6.3 Exponential Functions

Life savings:

You decide to start a piggy bank savings account. The first day you put one penny in the bank. Every following day you double the amount in the bank. How much money would you have after 30 days?

Identifying and Evaluating Exponential Functions

An exponential function is a nonlinear function of the form $y = ab^x$, where $a \neq 0, b \neq 1, and\ b > 0$

Example 1:

Does the table represent a linear or an exponential function?

a.

x	0	1	2	3
y	2	4	6	8

b.

x	0	1	2	3
y	4	8	16	32

Example 2: Evaluating Exponential Functions

Evaluate each function for the given value of x.

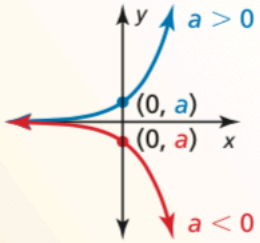
a. $y = -2(5)^x; x = 3$

b. $y = 3(0.5)^x; x = -2$

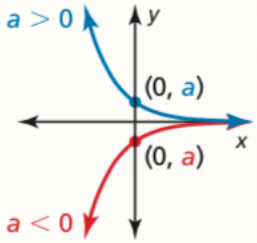
Graphing exponential functions

Core Concept

Graphing $y = ab^x$ When $b > 1$



Graphing $y = ab^x$ When $0 < b < 1$



Example 3: graphing $y = ab^x$ *when* $b > 1$

Graph $f(x) = 4(2)^x$. Compare the graph to the graph of the parent function. Describe the domain and range of $f(x)$.

Example 4: graphing $y = ab^x$ *when* $0 < b < 1$

Graph $f(x) = -\left(\frac{1}{2}\right)^x$. Compare the graph to the graph of the parent function. Describe the domain and range of $f(x)$.

Example 5: graphing $y = ab^{x-h} + k$

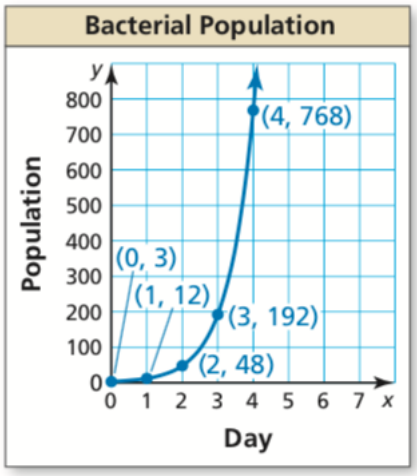
Graph $f(x) = 4(2)^{x-3} + 2$. Describe the domain and range of $f(x)$.

Example 6: Modeling with Mathematics

The graph represents a bacterial population y after x days.

a) Write an exponential function that represents the population

b) Fine the population after 12 hours and after 5 days.



Homework
5-10, 11, 13, 15, 17, 20, 31, 32, 36, 37, 39, 46

6.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **OPEN-ENDED** Sketch an increasing exponential function whose graph has a y-intercept of 2.
2. **REASONING** Why is a the y-intercept of the graph of the function $y = ab^x$?
3. **WRITING** Compare the graph of $y = 2(5)^x$ with the graph of $y = 5^x$.
4. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$y = 3^x$ $\quad f(x) = 2(4)^x$ $\quad f(x) = (-3)^x$ $\quad y = 5(3)^x$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, determine whether the equation represents an exponential function. Explain.

5. $y = 4(7)^x$
6. $y = -6x$
7. $y = 2x^3$
8. $y = -3^x$
9. $y = 9(-5)^x$
10. $y = \frac{1}{2}(1)^x$

In Exercises 11–14, determine whether the table represents a *linear* or an *exponential* function. Explain. *(See Example 1.)*

11.

x	y
1	−2
2	0
3	2
4	4

12.

x	y
1	6
2	12
3	24
4	48

13.

x	−1	0	1	2	3
y	0.25	1	4	16	64

14.

x	−3	0	3	6	9
y	10	1	−8	−17	−26

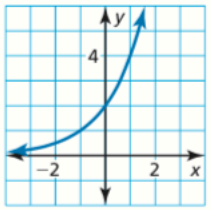
In Exercises 15–20, evaluate the function for the given value of x. *(See Example 2.)*

15. $y = 3^x; x = 2$
16. $f(x) = 3(2)^x; x = -1$
17. $y = -4(5)^x; x = 2$
18. $f(x) = 0.5^x; x = -3$
19. $f(x) = \frac{1}{3}(6)^x; x = 3$
20. $y = \frac{1}{4}(4)^x; x = \frac{3}{2}$

USING STRUCTURE In Exercises 21–24, match the function with its graph.

