6.2 Bisectors of a Triangle

Perpendicular Bisectors

When three or more lines intersect in the same point, they are called **concurrent** lines.
The point of intersection of the lines is called the **point on concurrency.**
In a triangle, the three perpendicular bisectors are concurrent.
The point of concurrency is the **circumcenter** of the triangle.

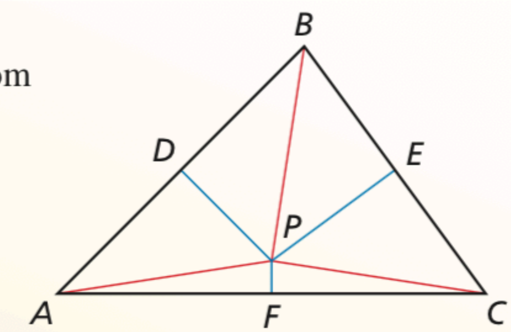
Theorems

Theorem 6.5 Circumcenter Theorem

The circumcenter of a triangle is equidistant from the vertices of the triangle.

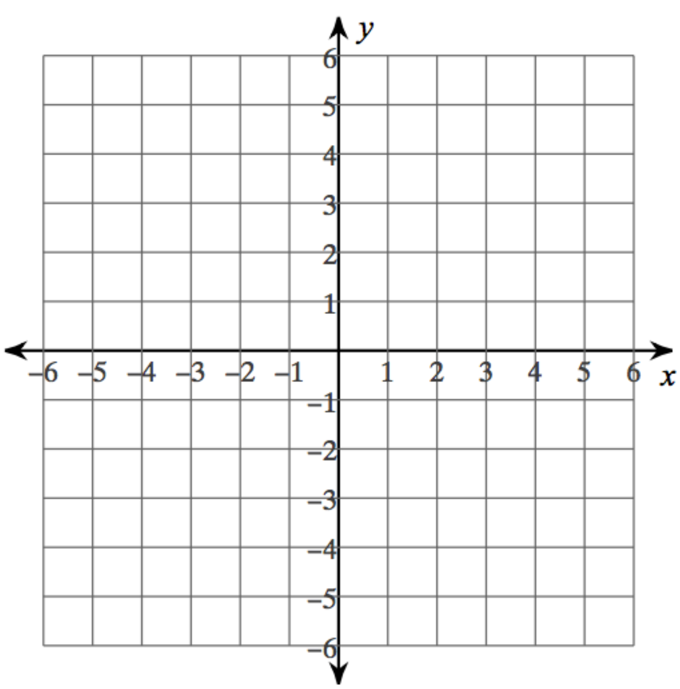
If $\overline{PD}$, $\overline{PE}$, and $\overline{PF}$ are perpendicular bisectors, then $PA = PB = PC$.

Proof p. 310



Example 2: Finding the Circumcenter of a Triangle

Find the coordinates of the circumcenter of ΔABC with vertices $A(0,3), B(0,-1), and\ C(6,-1)$



Using the Incenter of a triangle

Lets bisect each angle. The point of concurrency is the **incenter** of the triangle.

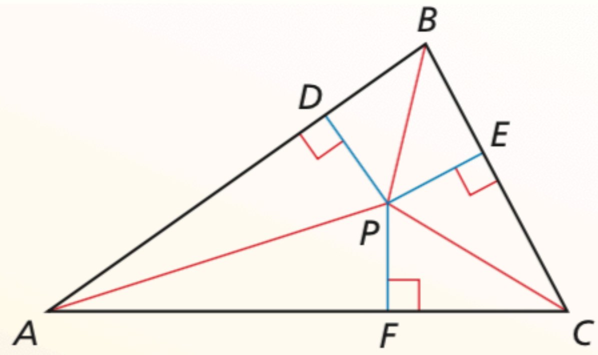
Theorem

Theorem 6.6 Incenter Theorem

The incenter of a triangle is equidistant from the sides of the triangle.

