Chapter 6
Relationships Within Triangles

6.1 Perpendicular and Angle Bisectors

Using perpendicular bisectors.

What does it mean to be perpendicular?

What does it mean to be a bisector?

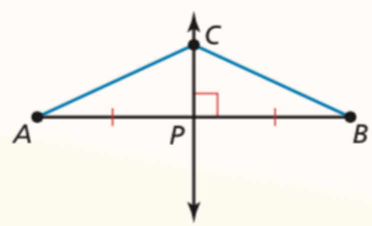
Using Perpendicular Bisectors

Theorems

Theorem 6.1 Perpendicular Bisector Theorem

In a plane, if a point lies on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

If $\overleftrightarrow{CP}$ is the $\perp$ bisector of $\overline{AB}$, then $CA = CB$.

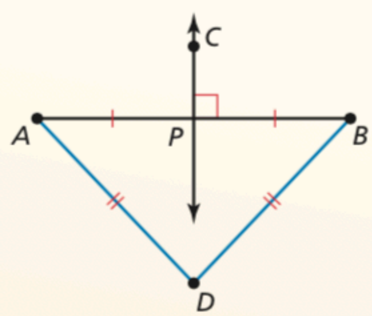


Proof p. 302

Theorem 6.2 Converse of the Perpendicular Bisector Theorem

In a plane, if a point is equidistant from the endpoints of a segment, then it lies on the perpendicular bisector of the segment.

If $DA = DB$, then point D lies on the $\perp$ bisector of $\overline{AB}$.

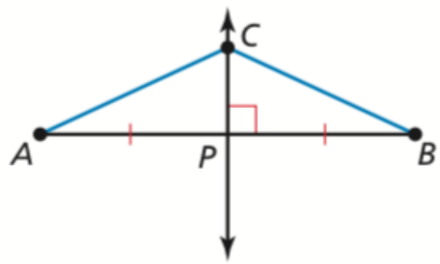


Proof Ex. 32, p. 308

Proving the perpendicular bisector theorem:

Given $\overleftrightarrow{CP}$ is the perpendicular bisector of $\overline{AB}$.

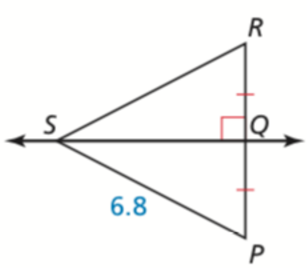
Prove $CA = CB$



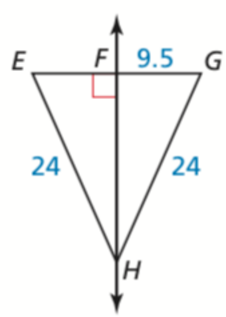
Example 1: Using the perpendicular Bisector Theorems

Find the measure of:

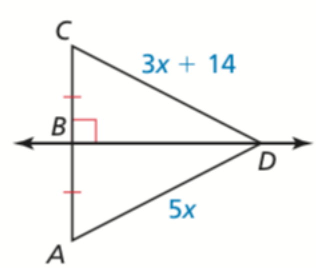
RS=



EG=

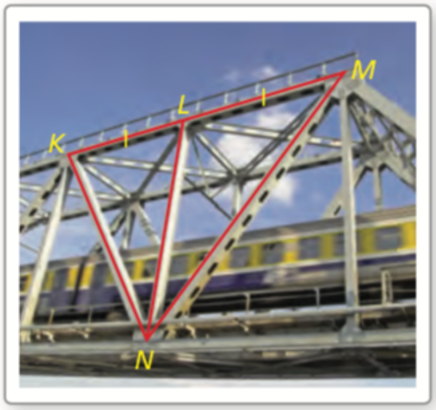


AD=



Example 2: Solving a Real World Problem

Is there enough information to conclude that point N lies on the perpendicular bisector of KM?



Using angle bisectors:

We have learned that an Angle Bisector divides an angle into ______________________________

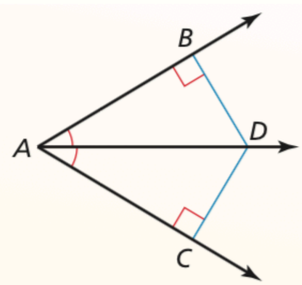
We have also learned that the shortest distance from a point to a line is ______________________

Theorems

Theorem 6.3 Angle Bisector Theorem

If a point lies on the bisector of an angle, then it is equidistant from the two sides of the angle.

