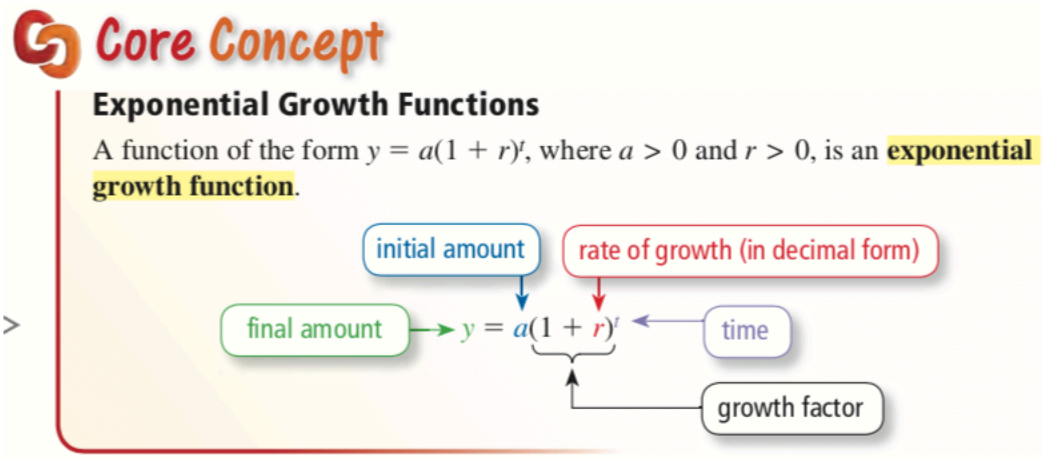
6.4 Exponential Growth and Decay

Exponential Growth and Decay Functions

Exponential growth occurs when a quantity increases by the same factor over intervals of time.


Core Concept

Exponential Growth Functions

A function of the form $y = a(1 + r)^t$, where $a > 0$ and $r > 0$, is an **exponential growth function**.

Example 1: Using an exponential growth factor

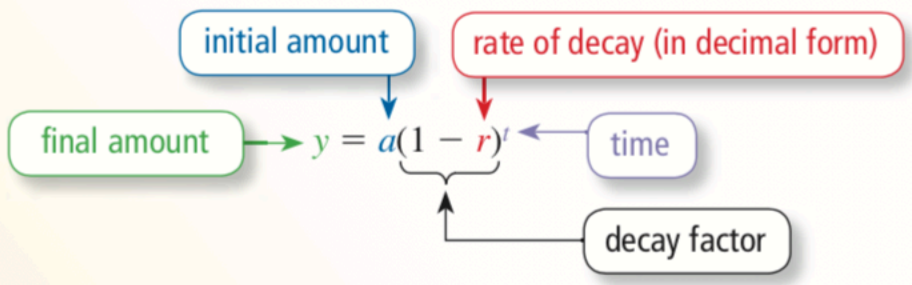
The inaugural attendance of an annual music festival is 150,000.
The attendance y increases by 8% each year.

Exponential decay occurs when a quantity decreases by the same factor over intervals of time.

Core Concept

Exponential Decay Functions

A function of the form $y = a(1 - r)^t$, where $a > 0$ and $0 < r < 1$, is an **exponential decay function**.



Example 2: Identifying exponential growth and decay

Determine whether each table represents an exponential growth or decay function or neither.

a.

x	*y*
0	270
1	90
2	30
3	10

b.

x	0	1	2	3
y	5	10	20	40

Example 3: Interpreting exponential functions

Determine whether each function represents an exponential growth or decay function or neither. Identify the rate of change.

a) $y = 5(1.07)^t$

b) $y = 0.2(0.98)^t$

c) $y = 5(-0.6)^t$

Example 4: Rewriting exponential functions

a) $y = 100(0.96)^{t/4}$

b) $y = (1.1)^{t-3}$

Real World Situations

Core Concept

Compound Interest

Compound interest is the interest earned on the principal *and* on previously earned interest. The balance y of an account earning compound interest is

$$y = P\left(1 + \frac{r}{n}\right)^{nt}.$$

P = principal (initial amount)
r = annual interest rate (in decimal form)
t = time (in years)
n = number of times interest is compounded per year

Example 5: Writing a Function

You deposit $100 in a savings account (not the same piggy bank!) that earns 6% annual interest compounded monthly. Write a function that represents the balance after t years.