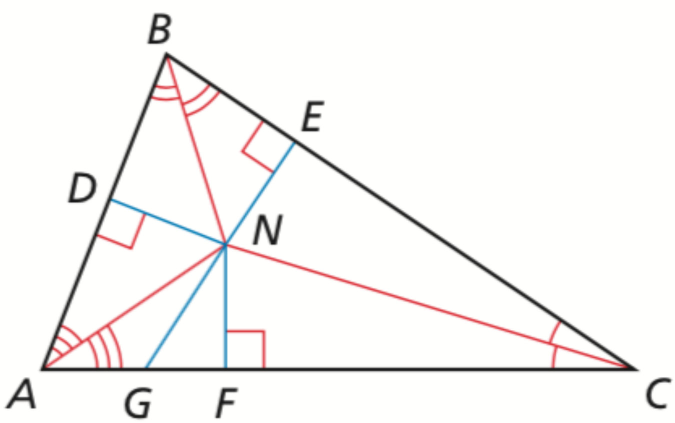
If $\overline{AP}$, $\overline{BP}$, and $\overline{CP}$ are angle bisectors of $\triangle ABC$, then $PD = PE = PF$.

Proof Ex. 38, p. 317



Example 3: Using the Incert of a triangle

In the figure shown, $ND = 5x - 1$ and $NE = 2x + 11$ a) Find NF   b) Can NG be equal to 18?



Homework
3-15 odd, 29-32, 35, 36

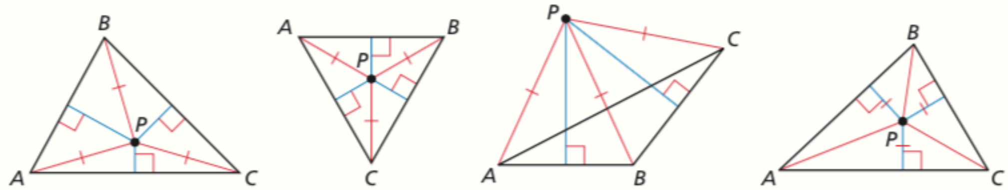
6.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** When three or more lines, rays, or segments intersect in the same point, they are called ___________ lines, rays, or segments.

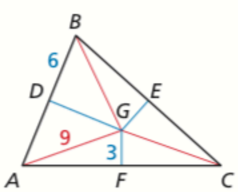
2. **WHICH ONE DOESN'T BELONG?** Which triangle does *not* belong with the other three? Explain your reasoning.



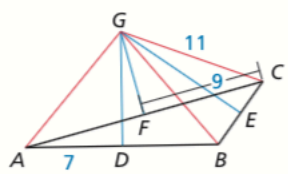
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, the perpendicular bisectors of $\triangle ABC$ intersect at point G and are shown in blue. Find the indicated measure.

3. Find BG.

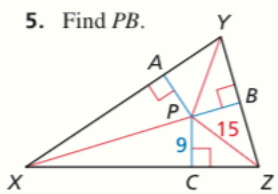


4. Find GA.

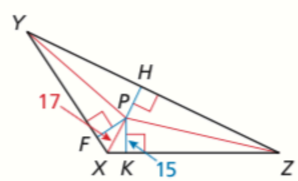


In Exercises 5 and 6, the angle bisectors of $\triangle XYZ$ intersect at point P and are shown in red. Find the indicated measure.

5. Find PB.



6. Find HP.



In Exercises 7–10, find the coordinates of the circumcenter of the triangle with the given vertices. *(See Example 2.)*

7. $A(2, 6), B(8, 6), C(8, 10)$

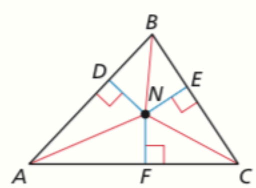
8. $D(-7, -1), E(-1, -1), F(-7, -9)$

9. $H(-10, 7), J(-6, 3), K(-2, 3)$

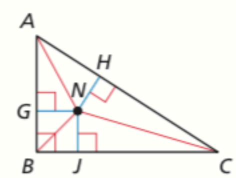
10. $L(3, -6), M(5, -3), N(8, -6)$

In Exercises 11–14, N is the incenter of $\triangle ABC$. Use the given information to find the indicated measure. *(See Example 3.)*

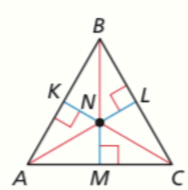
11. $ND = 6x - 2$
    $NE = 3x + 7$
    Find NF.



