5.5 Proving triangle congruence by SSS

Using the Side-Side-Side Congruence Theorem

Theorem

Theorem 5.8 Side-Side-Side (SSS) Congruence Theorem

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, then $\triangle ABC \cong \triangle DEF$.
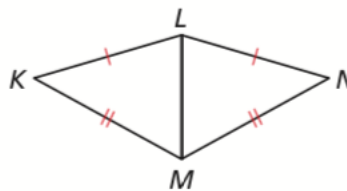

Example 1: Using the SSS congruence theorem

Write a proof.

Given $\overline{KL} \cong \overline{NL}, \ \overline{KM} \cong \overline{NM}$

Prove $\triangle KLM \cong \triangle NLM$

SOLUTION

Example 2: Real World Modeling

Explain why the bench with the diagonal support is stable, while the one without the support can collapse.
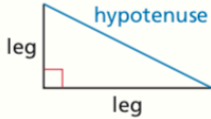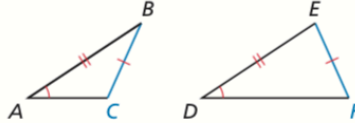
Don't be an Angle Side Side!

Using the Hypotenuse-Leg Congruence Theorem

You know that SAS and SSS are valid methods for proving that triangles are congruent. What about SSA?

In general, SSA is *not* a valid method for proving that triangles are congruent. In the triangles below, two pairs of sides and a pair of angles not included between them are congruent, but the triangles are not congruent.

While SSA is not valid in general, there is a special case for right triangles.

In a right triangle, the sides adjacent to the right angle are called the **legs**. The side opposite the right angle is called the **hypotenuse** of the right triangle.
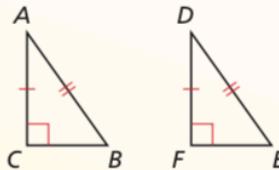

Key Notes

Theorem

Theorem 5.9   Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $m\angle C = m\angle F = 90°$, then $\triangle ABC \cong \triangle DEF$.
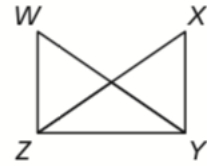
Proof Ex. 38, p. 470; *BigIdeasMath.com*

Example 3: Using the HL Congruence theorem

Write a proof.

Given $WY \cong XZ$, $WZ \perp ZY$, $XY \perp ZY$
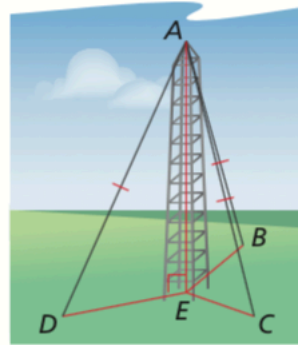
Prove $\triangle WYZ \cong \triangle XZY$

Example 4: Using the HL congruence theorem

The television antenna is perpendicular to the plane containing points B, C, D, and E. Each of the cables running from the top of the antenna to B, C, and D has the same length. Prove that $\triangle AEB$, $\triangle AEC$, and $\triangle AED$ are congruent.

Given $AE \perp EB$, $AE \perp EC$, $AE \perp ED$, $AB \cong AC \cong AD$

Prove $\triangle AEB \cong \triangle AEC \cong \triangle AED$

SOLUTION

Classwork/Homework
3-8, 11-13, 15, 23, 31, 36
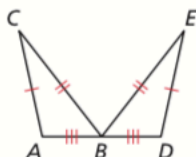
Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The side opposite the right angle is called the _____ of the right triangle.

2. **WHICH ONE DOESN'T BELONG?** Which triangle's legs do *not* belong with the other three? Explain your reasoning.

Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, decide whether enough information is given to prove that the triangles are congruent using the SSS Congruence Theorem (Theorem 5.8). Explain.

3. △ABC, △DBE
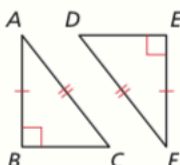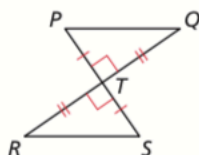
4. △PQS, △RQS

In Exercises 5 and 6, decide whether enough information is given to prove that the triangles are congruent using the HL Congruence Theorem (Theorem 5.9). Explain.