Example 6: Solving a real world problem

The table shows the balance of a money market account over time.

a) Write a function that represents the balance after t years
b) Graph the function. Then graph the function from example 5.
c) Compare the account balances

Year, *t*	Balance
0	$100
1	$110
2	$121
3	$133.10
4	$146.41
5	$161.05

Example 7: Solving a real world problem.

The value of a car is $21,500. It loses 12% of its value every year. (a) write a function that represents the value y (in dollars) of the car after t years. (b) Find the approximate monthly percent decrease in value. (c) Graph the function from part A. Use the graph to estimate the value of the car after 6 years.

Homework
7, 9, 13-18, 21, 24-30 even, 33, 34, 38, 57, 62, 63

6.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** In the exponential growth function $y = a(1 + r)^t$, the quantity r is called the ________.

2. **VOCABULARY** What is the decay factor in the exponential decay function $y = a(1 - r)^t$?

3. **VOCABULARY** Compare exponential growth and exponential decay.

4. **WRITING** When does the function $y = ab^x$ represent exponential growth? exponential decay?

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, identify the initial amount a and the rate of growth r (as a percent) of the exponential function. Evaluate the function when $t = 5$. Round your answer to the nearest tenth.

5. $y = 350(1 + 0.75)^t$
6. $y = 10(1 + 0.4)^t$
7. $y = 25(1.2)^t$
8. $y = 12(1.05)^t$
9. $f(t) = 1500(1.074)^t$
10. $h(t) = 175(1.028)^t$
11. $g(t) = 6.72(2)^t$
12. $p(t) = 1.8^t$

In Exercises 13–16, write a function that represents the situation.

13. Sales of \$10,000 increase by 65% each year.

14. Your starting annual salary of \$35,000 increases by 4% each year.

15. A population of 210,000 increases by 12.5% each year.

16. An item costs \$4.50, and its price increases by 3.5% each year.

17. **MODELING WITH MATHEMATICS** The population of a city has been increasing by 2% annually. The sign shown is from the year 2000. *(See Example 1.)*

    a. Write an exponential growth function that represents the population t years after 2000.

    b. What will the population be in 2020? Round your answer to the nearest thousand.

18. **MODELING WITH MATHEMATICS** A young channel catfish weighs about 0.1 pound. During the next 8 weeks, its weight increases by about 23% each week.

    a. Write an exponential growth function that represents the weight of the catfish after t weeks during the 8-week period.

    b. About how much will the catfish weigh after 4 weeks? Round your answer to the nearest thousandth.

In Exercises 19–26, identify the initial amount a and the rate of decay r (as a percent) of the exponential function. Evaluate the function when $t = 3$. Round your answer to the nearest tenth.

19. $y = 575(1 - 0.6)^t$
20. $y = 8(1 - 0.15)^t$
21. $g(t) = 240(0.75)^t$
22. $f(t) = 475(0.5)^t$
23. $w(t) = 700(0.995)^t$
24. $h(t) = 1250(0.865)^t$
25. $y = \left(\frac{7}{8}\right)^t$
26. $y = 0.5\left(\frac{3}{4}\right)^t$

In Exercises 27–30, write a function that represents the situation.

27. A population of 100,000 decreases by 2% each year.

28. A \$900 sound system decreases in value by 9% each year.

29. A stock valued at \$100 decreases in value by 9.5% each year.

30. A company profit of \$20,000 decreases by 13.4% each year.

31. **ERROR ANALYSIS** The growth rate of a bacterial culture is 150% each hour. Initially, there are 10 bacteria. Describe and correct the error in finding the number of bacteria in the culture after 8 hours.

> ✗
> $b(t) = 10(1.5)^t$
> $b(8) = 10(1.5)^8 \approx 256.3$
> After 8 hours, there are about 256 bacteria in the culture.

32. **ERROR ANALYSIS** You purchase a car in 2010 for \$25,000. The value of the car decreases by 14% annually. Describe and correct the error in finding the value of the car in 2015.

> ✗
> $v(t) = 25{,}000(1.14)^t$
> $v(5) = 25{,}000(1.14)^5 \approx 48{,}135$
> The value of the car in 2015 is about \$48,000.