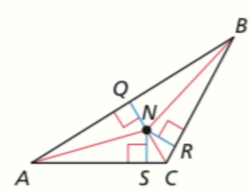
12. $NG = x + 3$
    $NH = 2x - 3$
    Find NJ.



13. $NK = 2x - 2$
    $NL = -x + 10$
    Find NM.

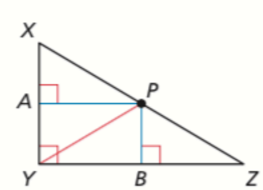


14. $NQ = 2x$
    $NR = 3x - 2$
    Find NS.



15. P is the circumcenter of $\triangle XYZ$. Use the given information to find PZ.

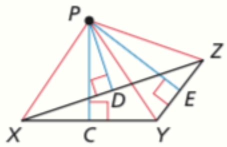
    $PX = 3x + 2$
    $PY = 4x - 8$



16. *P* is the circumcenter of $\triangle XYZ$. Use the given information to find *PY*.

$PX = 4x + 3$

$PZ = 6x - 11$



CONSTRUCTION **In Exercises 17–20, draw a triangle of the given type. Find the circumcenter. Then construct the circumscribed circle.**

17. right

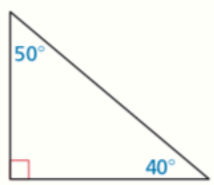
18. obtuse

19. acute isosceles

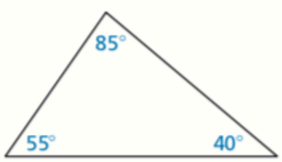
20. equilateral

CONSTRUCTION **In Exercises 21–24, copy the triangle with the given angle measures. Find the incenter. Then construct the inscribed circle.**

21.



22.



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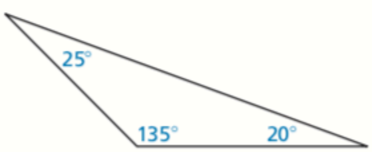

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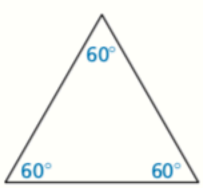
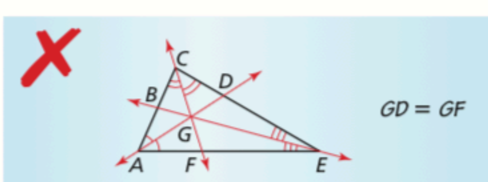
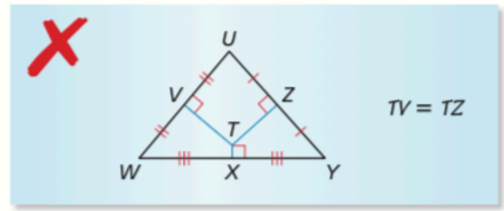


ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in identifying equal distances inside the triangle.**

25.

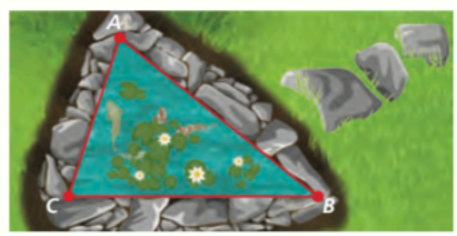


26.



27. **MODELING WITH MATHEMATICS** You and two friends plan to meet to walk your dogs together. You want the meeting place to be the same distance from each person's house. Explain how you can use the diagram to locate the meeting place. *(See Example 1.)*

28. **MODELING WITH MATHEMATICS** You are placing a fountain in a triangular koi pond. You want the fountain to be the same distance from each edge of the pond. Where should you place the fountain? Explain your reasoning. Use a sketch to support your answer. *(See Example 4.)*



CRITICAL THINKING **In Exercises 29–32, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

29. The circumcenter of a scalene triangle is _________ inside the triangle.

30. If the perpendicular bisector of one side of a triangle intersects the opposite vertex, then the triangle is _________ isosceles.

