6.3 Medians and Altitudes of Triangles

Using the median of a triangle

A **median of a triangle** is a segment from a vertex to the midpoint of the opposite side.

The point where all three of the medians intersect is called **the centroid.**

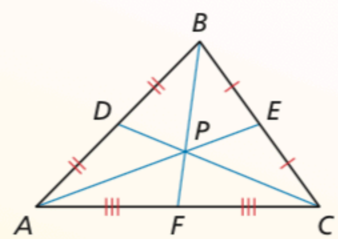
Theorem

Theorem 6.7 Centroid Theorem

The centroid of a triangle is two-thirds of the distance from each vertex to the midpoint of the opposite side.

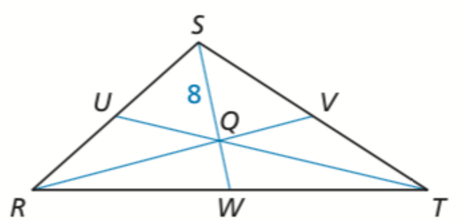
The medians of $\triangle ABC$ meet at point P, and $AP = \frac{2}{3}AE$, $BP = \frac{2}{3}BF$, and $CP = \frac{2}{3}CD$.

Proof *BigIdeasMath.com*



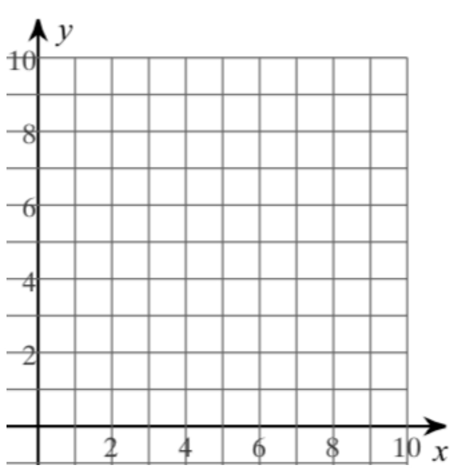
Example 1: Using the centroid of a triangle

Point Q is the centroid. SQ=8. Find QW and SW.



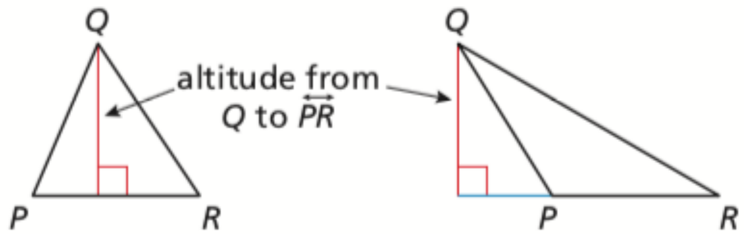
Example 2: Find the Centroid of a triangle.

Find the centroid of triangle RST with the vertices $R(2,1), S(5,8), and\ T(8,3)$



Using the Altitude of a Triangle

An **altitude** of a triangle is the perpendicular segment from a vertex to the opposite side of the line or the line that contains the opposite side.

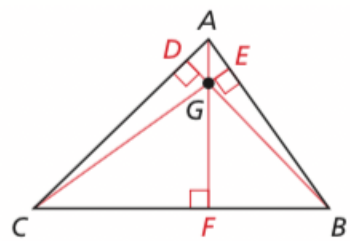


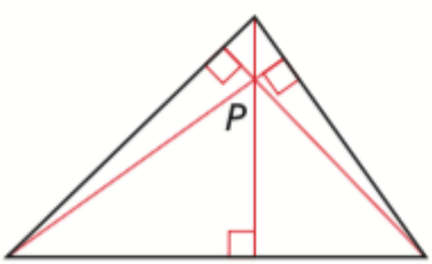
Core Concept

Orthocenter

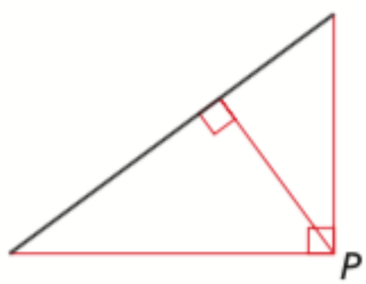
The lines containing the altitudes of a triangle are concurrent. This point of concurrency is the **orthocenter** of the triangle.

The lines containing $\overline{AF}$, $\overline{BD}$, and $\overline{CE}$ meet at the orthocenter G of $\triangle ABC$.

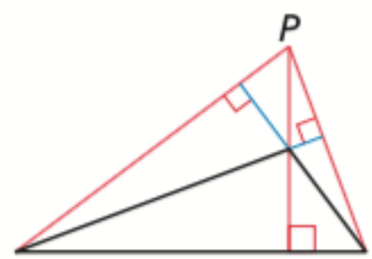




Acute triangle
P is inside triangle.