5. △ABC, △FED

6. △PQT, △SRT

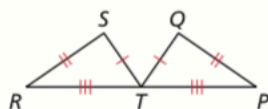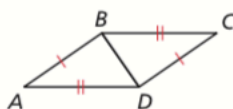In Exercises 7–10, decide whether the congruence statement is true. Explain your reasoning. *(See Example 1.)*

7. △RST ≅ △TQP

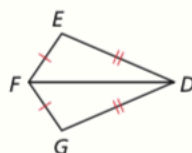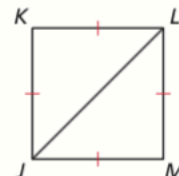8. △ABD ≅ △CDB

9. △DEF ≅ △DGF

10. △JKL ≅ △LJM

In Exercises 11 and 12, determine whether the figure is stable. Explain your reasoning. *(See Example 2.)*

11.

12.

In Exercises 13 and 14, redraw the triangles so they are side by side with corresponding parts in the same position. Then write a proof. *(See Example 3.)*

13. **Given** $\overline{AC} \cong \overline{BD}$,
$\overline{AB} \perp \overline{AD}$,
$\overline{CD} \perp \overline{AD}$

Prove △BAD ≅ △CDA

14. **Given** G is the midpoint of $\overline{EH}$, $\overline{FG} \cong \overline{GI}$, ∠E and ∠H are right angles.

Prove △EFG ≅ △HIG

In Exercises 15 and 16, write a proof.
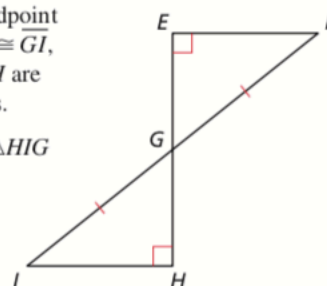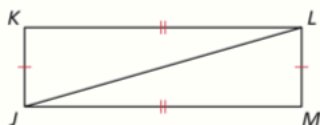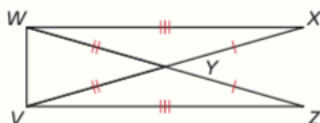
15. Given $\overline{LM} \cong \overline{JK}, \overline{MJ} \cong \overline{KL}$
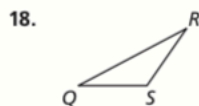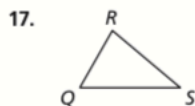**Prove** $\triangle LMJ \cong \triangle JKL$

16. Given $\overline{WX} \cong \overline{VZ}, \overline{WY} \cong \overline{VY}, \overline{YZ} \cong \overline{YX}$
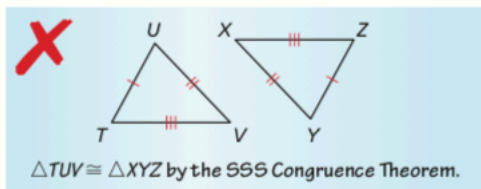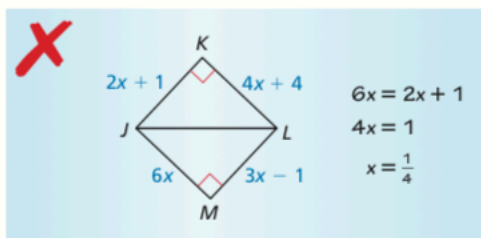**Prove** $\triangle VWX \cong \triangle WVZ$

CONSTRUCTION In Exercises 17 and 18, construct a triangle that is congruent to $\triangle QRS$ using the SSS Congruence Theorem (Theorem 5.8).

17.

18.

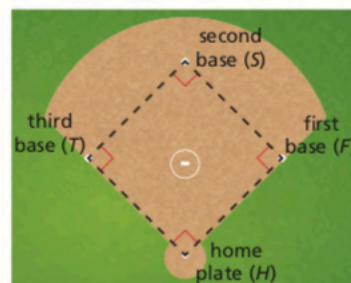**19. ERROR ANALYSIS** Describe and correct the error in identifying congruent triangles.

$\triangle TUV \cong \triangle XYZ$ by the SSS Congruence Theorem.

20. ERROR ANALYSIS Describe and correct the error in determining the value of x that makes the triangles congruent.
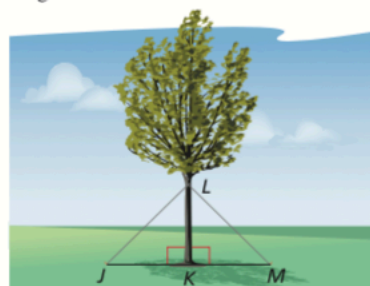

$6x = 2x + 1$
$4x = 1$
$x = \frac{1}{4}$

21. MAKING AN ARGUMENT Your friend claims that in order to use the SSS Congruence Theorem (Theorem 5.8) to prove that two triangles are congruent, both triangles must be equilateral triangles. Is your friend correct? Explain your reasoning.
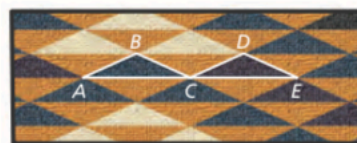
22. MODELING WITH MATHEMATICS The distances between consecutive bases on a softball field are the same. The distance from home plate to second base is the same as the distance from first base to third base. The angles created at each base are 90°. Prove $\triangle HFS \cong \triangle FST \cong \triangle STH$. *(See Example 4.)*