If $\overrightarrow{AD}$ bisects $\angle BAC$ and $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$, then $DB = DC$.

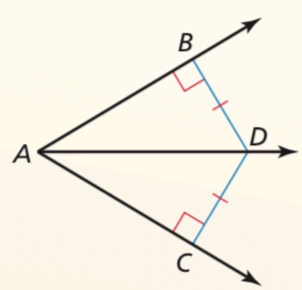


Proof Ex. 33(a), p. 308

Theorem 6.4 Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the angle.

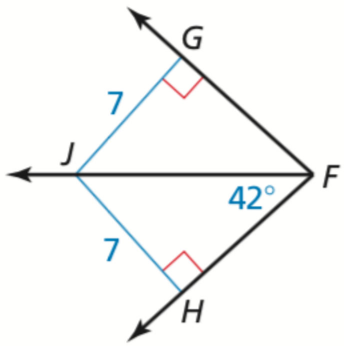
If $\overline{DB} \perp \overrightarrow{AB}$ and $\overline{DC} \perp \overrightarrow{AC}$ and $DB = DC$, then $\overrightarrow{AD}$ bisects $\angle BAC$.



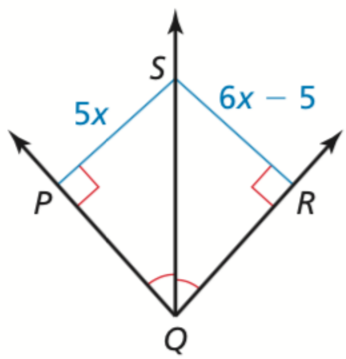
Proof Ex. 33(b), p. 308

Example 3: Find each measure

a) Measure of angle GFJ

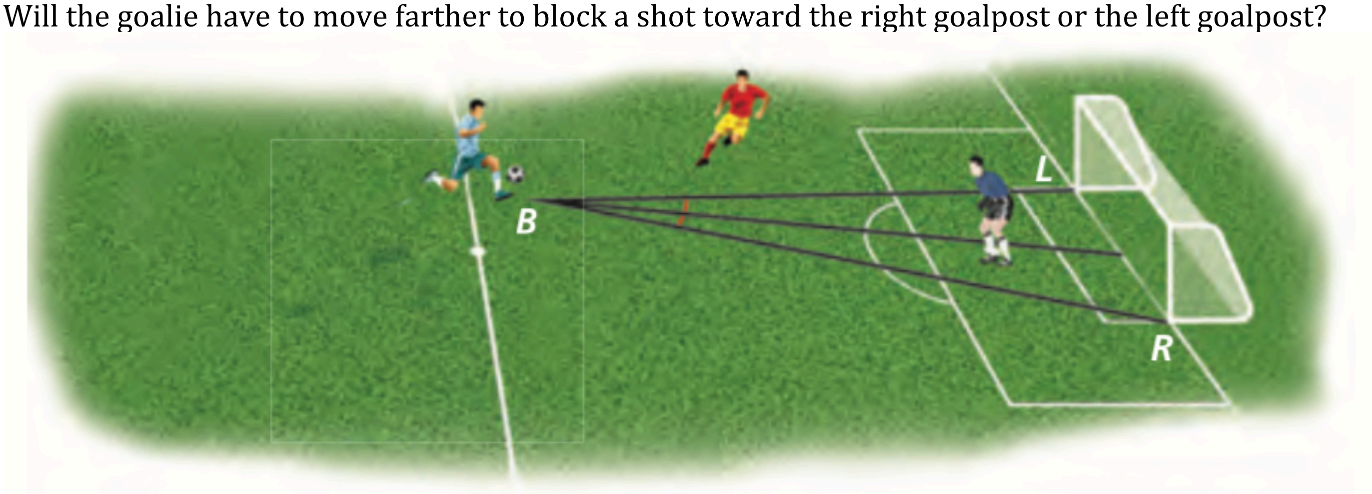


b) Measure of RS



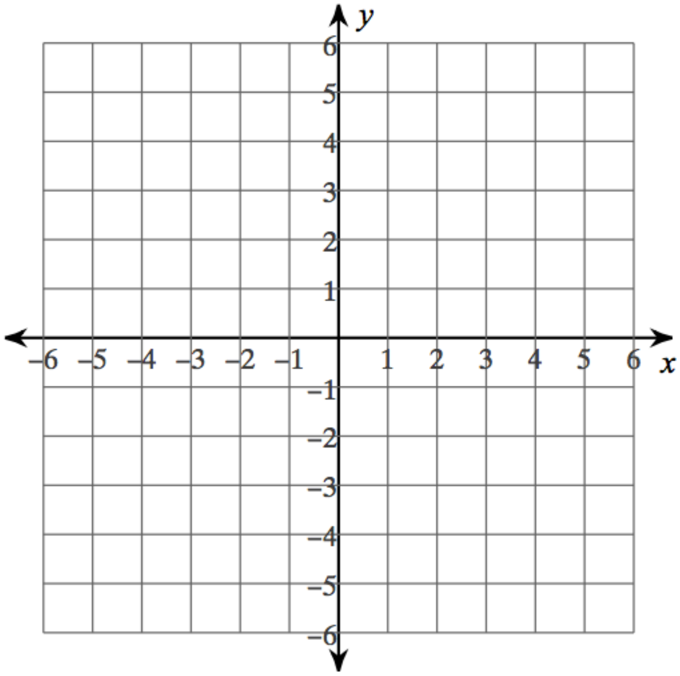
Example 4: Solving Real World Problems

Will the goalie have to move farther to block a shot toward the right goalpost or the left goalpost?



Example 5: Writing an equation for a bisector

Write an equation of the perpendicular bisector of the segment with endpoints P(-2,3) and Q(4,1).



Homework
3-14, 16, 18, 19, 21, 26, 29

6.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** Point C is in the interior of $\angle DEF$. If $\angle DEC$ and $\angle CEF$ are congruent, then $\overrightarrow{EC}$ is the ________ of $\angle DEF$.

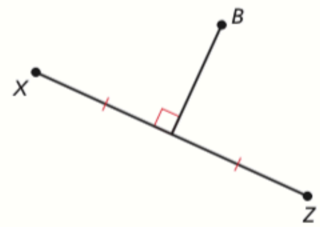
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

   Is point B the same distance from both X and Z?

   Is point B equidistant from X and Z?