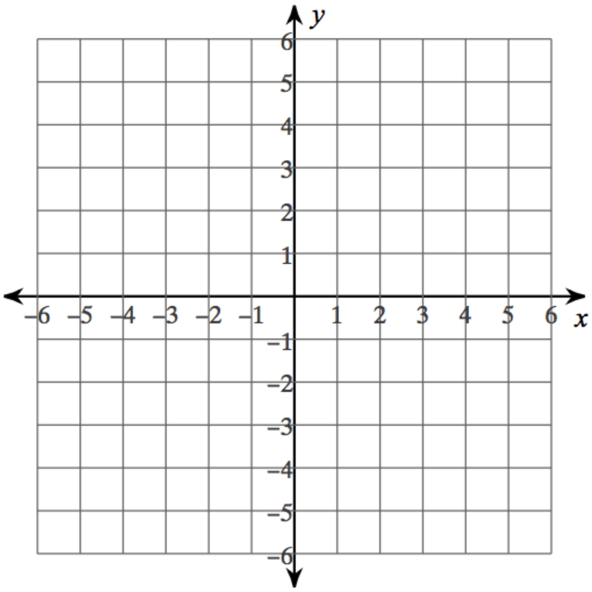
Right triangle
P is on triangle.



Obtuse triangle
P is outside triangle.

Example 3: Find the Orthocenter of a triangle

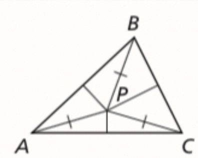
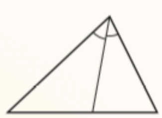
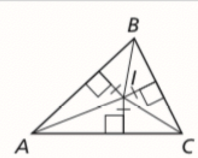
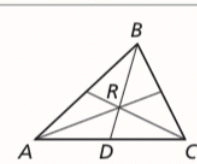
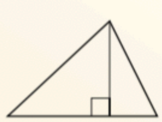
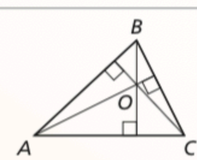
Find the centroid of triangle RST with the vertices $R(-5,-1), Y(-2,4), and\ T(3,-1)$



Chapter Summery:

Concept Summary

Segments, Lines, Rays, and Points in Triangles

	Example	Point of Concurrency	Property	Example
perpendicular bisector		circumcenter	The circumcenter P of a triangle is equidistant from the vertices of the triangle.	
angle bisector		incenter	The incenter I of a triangle is equidistant from the sides of the triangle.	
median		centroid	The centroid R of a triangle is two thirds of the distance from each vertex to the midpoint of the opposite side.	
altitude		orthocenter	The lines containing the altitudes of a triangle are concurrent at the orthocenter O.	

Homework
3-13 odd, 17, 20, 23-26, 31-36, 41, 44

6.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

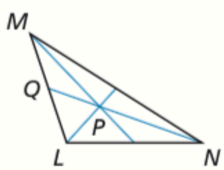
1. **VOCABULARY** Name the four types of points of concurrency. Which lines intersect to form each of the points?

2. **COMPLETE THE SENTENCE** The length of a segment from a vertex to the centroid is ____________ the length of the median from that vertex.

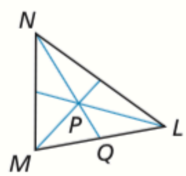
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, point P is the centroid of $\triangle LMN$. Find PN and QP. *(See Example 1.)*

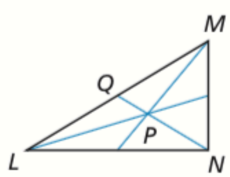
3. $QN = 9$



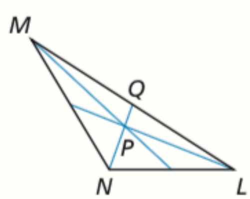
4. $QN = 21$



5. $QN = 30$

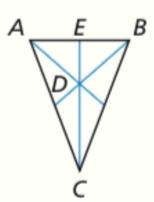


6. $QN = 42$

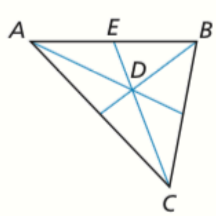


In Exercises 7–10, point D is the centroid of $\triangle ABC$. Find CD and CE.

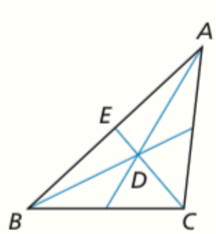
7. $DE = 5$



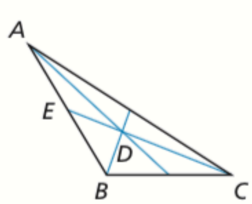
8. $DE = 11$



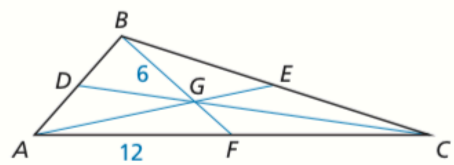
9. $DE = 9$



10. $DE = 15$



In Exercises 11–14, point G is the centroid of $\triangle ABC$. $BG = 6$, $AF = 12$, and $AE = 15$. Find the length of the segment.



