## 6.5 Solving Exponential Equations

Solving Exponential Equations with the **SAME BASE**.

Exponential equations are equations in which variable expressions occur as exponents.

### Core Concept

**Property of Equality for Exponential Equations**

**Words** Two powers with the *same positive base b*, where $b \neq 1$, are equal if and only if their exponents are equal.

**Numbers** If $2^x = 2^5$, then $x = 5$. If $x = 5$, then $2^x = 2^5$.

**Algebra** If $b > 0$ and $b \neq 1$, then $b^x = b^y$ if and only if $x = y$.

**Example 1:** Solving exponential equations with the same base.

Solve each equation:

a) $3^{x+1} = 3^5$ b) $6 = 6^{2x-3}$ c) $10^{3x} = 10^{2x+3}$

**Example 2:** Solving Exponential Equations with Unlike Bases

Solve:

a) $5^x = 125$ b) $4^x = 2^{x-3}$ c) $9^{x+2} = 27^x$

**Example 3:** Solving Exponential Equations when 0 < b < 1

Solve :

a) $\left(\frac{1}{2}\right)^{x} = 4$ b) $4^{x+1} = \left(\frac{1}{64}\right)$

**Example 4:** Solving Exponential Equations by Graphing

Use a graphing calculator to solve:

a) $\left(\frac{1}{2}\right)^{x-1} = 7$ b) $3^{x+2} = x + 1$

Homework
3-17 odd, 21-24, 25-35 odd, 38, 40, 42, 43, 47, 51

# 6.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **WRITING** Describe how to solve an exponential equation with unlike bases.

2. **WHICH ONE DOESN'T BELONG?** Which equation does *not* belong with the other three? Explain your reasoning.

$2^x = 4^{x+6}$ $\quad 5^{3x+8} = 5^{2x}$ $\quad 3^4 = x + 4^2$ $\quad 2^{x-7} = 2^7$

## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–12, solve the equation. Check your solution.** *(See Examples 1 and 2.)*

3. $4^{5x} = 4^{10}$
4. $7^{x-4} = 7^8$
5. $3^{9x} = 3^{7x+8}$
6. $2^{4x} = 2^{x+9}$
7. $2^x = 64$
8. $3^x = 243$
9. $7^{x-5} = 49^x$
10. $216^x = 6^{x+10}$
11. $64^{2x+4} = 16^{5x}$
12. $27^x = 9^{x-2}$

**In Exercises 13–18, solve the equation. Check your solution.** *(See Example 3.)*

13. $\left(\frac{1}{5}\right)^x = 125$
14. $\left(\frac{1}{4}\right)^x = 256$
15. $\frac{1}{128} = 2^{5x+3}$
16. $3^{4x-9} = \frac{1}{243}$
17. $36^{-3x+3} = \left(\frac{1}{216}\right)^{x+1}$
18. $\left(\frac{1}{27}\right)^{4-x} = 9^{2x-1}$

**ERROR ANALYSIS** **In Exercises 19 and 20, describe and correct the error in solving the exponential equation.**

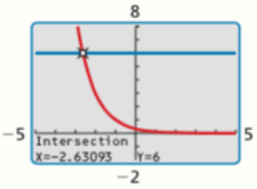
19. ✗
$$5^{3x+2} = 25^{x-8}$$
$$3x + 2 = x - 8$$
$$x = -5$$

20. ✗
$$\left(\tfrac{1}{8}\right)^{5x} = 32^{x+8}$$
$$(2^3)^{5x} = (2^5)^{x+8}$$
$$2^{15x} = 2^{5x+40}$$
$$15x = 5x + 40$$
$$x = 4$$

**In Exercises 21–24, match the equation with the graph that can be used to solve it. Then solve the equation.**

21. $2^x = 6$
22. $4^{2x-5} = 6$
23. $5^{x+2} = 6$
24. $3^{-x-1} = 6$

A.

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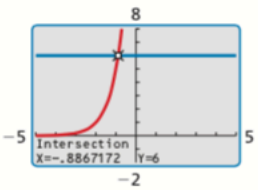


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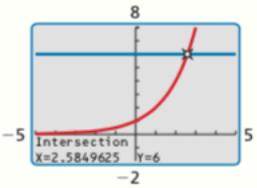


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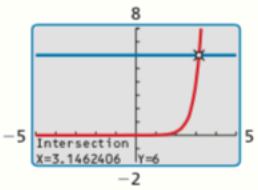