   Is point B collinear with X and Z?

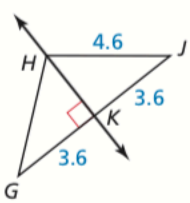
   Is point B on the perpendicular bisector of $\overline{XZ}$?



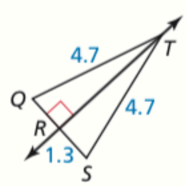
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the indicated measure. Explain your reasoning. *(See Example 1.)*

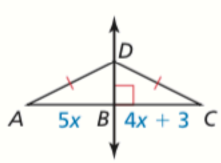
3. GH



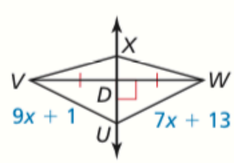
4. QR



5. AB

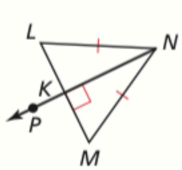


6. UW

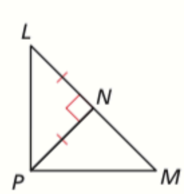


In Exercises 7–10, tell whether the information in the diagram allows you to conclude that point P lies on the perpendicular bisector of $\overline{LM}$. Explain your reasoning. *(See Example 2.)*

7.



8.



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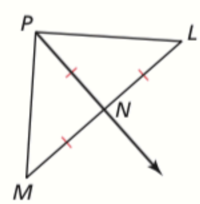
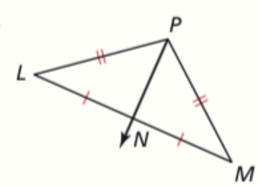
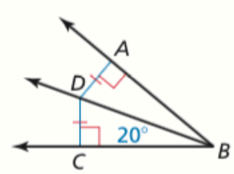


10.

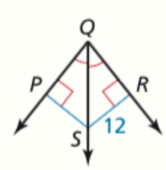


In Exercises 11–14, find the indicated measure. Explain your reasoning. *(See Example 3.)*

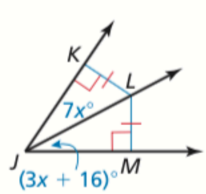
11. $m\angle ABD$



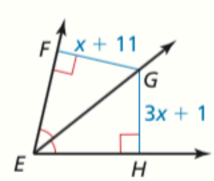
12. PS



13. $m\angle KJL$

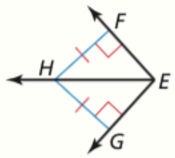


14. FG

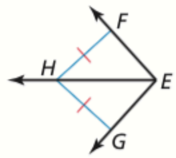


In Exercises 15 and 16, tell whether the information in the diagram allows you to conclude that $\overrightarrow{EH}$ bisects $\angle FEG$. Explain your reasoning. *(See Example 4.)*

15.