11. $\overline{FC}$

12. $\overline{BF}$

13. $\overline{AG}$

14. $\overline{GE}$

In Exercises 15–18, find the coordinates of the centroid of the triangle with the given vertices. *(See Example 2.)*

15. $A(2, 3)$, $B(8, 1)$, $C(5, 7)$

16. $F(1, 5)$, $G(-2, 7)$, $H(-6, 3)$

17. $S(5, 5)$, $T(11, -3)$, $U(-1, 1)$

18. $X(1, 4)$, $Y(7, 2)$, $Z(2, 3)$

In Exercises 19–22, tell whether the orthocenter is *inside*, *on*, or *outside* the triangle. Then find the coordinates of the orthocenter. *(See Example 3.)*

19. $L(0, 5)$, $M(3, 1)$, $N(8, 1)$

20. $X(-3, 2)$, $Y(5, 2)$, $Z(-3, 6)$

21. $A(-4, 0)$, $B(1, 0)$, $C(-1, 3)$

22. $T(-2, 1)$, $U(2, 1)$, $V(0, 4)$

CONSTRUCTION In Exercises 23–26, draw the indicated triangle and find its centroid and orthocenter.

23. isosceles right triangle

24. obtuse scalene triangle

25. right scalene triangle

26. acute isosceles triangle

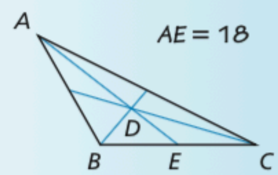
ERROR ANALYSIS **In Exercises 27 and 28, describe and correct the error in finding *DE*. Point *D* is the centroid of $\triangle ABC$.**

27. ✗

$DE = \frac{2}{3}AE$

$DE = \frac{2}{3}(18)$

$DE = 12$

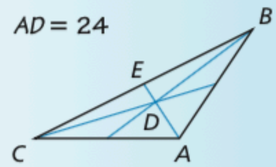


28. ✗

$DE = \frac{2}{3}AD$

$DE = \frac{2}{3}(24)$

$DE = 16$



PROOF **In Exercises 29 and 30, write a proof of the statement.** *(See Example 4.)*

29. The angle bisector from the vertex angle to the base of an isosceles triangle is also a median.

30. The altitude from the vertex angle to the base of an isosceles triangle is also a perpendicular bisector.

CRITICAL THINKING **In Exercises 31–36, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

31. The centroid is __________ on the triangle.

32. The orthocenter is __________ outside the triangle.

33. A median is __________ the same line segment as a perpendicular bisector.

34. An altitude is __________ the same line segment as an angle bisector.

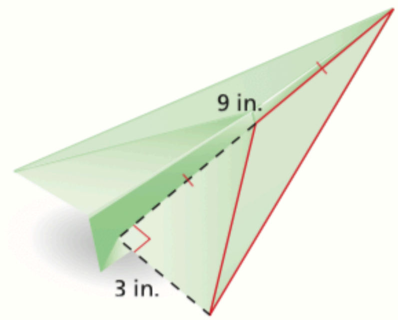
35. The centroid and orthocenter are __________ the same point.

36. The centroid is __________ formed by the intersection of the three medians.

37. **WRITING** Compare an altitude of a triangle with a perpendicular bisector of a triangle.

38. **WRITING** Compare a median, an altitude, and an angle bisector of a triangle.

39. **MODELING WITH MATHEMATICS** Find the area of the triangular part of the paper airplane wing that is outlined in red. Which special segment of the triangle did you use?

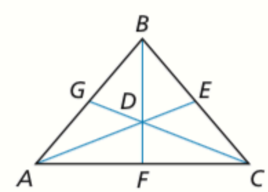


40. **ANALYZING RELATIONSHIPS** Copy and complete the statement for $\triangle DEF$ with centroid K and medians $\overline{DH}$, $\overline{EJ}$, and $\overline{FG}$.

a. $EJ =$ _____ KJ **b.** $DK =$ _____ KH

c. $FG =$ _____ KF **d.** $KG =$ _____ FG

MATHEMATICAL CONNECTIONS **In Exercises 41–44, point *D* is the centroid of $\triangle ABC$. Use the given information to find the value of *x*.**



41. $BD = 4x + 5$ and $BF = 9x$

42. $GD = 2x - 8$ and $GC = 3x + 3$

43. $AD = 5x$ and $DE = 3x - 2$

44. $DF = 4x - 1$ and $BD = 6x + 4$

45. **MATHEMATICAL CONNECTIONS** Graph the lines on the same coordinate plane. Find the centroid of the triangle formed by their intersections.

$y_1 = 3x - 4$

$y_2 = \frac{3}{4}x + 5$

$y_3 = -\frac{3}{2}x - 4$

46. **CRITICAL THINKING** In what type(s) of triangles can a vertex be one of the points of concurrency of the triangle? Explain your reasoning.