31. The perpendicular bisectors of a triangle intersect at a point that is ________ equidistant from the midpoints of the sides of the triangle.

32. The angle bisectors of a triangle intersect at a point that is ________ equidistant from the sides of the triangle.

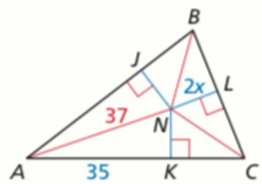
CRITICAL THINKING **In Exercises 33 and 34, find the coordinates of the circumcenter of the triangle with the given vertices.**

33. $A(2, 5)$, $B(6, 6)$, $C(12, 3)$

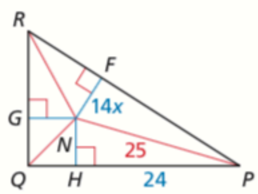
34. $D(-9, -5)$, $E(-5, -9)$, $F(-2, -2)$

MATHEMATICAL CONNECTIONS **In Exercises 35 and 36, find the value of x that makes N the incenter of the triangle.**

35.



36.

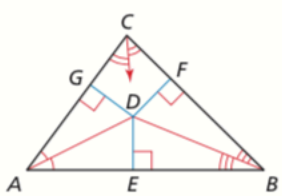


37. **PROOF** Where is the circumcenter located in any right triangle? Write a coordinate proof of this result.

38. **PROVING A THEOREM** Write a proof of the Incenter Theorem (Theorem 6.6).

Given $\triangle ABC$, $\overline{AD}$ bisects $\angle CAB$, $\overline{BD}$ bisects $\angle CBA$, $\overline{DE} \perp \overline{AB}$, $\overline{DF} \perp \overline{BC}$, and $\overline{DG} \perp \overline{CA}$

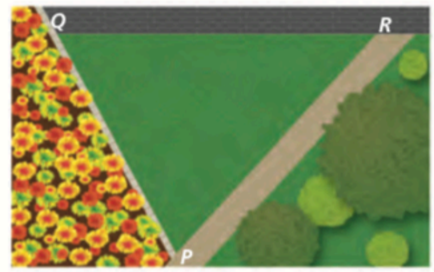
Prove The angle bisectors intersect at D, which is equidistant from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$.



39. **WRITING** Explain the difference between the circumcenter and the incenter of a triangle.

40. **REASONING** Is the incenter of a triangle ever located outside the triangle? Explain your reasoning.

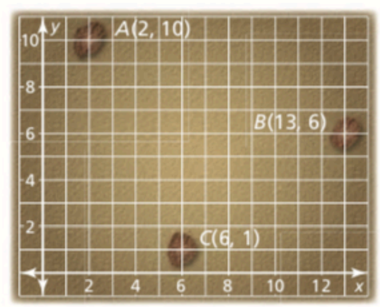
41. **MODELING WITH MATHEMATICS** You are installing a circular pool in the triangular courtyard shown. You want to have the largest pool possible on the site without extending into the walkway.



a. Copy the triangle and show how to install the pool so that it just touches each edge. Then explain how you can be sure that you could not fit a larger pool on the site.

b. You want to have the largest pool possible while leaving at least 1 foot of space around the pool. Would the center of the pool be in the same position as in part (a)? Justify your answer.

42. **MODELING WITH MATHEMATICS** Archaeologists find three stones. They believe that the stones were once part of a circle of stones with a community fire pit at its center. They mark the locations of stones A, B, and C on a graph, where distances are measured in feet.



a. Explain how archaeologists can use a sketch to estimate the center of the circle of stones.

b. Copy the diagram and find the approximate coordinates of the point at which the archaeologists should look for the fire pit.

43. **REASONING** Point P is inside $\triangle ABC$ and is equidistant from points A and B. On which of the following segments must P be located?

Ⓐ $\overline{AB}$

Ⓑ the perpendicular bisector of $\overline{AB}$

Ⓒ $\overline{AC}$

Ⓓ the perpendicular bisector of $\overline{AC}$