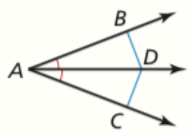


16.

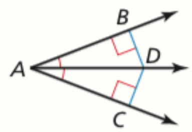


In Exercises 17 and 18, tell whether the information in the diagram allows you to conclude that $DB = DC$. Explain your reasoning.

17.



18.



In Exercises 19–22, write an equation of the perpendicular bisector of the segment with the given endpoints. *(See Example 5.)*

19. $M(1, 5), N(7, -1)$

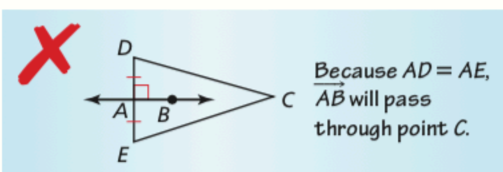
20. $Q(-2, 0), R(6, 12)$

21. $U(-3, 4), V(9, 8)$

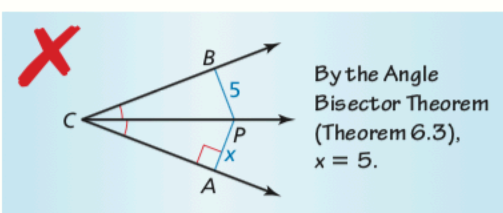
22. $Y(10, -7), Z(-4, 1)$

ERROR ANALYSIS **In Exercises 23 and 24, describe and correct the error in the student's reasoning.**

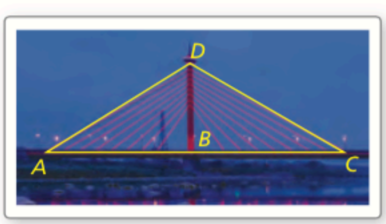
23.



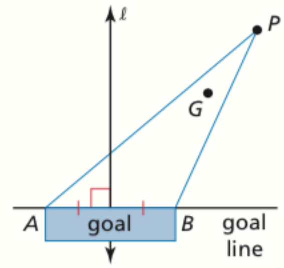
24.



25. **MODELING MATHEMATICS** In the photo, the road is perpendicular to the support beam and $\overline{AB} \cong \overline{CB}$. Which theorem allows you to conclude that $\overline{AD} \cong \overline{CD}$?



26. **MODELING WITH MATHEMATICS** The diagram shows the position of the goalie and the puck during a hockey game. The goalie is at point G, and the puck is at point P.



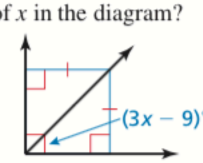
a. What should be the relationship between $\overrightarrow{PG}$ and $\angle APB$ to give the goalie equal distances to travel on each side of $\overrightarrow{PG}$?

b. How does $m\angle APB$ change as the puck gets closer to the goal? Does this change make it easier or more difficult for the goalie to defend the goal? Explain your reasoning.

27. **CONSTRUCTION** Use a compass and straightedge to construct a copy of $\overline{XY}$. Construct a perpendicular bisector and plot a point Z on the bisector so that the distance between point Z and $\overline{XY}$ is 3 centimeters. Measure $\overline{XZ}$ and $\overline{YZ}$. Which theorem does this construction demonstrate?

28. **WRITING** Explain how the Converse of the Perpendicular Bisector Theorem (Theorem 6.2) is related to the construction of a perpendicular bisector.

29. **REASONING** What is the value of x in the diagram?



Ⓐ 13

Ⓑ 18

Ⓒ 33

Ⓓ not enough information

30. **REASONING** Which point lies on the perpendicular bisector of the segment with endpoints $M(7, 5)$ and $N(-1, 5)$?

Ⓐ (2, 0) Ⓑ (3, 9)

Ⓒ (4, 1) Ⓓ (1, 3)

31. **MAKING AN ARGUMENT** Your friend says it is impossible for an angle bisector of a triangle to be the same line as the perpendicular bisector of the opposite side. Is your friend correct? Explain your reasoning.