5.6 Proving Triangle Congruence by ASA and AAS

Spiral Review: What are all the ways that we have learned to prove triangles are congruent?

Using ASA and AAS Congruence Theorems

Theorem

Theorem 5.10 Angle-Side-Angle (ASA) Congruence Theorem

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

If $\angle A \cong \angle D$, $\overline{AC} \cong \overline{DF}$, and $\angle C \cong \angle F$, then $\triangle ABC \cong \triangle DEF$.

Proof p. 270

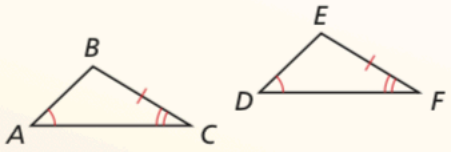
Theorem

Theorem 5.11 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

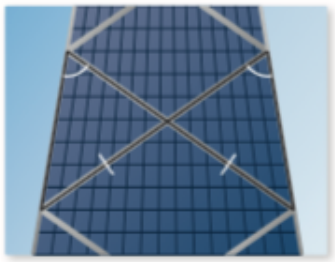
If $\angle A \cong \angle D$, $\angle C \cong \angle F$, and $\overline{BC} \cong \overline{EF}$, then $\triangle ABC \cong \triangle DEF$.

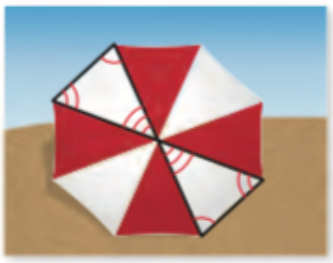
Proof p. 271

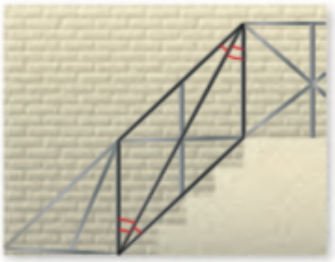


Example 1: Identifying congruent triangles

Is there enough information to determine if the triangles are congruent?

a. 

b. 

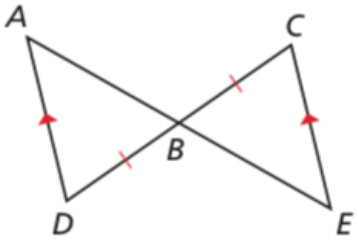
c. 

Example 2: Using the ASA congruence theorem

Write a proof.

Given $\overline{AD} \parallel \overline{EC}$, $\overline{BD} \cong \overline{BC}$

Prove $\triangle ABD \cong \triangle EBC$



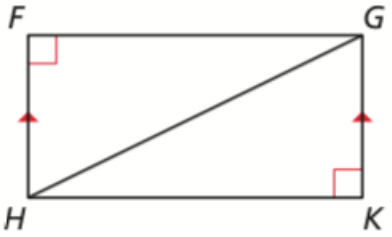
SOLUTION

Example 3: Using the AAS congruence theorem

Write a proof.

Given $\overline{HF} \parallel \overline{GK}$, $\angle F$ and $\angle K$ are right angles.

Prove $\triangle HFG \cong \triangle GKH$

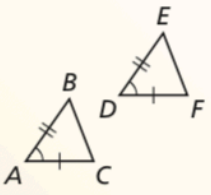
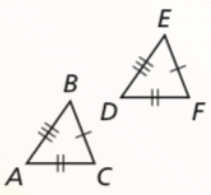
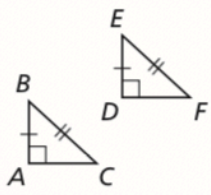
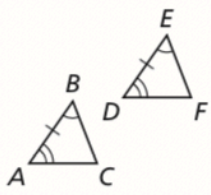
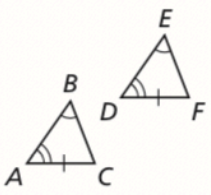


CHAPTER RECAP!!!

Concept Summary

Triangle Congruence Theorems

You have learned five methods for proving that triangles are congruent.