21. $f(x) = 2(0.5)^x$
22. $y = -2(0.5)^x$
23. $y = 2(2)^x$
24. $f(x) = -2(2)^x$

A.



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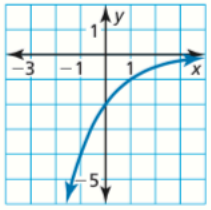
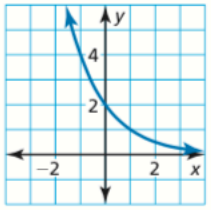
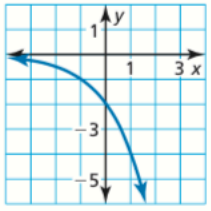


C.



D.



In Exercises 25–30, graph the function. Compare the graph to the graph of the parent function. Describe the domain and range of f. *(See Examples 3 and 4.)*

25. $f(x) = 3(0.5)^x$
26. $f(x) = -4^x$
27. $f(x) = -2(7)^x$
28. $f(x) = 6\left(\frac{1}{3}\right)^x$
29. $f(x) = \frac{1}{2}(8)^x$
30. $f(x) = \frac{3}{2}(0.25)^x$

In Exercises 31–36, graph the function. Describe the domain and range. *(See Example 5.)*

31. $f(x) = 3^x - 1$
32. $f(x) = 4^{x+3}$

33. $y = 5^{x-2} + 7$

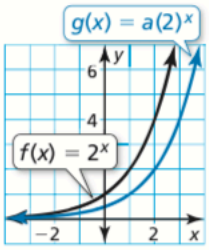
34. $y = -\left(\frac{1}{2}\right)^{x+1} - 3$

35. $y = -8(0.75)^{x+2} - 2$

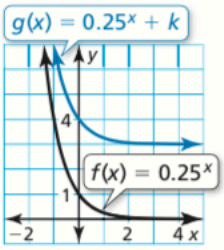
36. $f(x) = 3(6)^{x-1} - 5$

In Exercises 37–40, compare the graphs. Find the value of h, k, or a.

37.



38.



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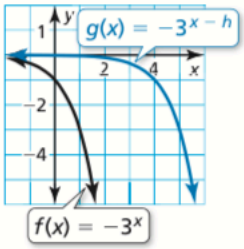
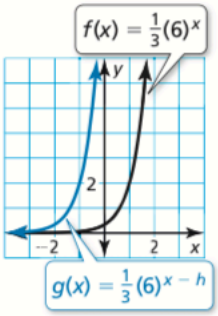


40.

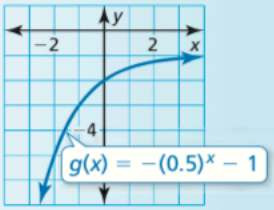


41. ERROR ANALYSIS Describe and correct the error in evaluating the function.

$$g(x) = 6(0.5)^x; x = -2$$
$$g(-2) = 6(0.5)^{-2}$$
$$= 3^{-2}$$
$$= \frac{1}{9}$$

42. ERROR ANALYSIS Describe and correct the error in finding the domain and range of the function.

The domain is all real numbers, and the range is $y < 0$.



In Exercises 43 and 44, graph the function with the given description. Compare the function to $f(x) = 0.5(4)^x$ over the interval $x = 0$ to $x = 2$. *(See Example 6.)*

43. An exponential function g models a relationship in which the dependent variable is multiplied by 2.5 for every 1 unit the independent variable x increases. The value of the function at 0 is 8.

44. An exponential function h models a relationship in which the dependent variable is multiplied by $\frac{1}{2}$ for every 1 unit the independent variable x increases. The value of the function at 0 is 32.

45. MODELING WITH MATHEMATICS You graph an exponential function on a calculator. You zoom in repeatedly to 25% of the screen size. The function $y = 0.25^x$ represents the percent (in decimal form) of the original screen display that you see, where x is the number of times you zoom in.

a. Graph the function. Describe the domain and range.

b. Find and interpret the y-intercept.

c. You zoom in twice. What percent of the original screen do you see?

46. MODELING WITH MATHEMATICS A population y of coyotes in a national park triples every 20 years. The function $y = 15(3)^x$ represents the population, where x is the number of 20-year periods.

a. Graph the function. Describe the domain and range.

b. Find and interpret the y-intercept.

c. How many coyotes are in the national park in 40 years?

In Exercises 47–50, write an exponential function represented by the table or graph. *(See Example 7.)*

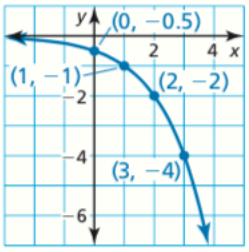
47.

x	0	1	2	3
y	2	14	98	686

48.

x	0	1	2	3
y	−50	−10	−2	−0.4

49.



50.