the relationship between the base angles of an isosceles triangle? Explain.

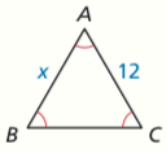
Monitoring Progress and Modeling with Mathematics

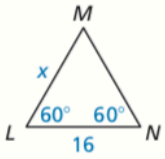
In Exercises 3–6, copy and complete the statement. State which theorem you used. *(See Example 1.)*

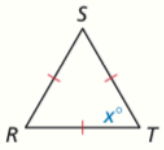


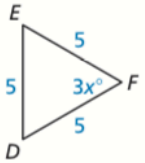
3. If $\overline{AE} \cong \overline{DE}$, then $\angle$___ $\cong \angle$___.
4. If $\overline{AB} \cong \overline{EB}$, then $\angle$___ $\cong \angle$___.
5. If $\angle D \cong \angle CED$, then ___ $\cong$ ___.
6. If $\angle EBC \cong \angle ECB$, then ___ $\cong$ ___.

In Exercises 7–10, find the value of x. *(See Example 2.)*

7. 

8. 

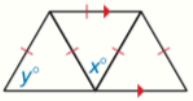
9. 

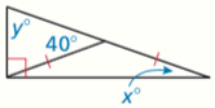
10. 

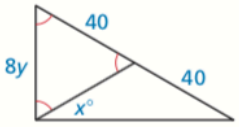
11. **MODELING WITH MATHEMATICS** The dimensions of a sports pennant are given in the diagram. Find the values of x and y.

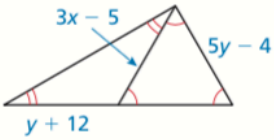
12. **MODELING WITH MATHEMATICS** A logo in an advertisement is an equilateral triangle with a side length of 7 centimeters. Sketch the logo and give the measure of each side.

In Exercises 13–16, find the values of x and y. *(See Example 3.)*

13. 

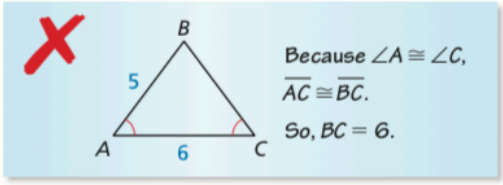
14. 

15. 

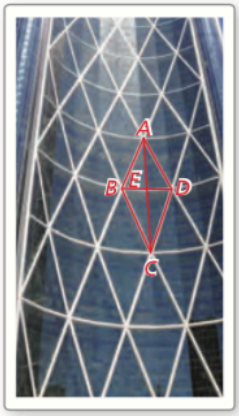
16. 

CONSTRUCTION In Exercises 17 and 18, construct an equilateral triangle whose sides are the given length.

17. 3 inches
18. 1.25 inches
19. **ERROR ANALYSIS** Describe and correct the error in finding the length of $\overline{BC}$.



20. PROBLEM SOLVING The diagram represents part of the exterior of the Bow Tower in Calgary, Alberta, Canada. In the diagram, $\triangle ABD$ and $\triangle CBD$ are congruent equilateral triangles. *(See Example 4.)*



a. Explain why $\triangle ABC$ is isosceles.

b. Explain why $\angle BAE \cong \angle BCE$.

c. Show that $\triangle ABE$ and $\triangle CBE$ are congruent.

d. Find the measure of $\angle BAE$.

21. FINDING A PATTERN In the pattern shown, each small triangle is an equilateral triangle with an area of 1 square unit.

Triangle	Area
	1 square unit
	
	
	

a. Explain how you know that any triangle made out of equilateral triangles is equilateral.

b. Find the areas of the first four triangles in the pattern.

c. Describe any patterns in the areas. Predict the area of the seventh triangle in the pattern. Explain your reasoning.

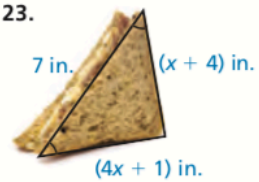
22. REASONING The base of isosceles $\triangle XYZ$ is $\overline{YZ}$. What can you prove? Select all that apply.

Ⓐ $\overline{XY} \cong \overline{XZ}$ Ⓑ $\angle X \cong \angle Y$

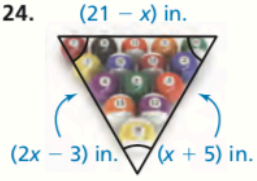
Ⓒ $\angle Y \cong \angle Z$ Ⓓ $\overline{YZ} \cong \overline{ZX}$

In Exercises 23 and 24, find the perimeter of the triangle.

23.



24.



MODELING WITH MATHEMATICS In Exercises 25–28, use the diagram based on the color wheel. The 12 triangles in the diagram are isosceles triangles with congruent vertex angles.

