

Chapter 6

Exponential Functions and Sequences

6.1 Properties of Exponents

Do Now: What are some important properties of exponents?

Using Zero and Negative Exponents

Example1:

Zero Exponents:

$$4^0 =$$

$$(2x)^0 =$$

$$(5^2)^0 =$$

Negative Exponents

$$4^{-2} =$$

$$8^{-3} =$$

Example 2: Simplifying an Expression

Simplify the expression $\frac{4x^0}{y^{-3}}$

Try on your own!

Simplify the expression $\frac{3^{-2}x^{-5}}{y^0}$

Using the properties of exponents

Core Concept

Product of Powers Property

Let a be a real number, and let m and n be integers.

Words To multiply powers with the same base, add their exponents.

Numbers $4^6 \cdot 4^3 = 4^{6+3} = 4^9$ **Algebra** $a^m \cdot a^n = a^{m+n}$

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be integers.

Words To divide powers with the same base, subtract their exponents.

Numbers $\frac{4^6}{4^3} = 4^{6-3} = 4^3$ **Algebra** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$

Power of a Power Property

Let a be a real number, and let m and n be integers.

Words To find a power of a power, multiply the exponents.

Numbers $(4^6)^3 = 4^6 \cdot 3 = 4^{18}$ **Algebra** $(a^m)^n = a^{mn}$

Example 3: Using properties of exponents

Simplify each expression. Write your answer using only **POSITIVE EXPONENTS**

a. $3^2 \cdot 3^6$

b. $\frac{(-4)^2}{(-4)^7}$

c. $(3^2)^{-3}$

Try on your own:

5. $10^4 \cdot 10^{-6}$

6. $x^9 \cdot x^{-9}$

7. $\frac{-5^8}{-5^4}$

8. $\frac{y^6}{y^7}$

9. $(6^{-2})^{-1}$

10. $(w^{12})^5$

Key Notes!!!

Core Concept

Power of a Product Property

Let a and b be real numbers, and let m be an integer.

Words To find a power of a product, find the power of each factor and multiply.

Numbers $(3 \cdot 2)^5 = 3^5 \cdot 2^5$ **Algebra** $(ab)^m = a^m b^m$

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be an integer.

Words To find the power of a quotient, find the power of the numerator and the power of the denominator and divide.

Numbers $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5}$ **Algebra** $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$

Example 4:

Simplify each expression. Write your answer using only **POSITIVE EXPONENTS**

a. $(-1.5y)^2$

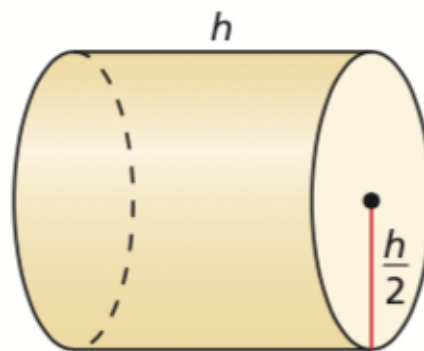
b. $\left(\frac{a}{-10}\right)^3$

c. $\left(\frac{3d}{2}\right)^4$

d. $\left(\frac{2x}{3}\right)^{-5}$

Example 5: Manipulating Expressions

Which of the expressions shown represents the volume of the cylinder?



Volume = ?

$$2\pi r^3$$

$$\pi h^3 2^{-2}$$

$$\pi h 4^{-1}$$

$$\frac{\pi h^2}{4}$$

$$\frac{\pi h^3}{4}$$

$$\frac{\pi h^3}{2}$$

Example 6: Solving real-life problems.

A jellyfish emits about 1.25×10^8 particles of light, or photons, in 6.25×10^{-4} seconds. How many photons does the jellyfish emit each second? **Write your answer in scientific notation and in standard form.**



Homework:
5-31 Odd, 33, 34, 37-55 Odd

6.1 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** Which definitions or properties would you use to simplify the expression $(4^8 \cdot 4^{-4})^{-2}$? Explain.
- WRITING** Explain when and how to use the Power of a Product Property.
- WRITING** Explain when and how to use the Quotient of Powers Property.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

Simplify $3^3 \cdot 3^6$.

Simplify $3^3 + 6$.

Simplify $3^6 \cdot 3$.

Simplify $3^6 \cdot 3^3$.

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, evaluate the expression.
(See Example 1.)

- | | |
|------------------------------|--------------------------------|
| 5. $(-7)^0$ | 6. 4^0 |
| 7. 5^{-4} | 8. $(-2)^{-5}$ |
| 9. $\frac{2^{-4}}{4^0}$ | 10. $\frac{5^{-1}}{-9^0}$ |
| 11. $\frac{-3^{-3}}{6^{-2}}$ | 12. $\frac{(-8)^{-2}}{3^{-4}}$ |

In Exercises 13–22, simplify the expression. Write your answer using only positive exponents. (See Example 2.)

- | | |
|---|---|
| 13. x^{-7} | 14. y^0 |
| 15. $9x^0y^{-3}$ | 16. $15c^{-8}d^0$ |
| 17. $\frac{2^{-2}m^{-3}}{n^0}$ | 18. $\frac{10^0r^{-11}s}{3^2}$ |
| 19. $\frac{4^{-3}d^0}{b^{-7}}$ | 20. $\frac{p^{-8}}{7^{-2}q^{-9}}$ |
| 21. $\frac{2^2y^{-6}}{8^{-1}z^0x^{-7}}$ | 22. $\frac{13x^{-5}y^0}{5^{-3}z^{-10}}$ |

In Exercises 23–32, simplify the expression. Write your answer using only positive exponents. (See Example 3.)