SAS	SSS	HL (right △ only)	ASA	AAS
				
Two sides and the included angle are congruent.	All three sides are congruent.	The hypotenuse and one of the legs are congruent.	Two angles and the included side are congruent.	Two angles and a non-included side are congruent.

In the Exercises, you will prove three additional theorems about the congruence of right triangles: **H**ypotenuse-**A**ngle, **L**eg-**L**eg, and **A**ngle-**L**eg.

Classwork / Homework

3-12, 17, 20, 24, 25, 31

5.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

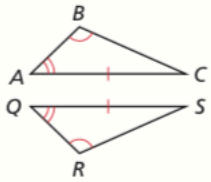
1. **WRITING** How are the AAS Congruence Theorem (Theorem 5.11) and the ASA Congruence Theorem (Theorem 5.10) similar? How are they different?

2. **WRITING** You know that a pair of triangles has two pairs of congruent corresponding angles. What other information do you need to show that the triangles are congruent?

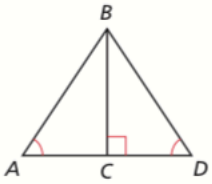
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, decide whether enough information is given to prove that the triangles are congruent. If so, state the theorem you would use. *(See Example 1.)*

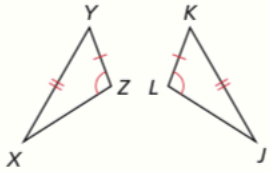
3. $\triangle ABC, \triangle QRS$



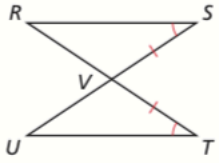
4. $\triangle ABC, \triangle DBC$



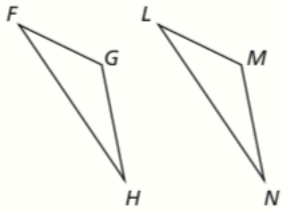
5. $\triangle XYZ, \triangle JKL$



6. $\triangle RSV, \triangle UTV$



In Exercises 7 and 8, state the third congruence statement that is needed to prove that $\triangle FGH \cong \triangle LMN$ using the given theorem.



7. **Given** $\overline{GH} \cong \overline{MN}, \angle G \cong \angle M, \_\_\_ \cong \_\_\_$

   Use the AAS Congruence Theorem (Thm. 5.11).

8. **Given** $\overline{FG} \cong \overline{LM}, \angle G \cong \angle M, \_\_\_ \cong \_\_\_$

   Use the ASA Congruence Theorem (Thm. 5.10).

In Exercises 9–12, decide whether you can use the given information to prove that $\triangle ABC \cong \triangle DEF$. Explain your reasoning.

9. $\angle A \cong \angle D, \angle C \cong \angle F, \overline{AC} \cong \overline{DF}$

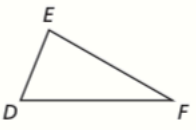
10. $\angle C \cong \angle F, \overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$

11. $\angle B \cong \angle E, \angle C \cong \angle F, \overline{AC} \cong \overline{DE}$

12. $\angle A \cong \angle D, \angle B \cong \angle E, \overline{BC} \cong \overline{EF}$

CONSTRUCTION **In Exercises 13 and 14, construct a triangle that is congruent to the given triangle using the ASA Congruence Theorem (Theorem 5.10). Use a compass and straightedge.**

13.



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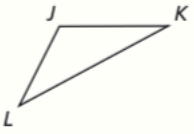
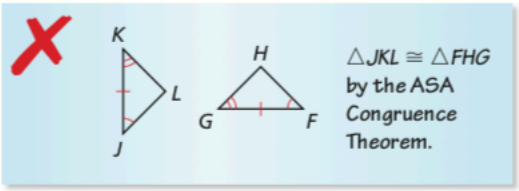


ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error.**

15.