25. Complementary colors lie directly opposite each other on the color wheel. Explain how you know that the yellow triangle is congruent to the purple triangle.

26. The measure of the vertex angle of the yellow triangle is 30°. Find the measures of the base angles.

27. Trace the color wheel. Then form a triangle whose vertices are the midpoints of the bases of the red, yellow, and blue triangles. (These colors are the *primary colors*.) What type of triangle is this?

28. Other triangles can be formed on the color wheel that are congruent to the triangle in Exercise 27. The colors on the vertices of these triangles are called *triads*. What are the possible triads?

29. CRITICAL THINKING Are isosceles triangles always acute triangles? Explain your reasoning.

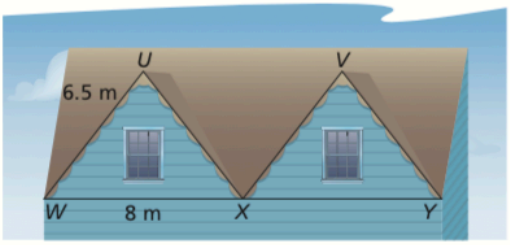
30. CRITICAL THINKING Is it possible for an equilateral triangle to have an angle measure other than 60°? Explain your reasoning.

31. MATHEMATICAL CONNECTIONS The lengths of the sides of a triangle are $3t$, $5t - 12$, and $t + 20$. Find the values of t that make the triangle isosceles. Explain your reasoning.

32. MATHEMATICAL CONNECTIONS The measure of an exterior angle of an isosceles triangle is $x°$. Write expressions representing the possible angle measures of the triangle in terms of x.

33. WRITING Explain why the measure of the vertex angle of an isosceles triangle must be an even number of degrees when the measures of all the angles of the triangle are whole numbers.

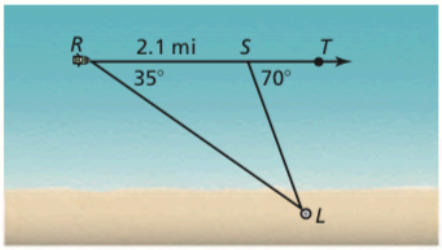
34. PROBLEM SOLVING The triangular faces of the peaks on a roof are congruent isosceles triangles with vertex angles U and V.



a. Name two angles congruent to $\angle WUX$. Explain your reasoning.

b. Find the distance between points U and V.

35. PROBLEM SOLVING A boat is traveling parallel to the shore along $\overrightarrow{RT}$. When the boat is at point R, the captain measures the angle to the lighthouse as 35°. After the boat has traveled 2.1 miles, the captain measures the angle to the lighthouse to be 70°.



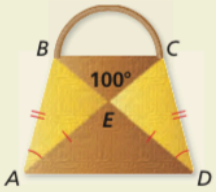
a. Find SL. Explain your reasoning.

b. Explain how to find the distance between the boat and the shoreline.

36. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, do all equiangular triangles have the same angle measures? Justify your answer.

37. PROVING A COROLLARY Prove that the Corollary to the Base Angles Theorem (Corollary 5.2) follows from the Base Angles Theorem (Theorem 5.6).

38. HOW DO YOU SEE IT? You are designing fabric purses to sell at the school fair.



a. Explain why $\triangle ABE \cong \triangle DCE$.

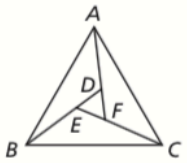
b. Name the isosceles triangles in the purse.

c. Name three angles that are congruent to $\angle EAD$.

39. PROVING A COROLLARY Prove that the Corollary to the Converse of the Base Angles Theorem (Corollary 5.3) follows from the Converse of the Base Angles Theorem (Theorem 5.7).

40. MAKING AN ARGUMENT The coordinates of two points are $T(0, 6)$ and $U(6, 0)$. Your friend claims that points T, U, and V will always be the vertices of an isosceles triangle when V is any point on the line $y = x$. Is your friend correct? Explain your reasoning.

41. PROOF Use the diagram to prove that $\triangle DEF$ is equilateral.



Given $\triangle ABC$ is equilateral.
$\angle CAD \cong \angle ABE \cong \angle BCF$

Prove $\triangle DEF$ is equilateral.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the given property to complete the statement. *(Section 2.5)*

42. Reflexive Property of Congruence (Theorem 2.1): ____ $\cong \overline{SE}$

43. Symmetric Property of Congruence (Theorem 2.1): If ____ $\cong$ ____, then $\overline{RS} \cong \overline{JK}$.

44. Transitive Property of Congruence (Theorem 2.1): If $\overline{EF} \cong \overline{PQ}$, and $\overline{PQ} \cong \overline{UV}$, then ____ $\cong$ ____.