**In Exercises 25–36, use a graphing calculator to solve the equation.** *(See Example 4.)*

25. $6^{x+2} = 12$
26. $5^{x-4} = 8$
27. $\left(\frac{1}{2}\right)^{7x+1} = -9$
28. $\left(\frac{1}{3}\right)^{x+3} = 10$
29. $2^{x+6} = 2x + 15$
30. $3x - 2 = 5^{x-1}$
31. $\frac{1}{2}x - 1 = \left(\frac{1}{3}\right)^{2x-1}$
32. $2^{-x+1} = -\frac{3}{4}x + 3$
33. $5^x = -4^{-x+4}$
34. $7^{x-2} = 2^{-x}$
35. $2^{-x-3} = 3^{x+1}$
36. $5^{-2x+3} = -6^{x+5}$

**In Exercises 37–40, solve the equation by using the Property of Equality for Exponential Equations.**

**37.** $30 \cdot 5^{x+3} = 150$

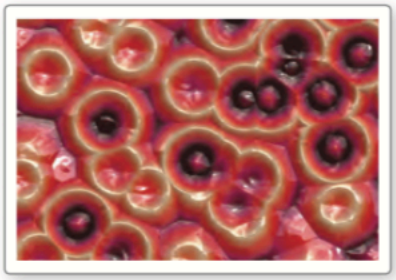
**38.** $12 \cdot 2^{x-7} = 24$

**39.** $4(3^{-2x-4}) = 36$

**40.** $2(4^{2x+1}) = 128$

**41. MODELING WITH MATHEMATICS** You scan a photo into a computer at four times its original size. You continue to increase its size repeatedly by 100% using the computer. The new size of the photo $y$ in comparison to its original size after $x$ enlargements on the computer is represented by $y = 2^{x+2}$. How many times must the photo be enlarged on the computer so the new photo is 32 times the original size?

**42. MODELING WITH MATHEMATICS** A bacterial culture quadruples in size every hour. You begin observing the number of bacteria 3 hours after the culture is prepared. The amount $y$ of bacteria $x$ hours after the culture is prepared is represented by $y = 192(4^{x-3})$. When will there be 200,000 bacteria?



**In Exercises 43–46, solve the equation.**

**43.** $3^{3x+6} = 27^{x+2}$

**44.** $3^{4x+3} = 81^{x}$

**45.** $4^{x+3} = 2^{2(x+1)}$

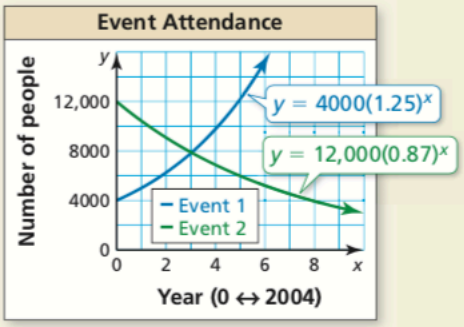
**46.** $5^{8(x-1)} = 625^{2x-2}$

**47. NUMBER SENSE** Explain how you can use mental math to solve the equation $8^{x-4} = 1$.

**48. PROBLEM SOLVING** There are a total of 128 teams at the start of a citywide 3-on-3 basketball tournament. Half the teams are eliminated after each round. Write and solve an exponential equation to determine after which round there are 16 teams left.

**49. PROBLEM SOLVING** You deposit \$500 in a savings account that earns 6% annual interest compounded yearly. Write and solve an exponential equation to determine when the balance of the account will be \$800.

**50. HOW DO YOU SEE IT?** The graph shows the annual attendance at two different events. Each event began in 2004.




**a.** Estimate when the events will have about the same attendance.

**b.** Explain how you can verify your answer in part (a).

**51. REASONING** Explain why the Property of Equality for Exponential Equations does not work when $b = 1$. Give an example to justify your answer.

**52. THOUGHT PROVOKING** Is it possible for an exponential equation to have two different solutions? If not, explain your reasoning. If so, give an example.

**USING STRUCTURE In Exercises 53–58, solve the equation.**

**53.** $8^{x-2} = \sqrt{8}$

**54.** $\sqrt{5} = 5^{x+4}$

**55.** $\left(\sqrt[5]{7}\right)^{x} = 7^{2x+3}$

**56.** $12^{2x-1} = \left(\sqrt[4]{12}\right)^{x}$

**57.** $\left(\sqrt[3]{6}\right)^{2x} = \left(\sqrt{6}\right)^{x+6}$

**58.** $\left(\sqrt[5]{3}\right)^{5x-10} = \left(\sqrt[8]{3}\right)^{4x}$

**59. MAKING AN ARGUMENT** Consider the equation $\left(\frac{1}{a}\right)^{x} = b$, where $a > 1$ and $b > 1$. Your friend says the value of $x$ will always be negative. Is your friend correct? Explain.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Determine whether the sequence is arithmetic. If so, find the common difference.** *(Section 4.6)*

**60.** $-20, -26, -32, -38, \ldots$

**61.** $9, 18, 36, 72, \ldots$

**62.** $-5, -8, -12, -17, \ldots$

**63.** $10, 20, 30, 40, \ldots$