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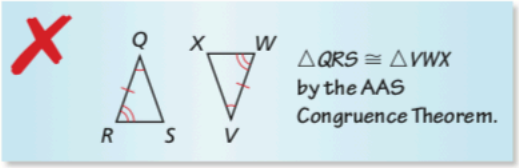
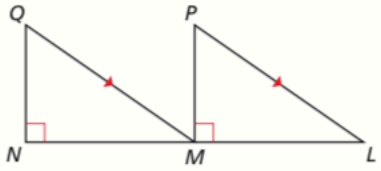


PROOF In Exercises 17 and 18, prove that the triangles are congruent using the ASA Congruence Theorem (Theorem 5.10). *(See Example 2.)*

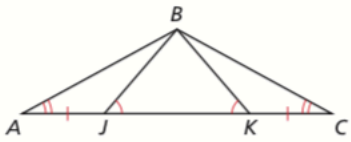
17. Given M is the midpoint of $\overline{NL}$.
$\overline{NL} \perp \overline{NQ}$, $\overline{NL} \perp \overline{MP}$, $\overline{QM} \parallel \overline{PL}$

Prove $\triangle NQM \cong \triangle MPL$



18. Given $\overline{AJ} \cong \overline{KC}$, $\angle BJK \cong \angle BKJ$, $\angle A \cong \angle C$

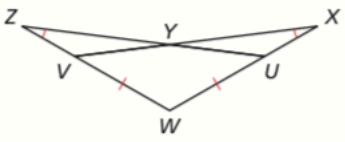
Prove $\triangle ABK \cong \triangle CBJ$



PROOF In Exercises 19 and 20, prove that the triangles are congruent using the AAS Congruence Theorem (Theorem 5.11). *(See Example 3.)*

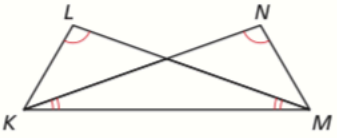
19. Given $\overline{VW} \cong \overline{UW}$, $\angle X \cong \angle Z$

Prove $\triangle XWV \cong \triangle ZWU$



20. Given $\angle NKM \cong \angle LMK$, $\angle L \cong \angle N$

Prove $\triangle NMK \cong \triangle LKM$



PROOF In Exercises 21–23, write a paragraph proof for the theorem about right triangles.

21. Hypotenuse-Angle (HA) Congruence Theorem If an angle and the hypotenuse of a right triangle are congruent to an angle and the hypotenuse of a second right triangle, then the triangles are congruent.

22. Leg-Leg (LL) Congruence Theorem If the legs of a right triangle are congruent to the legs of a second right triangle, then the triangles are congruent.

23. Angle-Leg (AL) Congruence Theorem If an angle and a leg of a right triangle are congruent to an angle and a leg of a second right triangle, then the triangles are congruent.

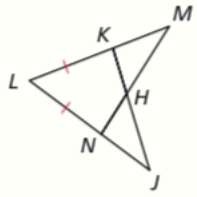
24. REASONING What additional information do you need to prove $\triangle JKL \cong \triangle MNL$ by the ASA Congruence Theorem (Theorem 5.10)?

Ⓐ $\overline{KM} \cong \overline{KJ}$

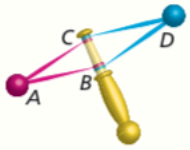
Ⓑ $\overline{KH} \cong \overline{NH}$

Ⓒ $\angle M \cong \angle J$

Ⓓ $\angle LKJ \cong \angle LNM$



25. MATHEMATICAL CONNECTIONS This toy contains $\triangle ABC$ and $\triangle DBC$. Can you conclude that $\triangle ABC \cong \triangle DBC$ from the given angle measures? Explain.



$m\angle ABC = (8x - 32)°$

$m\angle DBC = (4y - 24)°$

$m\angle BCA = (5x + 10)°$

$m\angle BCD = (3y + 2)°$

$m\angle CAB = (2x - 8)°$

$m\angle CDB = (y - 6)°$

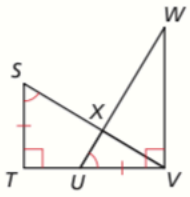
26. REASONING Which of the following congruence statements are true? Select all that apply.

