6.5 Indirect Proof and Inequalities in One Triangle

Indirect Proofs

What does it mean to be indirect?
(aka throwing subs)

Lets dive in!

Core Concept

How to Write an Indirect Proof (Proof by Contradiction)

Step 1 Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true.

Step 2 Reason logically until you reach a contradiction.

Step 3 Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

Example 1: Writing an Indirect Proof.

Write an indirect proof that in a given triangle, there can be at most one right angle.

Given:

Prove:

Solution:

Relating Sides and Angles of a Triangle
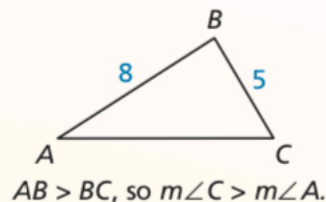
Example 2: Relating side lengths and angle measure

Using a protractor draw any triangle that is NOT equilateral on scrap paper. Labels the sides a,b, & c from least to greatest lengths respectively. Then measure the angles and label the anlges A, B, & C from least to greatest respectively.

Theorems

Theorem 6.9   Triangle Longer Side Theorem

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.
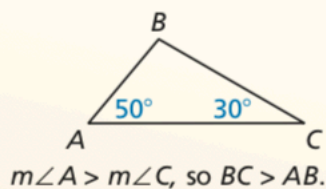
Proof Ex. 43, p. 342

$AB > BC$, so $m\angle C > m\angle A$.

Theorem 6.10   Triangle Larger Angle Theorem

If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Proof p. 337

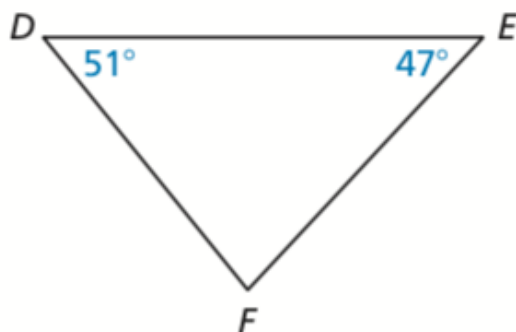$m\angle A > m\angle C$, so $BC > AB$.

Example 3: Ordering Angle Measures of a Triangle

You are constructing a stage prop that shows a large triangular mountain. The bottom edge of the mountain is about 32 feet long, the left slope is about 24 feet long, and the right slope is about 26 feet long. List the angles of $\triangle JKL$ in order from smallest to largest.

Example 4: Ordering Side Lengths of a Triangle

List the sides of triangle DEF in order from shortest to longest.

Using the Triangle Inequality Theorem

Use a compass to create the following triangles with the given segments.

1) 2cm, 4cm, 5cm

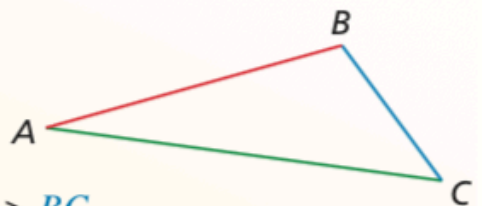2) 2cm, 3cm, 5cm

3) 2cm, 2cm, 5cm

Theorem

Theorem 6.11 Triangle Inequality Theorem

The sum of the lengths of any two sides of
a triangle is greater than the length of the
third side.

$AB + BC > AC$ $AC + BC > AB$ $AB + AC > BC$

Proof Ex. 47, p. 342

Example 5: Finding Possible Side Lengths

A triangle has one side of length 14 and another side of length 9.
Describe the possible lengths of the third side.

Vocabulary and Core Concept Check

1. **VOCABULARY** Why is an indirect proof also called a *proof by contradiction*?

2. **WRITING** How can you tell which side of a triangle is the longest from the angle measures of the triangle? How can you tell which side is the shortest?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, write the first step in an indirect proof of the statement. *(See Example 1.)*

3. If $WV + VU \neq 12$ inches and $VU = 5$ inches, then $WV \neq 7$ inches.

4. If x and y are odd integers, then xy is odd.

5. In $\triangle ABC$, if $m\angle A = 100°$, then $\angle B$ is not a right angle.

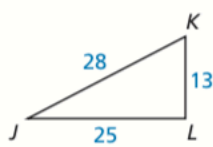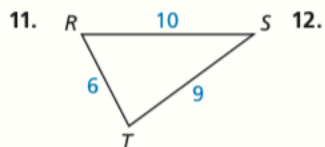6. In $\triangle JKL$, if M is the midpoint of $\overline{KL}$, then $\overline{JM}$ is a median.

In Exercises 7 and 8, determine which two statements contradict each other. Explain your reasoning.

7. Ⓐ $\triangle LMN$ is a right triangle.

   Ⓑ $\angle L \cong \angle N$

   Ⓒ $\triangle LMN$ is equilateral.

8. Ⓐ Both $\angle X$ and $\angle Y$ have measures greater than 20°.

