5.2 Properties of Rational Exponents and Radicals

Core Concept

Properties of Rational Exponents

Let a and b be real numbers and let m and n be rational numbers, such that the quantities in each property are real numbers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^{1/2} \cdot 5^{3/2} = 5^{(1/2+3/2)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^{5/2})^2 = 3^{(5/2 \cdot 2)} = 3^5 = 243$
Power of a Product	$(ab)^m = a^m b^m$	$(16 \cdot 9)^{1/2} = 16^{1/2} \cdot 9^{1/2} = 4 \cdot 3 = 12$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-1/2} = \frac{1}{36^{1/2}} = \frac{1}{6}$
Zero Exponent	$a^0 = 1, a \neq 0$	$213^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{4^{5/2}}{4^{1/2}} = 4^{(5/2-1/2)} = 4^2 = 16$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{27}{64}\right)^{1/3} = \frac{27^{1/3}}{64^{1/3}} = \frac{3}{4}$

Example 1: Using Properties of Exponents

Use the properties of rational exponents to simplify the following

1) $7^{1/4} * 7^{\frac{1}{2}} =$

2) $\left(6^{1/2} * 4^{1/3}\right)^2 =$

3) $(4^5 * 3^5)^{-1/5} =$

4) $\frac{5^4}{5^2} =$

5) $\left(\frac{24^{1/3}}{6^{1/3}}\right)^2 =$

Core Concept

Properties of Radicals

Let a and b be real numbers and let n be an integer greater than 1.

Property Name	Definition	Example
Product Property	$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$
Quotient Property	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$	$\frac{\sqrt[4]{162}}{\sqrt[4]{2}} = \sqrt[4]{\frac{162}{2}} = \sqrt[4]{81} = 3$

Example 2: Using Properties of Radicals

Use the properties of radicals to simplify each expression.

1) $\sqrt[3]{12} * \sqrt[3]{18} =$

2) $\frac{\sqrt[4]{80}}{\sqrt[4]{5}} =$

Simplest form: (VERY IMPORTANT!)

- No radicands have perfect nth powers as factors other than 1
- No radicands contain fractions
- No radicands appear in the denominator of a fraction

Example 3: Writing Radicals in Simplest Form

Write each expression in simplest form

a) $\sqrt[3]{135}$

b) $\frac{\sqrt[5]{7}}{\sqrt[5]{8}}$

<u>Conjugates!</u>

Example 4: Writing radical expressions in simplest form

Write $\frac{1}{5+\sqrt{3}}$ in simplest form.

Example 5: Adding and Subtracting Like Radicals and Roots

Simplify each expression:

a) $\sqrt[4]{10} + 7\sqrt[4]{10}$

b) $2(8^{1/5}) + 10(8^{1/5})$

c. $\sqrt[3]{54} - \sqrt[3]{2}$

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	Example
When *n* is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When *n* is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Absolute value is not needed when all variables are assumed to be positive.

Example 6: Simplify each expression

a) $\sqrt[3]{64y^6}$ b) $\sqrt[4]{\frac{x^4}{y^8}}$

Example 7: Writing Variable Expressions in Simplest form

a) $\sqrt[5]{4a^8b^{14}c^5}$ b) $\frac{x}{\sqrt[3]{y^8}}$ c) $\frac{14xy^{1/3}}{2x^{2/3}z^{-6}}$

Example 8: Adding and Subtracting Variable Expressions

Preform each indicated operation. Assume all variables are positive

a) $5\sqrt{y} + 6\sqrt{y}$ b) $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$

Homework: 3, 7, 11, 14, 17, 20, 23, 25, 26, 29-45odd, 51, 54, 57, 61, 66, 68, 71

5.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How do you know when a radical expression is in simplest form?

2. **WHICH ONE DOESN'T BELONG?** Which radical expression does *not* belong with the other three? Explain your reasoning.