In Exercises 33–38, determine whether the table represents an *exponential growth function*, an *exponential decay function*, or *neither*. Explain. *(See Example 2.)*

33.

x	y
−1	50
0	10
1	2
2	0.4

34.

x	y
0	32
1	28
2	24
3	20

35.

x	y
0	35
1	29
2	23
3	17

36.

x	y
1	17
2	51
3	153
4	459

37.

x	y
5	2
10	8
15	32
20	128

38.

x	y
3	432
5	72
7	12
9	2

39. **ANALYZING RELATIONSHIPS** The table shows the value of a camper t years after it is purchased.

a. Determine whether the table represents an exponential growth function, an exponential decay function, or neither.

b. What is the value of the camper after 5 years?

t	Value
1	\$37,000
2	\$29,600
3	\$23,680
4	\$18,944

40. **ANALYZING RELATIONSHIPS** The table shows the total numbers of visitors to a website t days after it is online.

t	42	43	44	45
Visitors	11,000	12,100	13,310	14,641

a. Determine whether the table represents an exponential growth function, an exponential decay function, or neither.

b. How many people will have visited the website after it is online 47 days?

In Exercises 41–48, determine whether each function represents *exponential growth* or *exponential decay*. Identify the percent rate of change. *(See Example 3.)*

41. $y = 4(0.8)^t$

42. $y = 15(1.1)^t$

43. $y = 30(0.95)^t$

44. $y = 5(1.08)^t$

45. $r(t) = 0.4(1.06)^t$

46. $s(t) = 0.65(0.48)^t$

47. $g(t) = 2\left(\frac{5}{4}\right)^t$

48. $m(t) = \left(\frac{4}{5}\right)^t$

In Exercises 49–56, rewrite the function to determine whether it represents *exponential growth* or *exponential decay*. *(See Example 4.)*

49. $y = (0.9)^{t-4}$

50. $y = (1.4)^{t+8}$

51. $y = 2(1.06)^{9t}$

52. $y = 5(0.82)^{t/5}$

53. $x(t) = (1.45)^{t/2}$

54. $f(t) = 0.4(1.16)^{t-1}$

55. $b(t) = 4(0.55)^{t+3}$

56. $r(t) = (0.88)^{4t}$

In Exercises 57–60, write a function that represents the balance after t years. *(See Example 5.)*

57. \$2000 deposit that earns 5% annual interest compounded quarterly

58. \$1400 deposit that earns 10% annual interest compounded semiannually

59. \$6200 deposit that earns 8.4% annual interest compounded monthly

60. \$3500 deposit that earns 9.2% annual interest compounded quarterly

61. **PROBLEM SOLVING** The cross-sectional area of a tree 4.5 feet from the ground is called its *basal area*. The table shows the basal areas (in square inches) of Tree A over time. *(See Example 6.)*

Year, t	0	1	2	3	4
Basal area, A	120	132	145.2	159.7	175.7

a. Write functions that represent the basal areas of the trees after t years.

b. Graph the functions from part (a) in the same coordinate plane. Compare the basal areas.

62. **PROBLEM SOLVING** You deposit \$300 into an investment account that earns 12% annual interest compounded quarterly. The graph shows the balance of a savings account over time.

a. Write functions that represent the balances of the accounts after t years.

b. Graph the functions from part (a) in the same coordinate plane. Compare the account balances.

63. **PROBLEM SOLVING** A city has a population of 25,000. The population is expected to increase by 5.5% annually for the next decade. *(See Example 7.)*

a. Write a function that represents the population y after t years.

b. Find the approximate monthly percent increase in population.

c. Graph the function from part (a). Use the graph to estimate the population after 4 years.

64. **PROBLEM SOLVING** Plutonium-238 is a material that generates steady heat due to decay and is used in power systems for some spacecraft. The function $y = a(0.5)^{t/x}$ represents the amount y of a substance remaining after t years, where a is the initial amount and x is the length of the half-life (in years).

a. A scientist is studying a 3-gram sample. Write a function that represents the amount y of plutonium-238 after t years.

b. What is the yearly percent decrease of plutonium-238?

c. Graph the function from part (a). Use the graph to estimate the amount remaining after 12 years.

65. **COMPARING FUNCTIONS** The three given functions describe the amount y of ibuprofen (in milligrams) in a person's bloodstream t hours after taking the dosage.

$y \approx 800(0.71)^t$

$y \approx 800(0.9943)^{60t}$

$y \approx 800(0.843)^{2t}$

a. Show that these expressions are approximately equivalent.

b. Describe the information given by each of the functions.