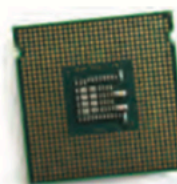
- | | |
|-------------------------------|---------------------------------|
| 23. $\frac{5^6}{5^2}$ | 24. $\frac{(-6)^8}{(-6)^5}$ |
| 25. $(-9)^2 \cdot (-9)^2$ | 26. $4^{-5} \cdot 4^5$ |
| 27. $(p^6)^4$ | 28. $(s^{-5})^3$ |
| 29. $6^{-8} \cdot 6^5$ | 30. $-7 \cdot (-7)^{-4}$ |
| 31. $\frac{x^5}{x^4} \cdot x$ | 32. $\frac{z^8 \cdot z^2}{z^5}$ |

33. USING PROPERTIES

A microscope magnifies an object 10^5 times. The length of an object is 10^{-7} meter. What is its magnified length?



34. **USING PROPERTIES** The area of the rectangular computer chip is $112a^3b^2$ square microns. What is the length?



width = $8ab$ microns

ERROR ANALYSIS In Exercises 35 and 36, describe and correct the error in simplifying the expression.

35. \times $2^4 \cdot 2^5 = (2 \cdot 2)^{4+5} = 4^9$

36. \times $\frac{x^5 \cdot x^3}{x^4} = \frac{x^8}{x^4} = x^{8/4} = x^2$

In Exercises 37–44, simplify the expression. Write your answer using only positive exponents. (See Example 4.)

37. $(-5z)^3$

38. $(4x)^{-4}$

39. $\left(\frac{6}{n}\right)^{-2}$

40. $\left(\frac{-t}{3}\right)^2$

41. $(3s^8)^{-5}$

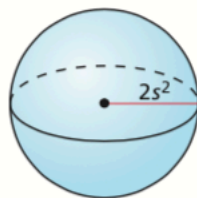
42. $(-5p^3)^3$

43. $\left(-\frac{w^3}{6}\right)^{-2}$

44. $\left(\frac{1}{2r^6}\right)^{-6}$

45. USING PROPERTIES

Which of the expressions represent the volume of the sphere? Explain. (See Example 5.)



(A) $\left(\frac{3s^2}{2^4\pi s^8}\right)^{-1}$

(B) $(2^5\pi s^6)(3^{-1})$

(C) $\frac{32\pi s^6}{3}$

(D) $(2s)^5 \cdot \frac{\pi s}{3}$

(E) $\left(\frac{3\pi s^6}{32}\right)^{-1}$

(F) $\frac{32}{3}\pi s^5$

- 46. MODELING WITH MATHEMATICS** Diffusion is the movement of molecules from one location to another. The time t (in seconds) it takes molecules to diffuse a distance of x centimeters is given by $t = \frac{x^2}{2D}$, where D is the diffusion coefficient. The diffusion coefficient for a drop of ink in water is about 10^{-5} square centimeters per second. How long will it take the ink to diffuse 1 micrometer (10^{-4} centimeter)?



In Exercises 47–50, simplify the expression. Write your answer using only positive exponents.

47. $\left(\frac{2x^{-2}y^3}{3xy^{-4}}\right)^4$

48. $\left(\frac{4s^5t^{-7}}{-2s^{-2}t^4}\right)^3$

49. $\left(\frac{3m^{-5}n^2}{4m^{-2}n^0}\right)^2 \cdot \left(\frac{mn^4}{9n}\right)^2$

50. $\left(\frac{3x^3y^0}{x^{-2}}\right)^4 \cdot \left(\frac{y^2x^{-4}}{5xy^{-8}}\right)^3$

In Exercises 51–54, evaluate the expression. Write your answer in scientific notation and standard form.

51. $(3 \times 10^2)(1.5 \times 10^{-5})$

52. $(6.1 \times 10^{-3})(8 \times 10^9)$

53. $\frac{(6.4 \times 10^7)}{(1.6 \times 10^5)}$

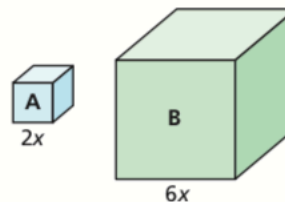
54. $\frac{(3.9 \times 10^{-5})}{(7.8 \times 10^{-8})}$

- 55. PROBLEM SOLVING** In 2012, on average, about 9.46×10^{-1} pound of potatoes was produced for every 2.3×10^{-5} acre harvested. How many pounds of potatoes on average were produced for each acre harvested? Write your answer in scientific notation and in standard form. (See Example 6.)

- 56. PROBLEM SOLVING** The speed of light is approximately 3×10^5 kilometers per second. How long does it take sunlight to reach Jupiter? Write your answer in scientific notation and in standard form.



- 57. MATHEMATICAL CONNECTIONS** Consider Cube A and Cube B.



- Which property of exponents should you use to simplify an expression for the volume of each cube?
- How can you use the Power of a Quotient Property to find how many times greater the volume of Cube B is than the volume of Cube A?

- 58. PROBLEM SOLVING** A byte is a unit used to measure a computer's memory. The table shows the numbers of bytes in several units of measure.

Unit	kilobyte	megabyte	gigabyte	terabyte
Number of bytes	2^{10}	2^{20}	2^{30}	2^{40}

- How many kilobytes are in 1 terabyte?
- How many megabytes are in 16 gigabytes?
- Another unit used to measure a computer's memory is a bit. There are 8 bits in a byte. How can you convert the number of bytes in each unit of measure given in the table to bits? Can you still use a base of 2? Explain.