$\sqrt[3]{\frac{4}{5}}$	$2\sqrt{x}$	$\sqrt[4]{11}$	$3\sqrt[5]{9x}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, use the properties of rational exponents to simplify the expression. *(See Example 1.)*

3. $(9^2)^{1/3}$
4. $(12^2)^{1/4}$
5. $\frac{6}{6^{1/4}}$
6. $\frac{7}{7^{1/3}}$
7. $\left(\frac{8^4}{10^4}\right)^{-1/4}$
8. $\left(\frac{9^3}{6^3}\right)^{-1/3}$
9. $(3^{-2/3} \cdot 3^{1/3})^{-1}$
10. $(5^{1/2} \cdot 5^{-3/2})^{-1/4}$
11. $\frac{2^{2/3} \cdot 16^{2/3}}{4^{2/3}}$
12. $\frac{49^{3/8} \cdot 49^{7/8}}{7^{5/4}}$

In Exercises 13–20, use the properties of radicals to simplify the expression. *(See Example 2.)*

13. $\sqrt{2} \cdot \sqrt{72}$
14. $\sqrt[3]{16} \cdot \sqrt[3]{32}$
15. $\sqrt[4]{6} \cdot \sqrt[4]{8}$
16. $\sqrt[4]{8} \cdot \sqrt[4]{8}$
17. $\frac{\sqrt[5]{486}}{\sqrt[5]{2}}$
18. $\frac{\sqrt{2}}{\sqrt{32}}$
19. $\frac{\sqrt[3]{6} \cdot \sqrt[3]{72}}{\sqrt[3]{2}}$
20. $\frac{\sqrt[3]{3} \cdot \sqrt[3]{18}}{\sqrt[6]{2} \cdot \sqrt[6]{2}}$

In Exercises 21–28, write the expression in simplest form. *(See Example 3.)*

21. $\sqrt[4]{567}$
22. $\sqrt[5]{288}$
23. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$
24. $\frac{\sqrt[4]{4}}{\sqrt[4]{27}}$
25. $\sqrt{\frac{3}{8}}$
26. $\sqrt[3]{\frac{7}{4}}$
27. $\sqrt[3]{\frac{64}{49}}$
28. $\sqrt[4]{\frac{1296}{25}}$

In Exercises 29–36, write the expression in simplest form. *(See Example 4.)*

29. $\frac{1}{1 + \sqrt{3}}$
30. $\frac{1}{2 + \sqrt{5}}$
31. $\frac{5}{3 - \sqrt{2}}$
32. $\frac{11}{9 - \sqrt{6}}$
33. $\frac{9}{\sqrt{3} + \sqrt{7}}$
34. $\frac{2}{\sqrt{8} + \sqrt{7}}$
35. $\frac{\sqrt{6}}{\sqrt{3} - \sqrt{5}}$
36. $\frac{\sqrt{7}}{\sqrt{10} - \sqrt{2}}$

In Exercises 37–46, simplify the expression. *(See Example 5.)*

37. $9\sqrt[3]{11} + 3\sqrt[3]{11}$
38. $8\sqrt[6]{5} - 12\sqrt[6]{5}$
39. $3(11^{1/4}) + 9(11^{1/4})$
40. $13(8^{3/4}) - 4(8^{3/4})$
41. $5\sqrt{12} - 19\sqrt{3}$
42. $27\sqrt{6} + 7\sqrt{150}$
43. $\sqrt[5]{224} + 3\sqrt[5]{7}$
44. $7\sqrt[3]{2} - \sqrt[3]{128}$
45. $5(24^{1/3}) - 4(3^{1/3})$
46. $5^{1/4} + 6(405^{1/4})$

47. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

$$\begin{aligned} 3\sqrt[3]{12} + 5\sqrt[3]{12} &= (3 + 5)\sqrt[3]{24} \\ &= 8\sqrt[3]{24} \\ &= 8\sqrt[3]{8 \cdot 3} \\ &= 8 \cdot 2\sqrt[3]{3} \\ &= 16\sqrt[3]{3} \end{aligned}$$

48. **MULTIPLE REPRESENTATIONS** Which radical expressions are like radicals?