Ⓐ $\overline{TU} \cong \overline{UV}$

Ⓑ $\triangle STV \cong \triangle XVW$

Ⓒ $\triangle TVS \cong \triangle VWU$

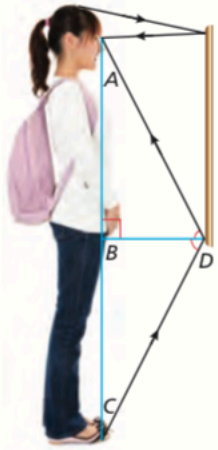
Ⓓ $\triangle VST \cong \triangle VUW$



27. PROVING A THEOREM Prove the Converse of the Base Angles Theorem (Theorem 5.7). (*Hint:* Draw an auxiliary line inside the triangle.)

28. MAKING AN ARGUMENT Your friend claims to be able to rewrite any proof that uses the AAS Congruence Theorem (Thm. 5.11) as a proof that uses the ASA Congruence Theorem (Thm. 5.10). Is this possible? Explain your reasoning.

29. MODELING WITH MATHEMATICS When a light ray from an object meets a mirror, it is reflected back to your eye. For example, in the diagram, a light ray from point C is reflected at point D and travels back to point A. The *law of reflection* states that the angle of incidence, $\angle CDB$, is congruent to the angle of reflection, $\angle ADB$.



a. Prove that $\triangle ABD$ is congruent to $\triangle CBD$.

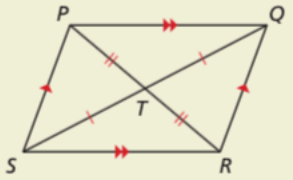
Given $\angle CDB \cong \angle ADB$, $\overline{DB} \perp \overline{AC}$

Prove $\triangle ABD \cong \triangle CBD$

b. Verify that $\triangle ACD$ is isosceles.

c. Does moving away from the mirror have any effect on the amount of his or her reflection a person sees? Explain.

30. HOW DO YOU SEE IT? Name as many pairs of congruent triangles as you can from the diagram. Explain how you know that each pair of triangles is congruent.



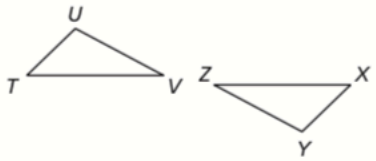
31. CONSTRUCTION Construct a triangle. Show that there is no AAA congruence rule by constructing a second triangle that has the same angle measures but is not congruent.

32. THOUGHT PROVOKING Graph theory is a branch of mathematics that studies vertices and the way they are connected. In graph theory, two polygons are *isomorphic* if there is a one-to-one mapping from one polygon's vertices to the other polygon's vertices that preserves adjacent vertices. In graph theory, are any two triangles isomorphic? Explain your reasoning.

33. MATHEMATICAL CONNECTIONS Six statements are given about $\triangle TUV$ and $\triangle XYZ$.

$\overline{TU} \cong \overline{XY}$ $\quad$ $\overline{UV} \cong \overline{YZ}$ $\quad$ $\overline{TV} \cong \overline{XZ}$

$\angle T \cong \angle X$ $\quad$ $\angle U \cong \angle Y$ $\quad$ $\angle V \cong \angle Z$



a. List all combinations of three given statements that would provide enough information to prove that $\triangle TUV$ is congruent to $\triangle XYZ$.

b. You choose three statements at random. What is the probability that the statements you choose provide enough information to prove that the triangles are congruent?

Maintaining Mathematical Proficiency

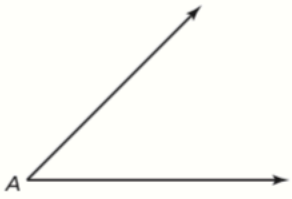
Reviewing what you learned in previous grades and lessons

Find the coordinates of the midpoint of the line segment with the given endpoints. *(Section 1.3)*

34. $C(1, 0)$ and $D(5, 4)$ **35.** $J(-2, 3)$ and $K(4, -1)$ **36.** $R(-5, -7)$ and $S(2, -4)$

Copy the angle using a compass and straightedge. *(Section 1.5)*

37.



38.