   Ⓑ Both $\angle X$ and $\angle Y$ have measures less than 30°.
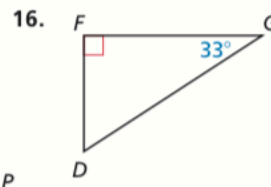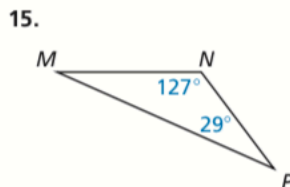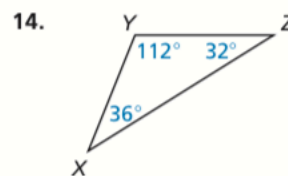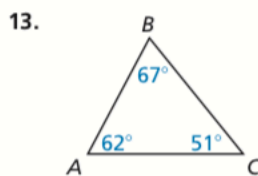
   Ⓒ $m\angle X + m\angle Y = 62°$

In Exercises 9 and 10, use a ruler and protractor to draw the given type of triangle. Mark the largest angle and longest side in red and the smallest angle and shortest side in blue. What do you notice? *(See Example 2.)*

9. acute scalene

10. right scalene

In Exercises 11 and 12, list the angles of the given triangle from smallest to largest. *(See Example 3.)*

11.

12.

In Exercises 13–16, list the sides of the given triangle from shortest to longest. *(See Example 4.)*

13.

14.

15.

16.

In Exercises 17–20, describe the possible lengths of the third side of the triangle given the lengths of the other two sides. *(See Example 5.)*

17. 5 inches, 12 inches

18. 12 feet, 18 feet

19. 2 feet, 40 inches

20. 25 meters, 25 meters

In Exercises 21–24, is it possible to construct a triangle with the given side lengths? If not, explain why not.

21. 6, 7, 11
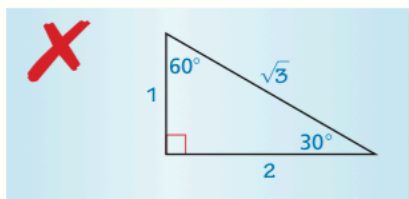
22. 3, 6, 9

23. 28, 17, 46

24. 35, 120, 125

25. **ERROR ANALYSIS** Describe and correct the error in writing the first step of an indirect proof.

Show that $\angle A$ is obtuse.

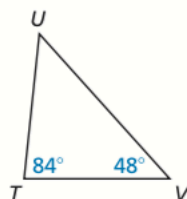Step 1   Assume temporarily that $\angle A$ is acute.

26. ERROR ANALYSIS Describe and correct the error in labeling the side lengths 1, 2, and $\sqrt{3}$ on the triangle.

27. REASONING You are a lawyer representing a client who has been accused of a crime. The crime took place in Los Angeles, California. Security footage shows your client in New York at the time of the crime. Explain how to use indirect reasoning to prove your client is innocent.

28. REASONING Your class has fewer than 30 students. The teacher divides your class into two groups. The first group has 15 students. Use indirect reasoning to show that the second group must have fewer than 15 students.
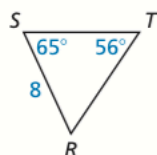
29. PROBLEM SOLVING Which statement about △*TUV* is false?

Ⓐ *UV* > *TU*

Ⓑ *UV* + *TV* > *TU*

Ⓒ *UV* < *TV*

Ⓓ △*TUV* is isosceles.

30. PROBLEM SOLVING In △*RST*, which is a possible side length for *ST*? Select all that apply.

Ⓐ 7

Ⓑ 8

Ⓒ 9

Ⓓ 10

31. PROOF Write an indirect proof that an odd number is not divisible by 4.

32. PROOF Write an indirect proof of the statement "In △*QRS*, if $m\angle Q + m\angle R = 90°$, then $m\angle S = 90°$."

33. WRITING Explain why the hypotenuse of a right triangle must always be longer than either leg.

34. CRITICAL THINKING Is it possible to decide if three side lengths form a triangle without checking all three inequalities shown in the Triangle Inequality Theorem (Theorem 6.11)? Explain your reasoning.

35. MODELING WITH MATHEMATICS You can estimate the width of the river from point *A* to the tree at point *B* by measuring the angle to the tree at several locations along the riverbank. The diagram shows the results for locations *C* and *D*.
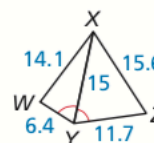

a. Using △*BCA* and △*BDA*, determine the possible widths of the river. Explain your reasoning.

b. What could you do if you wanted a closer estimate?

36. MODELING WITH MATHEMATICS You travel from Fort Peck Lake to Glacier National Park and from Glacier National Park to Granite Peak.

a. Write two inequalities to represent the possible distances from Granite Peak back to Fort Peck Lake.

b. How is your answer to part (a) affected if you know that $m\angle 2 < m\angle 1$ and $m\angle 2 < m\angle 3$?

37. REASONING In the figure, $\overline{XY}$ bisects $\angle WYZ$. List all six angles of △*XYZ* and △*WXY* in order from smallest to largest. Explain your reasoning.

38. MATHEMATICAL CONNECTIONS In △*DEF*, $m\angle D = (x + 25)°$, $m\angle E = (2x - 4)°$, and $m\angle F = 63°$. List the side lengths and angle measures of the triangle in order from least to greatest.