Ⓐ $(5^{2/9})^{3/2}$

Ⓑ $\dfrac{5^3}{(\sqrt[3]{5})^8}$

Ⓒ $\sqrt[3]{625}$

Ⓓ $\sqrt[3]{5145} - \sqrt[3]{875}$

Ⓔ $\sqrt[3]{5} + 3\sqrt[3]{5}$

Ⓕ $7\sqrt[4]{80} - 2\sqrt[4]{405}$

In Exercises 49–54, simplify the expression. *(See Example 6.)*

49. $\sqrt[4]{81y^8}$

50. $\sqrt[3]{64r^3t^6}$

51. $\sqrt[5]{\dfrac{m^{10}}{n^5}}$

52. $\sqrt[4]{\dfrac{k^{16}}{16z^4}}$

53. $\sqrt[6]{\dfrac{g^6h}{h^7}}$

54. $\sqrt[8]{\dfrac{n^{18}p^7}{n^2p^{-1}}}$

55. **ERROR ANALYSIS** Describe and correct the error in simplifying the expression.

$$\sqrt[6]{\frac{64h^{12}}{g^6}} = \frac{\sqrt[6]{64h^{12}}}{\sqrt[6]{g^6}} = \frac{\sqrt[6]{2^6 \cdot (h^2)^6}}{\sqrt[6]{g^6}} = \frac{2h^2}{g}$$

56. **OPEN-ENDED** Write two variable expressions involving radicals, one that needs absolute value in simplifying and one that does not need absolute value. Justify your answers.

In Exercises 57–64, write the expression in simplest form. Assume all variables are positive. *(See Example 7.)*

57. $\sqrt{81a^7b^{12}c^9}$

58. $\sqrt[3]{125r^4s^9t^7}$

59. $\sqrt[5]{\dfrac{160m^6}{n^7}}$

60. $\sqrt[4]{\dfrac{405x^3y^3}{5x^{-1}y}}$

61. $\dfrac{\sqrt[3]{w} \cdot \sqrt{w^5}}{\sqrt{25w^{16}}}$

62. $\dfrac{\sqrt[4]{v^6}}{\sqrt[7]{v^5}}$

63. $\dfrac{18w^{1/3}v^{5/4}}{27w^{4/3}v^{1/2}}$

64. $\dfrac{7x^{-3/4}y^{5/2}z^{-2/3}}{56x^{-1/2}y^{1/4}}$

In Exercises 65–70, perform the indicated operation. Assume all variables are positive. *(See Example 8.)*

65. $12\sqrt[3]{y} + 9\sqrt[3]{y}$

66. $11\sqrt{2z} - 5\sqrt{2z}$

67. $3x^{7/2} - 5x^{7/2}$

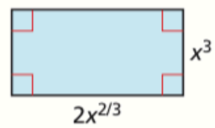
68. $7\sqrt[3]{m^7} + 3m^{7/3}$

69. $\sqrt[4]{16w^{10}} + 2w\sqrt[4]{w^6}$

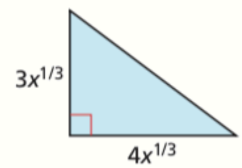
70. $(p^{1/2} \cdot p^{1/4}) - \sqrt[4]{16p^3}$

MATHEMATICAL CONNECTIONS **In Exercises 71 and 72, find simplified expressions for the perimeter and area of the given figure.**

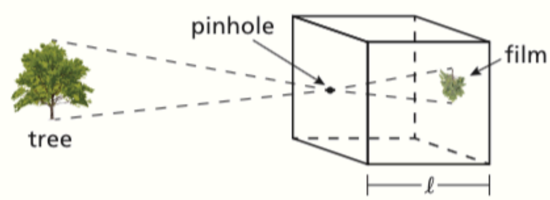
71.



72.



73. **MODELING WITH MATHEMATICS** The optimum diameter d (in millimeters) of the pinhole in a pinhole camera can be modeled by $d = 1.9[(5.5 \times 10^{-4})\ell]^{1/2}$, where ℓ is the length (in millimeters) of the camera box. Find the optimum pinhole diameter for a camera box with a length of 10 centimeters.



74. **MODELING WITH MATHEMATICS** The surface area S (in square centimeters) of a mammal can be modeled by $S = km^{2/3}$, where m is the mass (in grams) of the mammal and k is a constant. The table shows the values of k for different mammals.