23. REASONING To support a tree, you attach wires from the trunk of the tree to stakes in the ground, as shown in the diagram.

a. What additional information do you need to use the HL Congruence Theorem (Theorem 5.9) to prove that $\triangle JKL \cong \triangle MKL$?

b. Suppose K is the midpoint of JM. Name a theorem you could use to prove that $\triangle JKL \cong \triangle MKL$. Explain your reasoning.

24. REASONING Use the photo of the Navajo rug, where $\overline{BC} \cong \overline{DE}$ and $\overline{AC} \cong \overline{CE}$.

a. What additional information do you need to use the SSS Congruence Theorem (Theorem 5.8) to prove that $\triangle ABC \cong \triangle CDE$?

b. What additional information do you need to use the HL Congruence Theorem (Theorem 5.9) to prove that $\triangle ABC \cong \triangle CDE$?

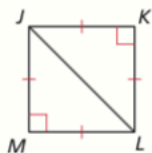**In Exercises 25–28, use the given coordinates to determine whether $\triangle ABC \cong \triangle DEF$.**

25. $A(-2, -2)$, $B(4, -2)$, $C(4, 6)$, $D(5, 7)$, $E(5, 1)$, $F(13, 1)$

26. $A(-2, 1)$, $B(3, -3)$, $C(7, 5)$, $D(3, 6)$, $E(8, 2)$, $F(10, 11)$

27. $A(0, 0)$, $B(6, 5)$, $C(9, 0)$, $D(0, -1)$, $E(6, -6)$, $F(9, -1)$

28. $A(-5, 7)$, $B(-5, 2)$, $C(0, 2)$, $D(0, 6)$, $E(0, 1)$, $F(4, 1)$

29. CRITICAL THINKING You notice two triangles in the tile floor of a hotel lobby. You want to determine whether the triangles are congruent, but you only have a piece of string. Can you determine whether the triangles are congruent? Explain.

30. HOW DO YOU SEE IT? There are several theorems you can use to show that the triangles in the "square" pattern are congruent. Name two of them.

31. MAKING AN ARGUMENT Your cousin says that $\triangle JKL$ is congruent to $\triangle LMJ$ by the SSS Congruence Theorem (Thm. 5.8). Your friend says that $\triangle JKL$ is congruent to $\triangle LMJ$ by the HL Congruence Theorem (Thm. 5.9). Who is correct? Explain your reasoning.
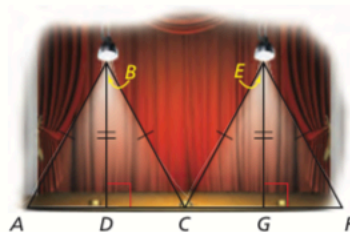

32. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. In spherical geometry, do you think that two triangles are congruent if their corresponding sides are congruent? Justify your answer.

USING TOOLS In Exercises 33 and 34, use the given information to sketch $\triangle LMN$ and $\triangle STU$. Mark the triangles with the given information.
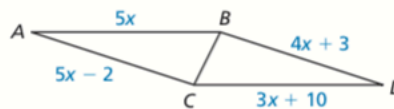
33. $\overline{LM} \perp \overline{MN}, \overline{ST} \perp \overline{TU}, \overline{LM} \cong \overline{NM} \cong \overline{UT} \cong \overline{ST}$

34. $\overline{LM} \perp \overline{MN}, \overline{ST} \perp \overline{TU}, \overline{LM} \cong \overline{ST}, \overline{LN} \cong \overline{SU}$

35. CRITICAL THINKING The diagram shows the light created by two spotlights. Both spotlights are the same distance from the stage.

a. Show that $\triangle ABD \cong \triangle CBD$. State which theorem or postulate you used and explain your reasoning.

b. Are all four right triangles shown in the diagram congruent? Explain your reasoning.

36. MATHEMATICAL CONNECTIONS Find all values of x that make the triangles congruent. Explain.

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Use the congruent triangles. *(Section 5.2)*

37. Name the segment in $\triangle DEF$ that is congruent to $\overline{AC}$.

38. Name the segment in $\triangle ABC$ that is congruent to $\overline{EF}$.

39. Name the angle in $\triangle DEF$ that is congruent to $\angle B$.

40. Name the angle in $\triangle ABC$ that is congruent to $\angle F$.