Mammal	Rabbit	Human	Bat
Value of k	9.75	11.0	57.5

a. Find the surface area of a bat whose mass is 32 grams.

b. Find the surface area of a rabbit whose mass is 3.4 kilograms (3.4×10^3 grams).

c. Find the surface area of a human whose mass is 59 kilograms.

75. MAKING AN ARGUMENT Your friend claims it is not possible to simplify the expression $7\sqrt{11} - 9\sqrt{44}$ because it does not contain like radicals. Is your friend correct? Explain your reasoning.

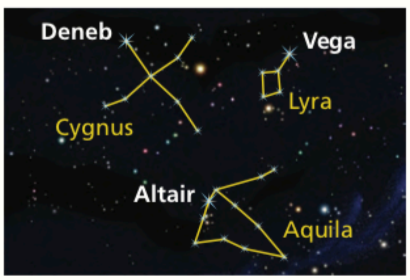
76. PROBLEM SOLVING The apparent magnitude of a star is a number that indicates how faint the star is in relation to other stars. The expression $\frac{2.512^{m_1}}{2.512^{m_2}}$ tells how many times fainter a star with apparent magnitude m_1 is than a star with apparent magnitude m_2.

Star	Apparent magnitude	Constellation
Vega	0.03	Lyra
Altair	0.77	Aquila
Deneb	1.25	Cygnus

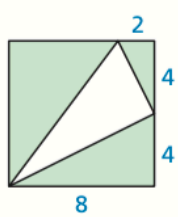
a. How many times fainter is Altair than Vega?

b. How many times fainter is Deneb than Altair?

c. How many times fainter is Deneb than Vega?

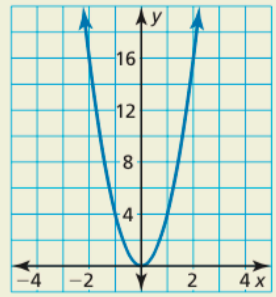


77. CRITICAL THINKING Find a radical expression for the perimeter of the triangle inscribed in the square shown. Simplify the expression.

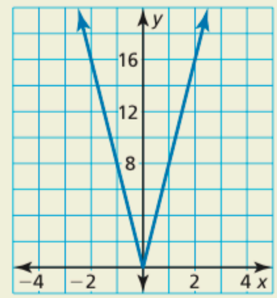


78. HOW DO YOU SEE IT? Without finding points, match the functions $f(x) = \sqrt{64x^2}$ and $g(x) = \sqrt[3]{64x^6}$ with their graphs. Explain your reasoning.

A.



B.



79. REWRITING A FORMULA You have filled two round balloons with water. One balloon contains twice as much water as the other balloon.

a. Solve the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, for r.

b. Substitute the expression for r from part (a) into the formula for the surface area of a sphere, $S = 4\pi r^2$. Simplify to show that $S = (4\pi)^{1/3}(3V)^{2/3}$.

c. Compare the surface areas of the two water balloons using the formula in part (b).

80. THOUGHT PROVOKING Determine whether the expressions $(x^2)^{1/6}$ and $(x^{1/6})^2$ are equivalent for all values of x.

81. DRAWING CONCLUSIONS Substitute different combinations of odd and even positive integers for m and n in the expression $\sqrt[n]{x^m}$. When you cannot assume x is positive, explain when absolute value is needed in simplifying the expression.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Identify the focus, directrix, and axis of symmetry of the parabola. Then graph the equation. *(Section 2.3)*

82. $y = 2x^2$ **83.** $y^2 = -x$ **84.** $y^2 = 4x$

Write a rule for g. Describe the graph of g as a transformation of the graph of f. *(Section 4.7)*

85. $f(x) = x^4 - 3x^2 - 2x,\ g(x) = -f(x)$

86. $f(x) = x^3 - x,\ g(x) = f(x) - 3$

87. $f(x) = x^3 - 4,\ g(x) = f(x - 2)$

88. $f(x) = x^4 + 2x^3 - 4x^2,\ g(x) = f(2x)$