5.4 Solving Radical Equations and Inequalities

What is a radical equation? Try to create an example.

Example 1: Solving Radical Equations

a) $2\sqrt{x+1} = 4$

b) $\sqrt[3]{2x-9} - 1 = 2$

Steps to solving radical equations.

Core Concept

Solving Radical Equations

To solve a radical equation, follow these steps:

Step 1 Isolate the radical on one side of the equation, if necessary.

Step 2 Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

Step 3 Solve the resulting equation using techniques you learned in previous chapters. Check your solution.

Example 2: Solving Real World Equations

In a hurricane, the mean sustained wind velocity v (in meters per second) can be modeled by $v(p) = 6.3\sqrt{1013 - p}$, where p is the air pressure (in millibars) at the center of the hurricane. Estimate the air pressure at the center of the hurricane when the mean sustained wind velocity is 54.5 meters per second.

Example 3: Solving Equations with an Extraneous Solution

Solve $x + 1 = \sqrt{7x + 15}$

Example 4: Solving an equation with two radicals

Solve $\sqrt{x + 2} + 1 = \sqrt{3 - x}$

Example 5: Solving an Equation with a Rational Exponent

Solve $(2x)^{3/4} + 2 = 10$

Example 6: Solving an equation with a rational exponent

Solve $(x + 30)^{1/2} = x$

Example 7: Solving a Radical Inequality

Solve $3\sqrt{x - 1} \leq 12$

Homework:
3, 7, 10, 13, 15, 17, 22, 26, 27, 31, 34, 37, 41, 44, (52 challenge), 54, 62

5.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** Is the equation $3x - \sqrt{2} = \sqrt{6}$ a radical equation? Explain your reasoning.

2. **WRITING** Explain the steps you should use to solve $\sqrt{x} + 10 < 15$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, solve the equation. Check your solution. *(See Example 1.)*

3. $\sqrt{5x + 1} = 6$

4. $\sqrt{3x + 10} = 8$

5. $\sqrt[3]{x - 16} = 2$

6. $\sqrt[3]{x} - 10 = -7$

7. $-2\sqrt{24x} + 13 = -11$

8. $8\sqrt[3]{10x} - 15 = 17$

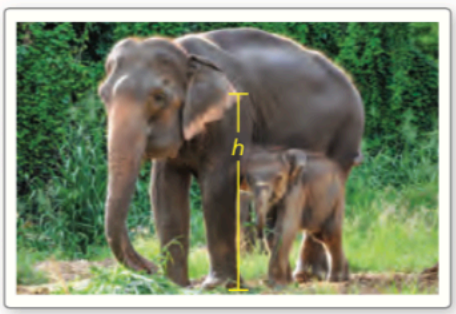
9. $\frac{1}{5}\sqrt[3]{3x} + 10 = 8$

10. $\sqrt{2x} - \frac{2}{3} = 0$

11. $2\sqrt[5]{x} + 7 = 15$

12. $\sqrt[4]{4x} - 13 = -15$

13. **MODELING WITH MATHEMATICS** Biologists have discovered that the shoulder height h (in centimeters) of a male Asian elephant can be modeled by $h = 62.5\sqrt[3]{t} + 75.8$, where t is the age (in years) of the elephant. Determine the age of an elephant with a shoulder height of 250 centimeters. *(See Example 2.)*



14. **MODELING WITH MATHEMATICS** In an amusement park ride, a rider suspended by cables swings back and forth from a tower. The maximum speed v (in meters per second) of the rider can be approximated by $v = \sqrt{2gh}$, where h is the height (in meters) at the top of each swing and g is the acceleration due to gravity ($g \approx 9.8$ m/sec^2). Determine the height at the top of the swing of a rider whose maximum speed is 15 meters per second.

In Exercises 15–26, solve the equation. Check your solution(s). *(See Examples 3 and 4.)*

15. $x - 6 = \sqrt{3x}$

16. $x - 10 = \sqrt{9x}$

17. $\sqrt{44 - 2x} = x - 10$

18. $\sqrt{2x + 30} = x + 3$

19. $\sqrt[3]{8x^3 - 1} = 2x - 1$

20. $\sqrt[4]{3 - 8x^2} = 2x$

21. $\sqrt{4x + 1} = \sqrt{x + 10}$

22. $\sqrt{3x - 3} - \sqrt{x + 12} = 0$

23. $\sqrt[3]{2x - 5} - \sqrt[3]{8x + 1} = 0$

24. $\sqrt[3]{x + 5} = 2\sqrt[3]{2x + 6}$

25. $\sqrt{3x - 8} + 1 = \sqrt{x + 5}$

26. $\sqrt{x + 2} = 2 - \sqrt{x}$

In Exercises 27–34, solve the equation. Check your solution(s). *(See Examples 5 and 6.)*

27. $2x^{2/3} = 8$

28. $4x^{3/2} = 32$

29. $x^{1/4} + 3 = 0$

30. $2x^{3/4} - 14 = 40$

31. $(x + 6)^{1/2} = x$

32. $(5 - x)^{1/2} - 2x = 0$

33. $2(x + 11)^{1/2} = x + 3$

34. $(5x^2 - 4)^{1/4} = x$

ERROR ANALYSIS **In Exercises 35 and 36, describe and correct the error in solving the equation.**

35.
$$\begin{aligned} \sqrt[3]{3x - 8} &= 4 \\ \left(\sqrt[3]{3x - 8}\right)^3 &= 4 \\ 3x - 8 &= 4 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

36.
$$\begin{aligned} 8x^{3/2} &= 1000 \\ 8(x^{3/2})^{2/3} &= 1000^{2/3} \\ 8x &= 100 \\ x &= \frac{25}{2} \end{aligned}$$

In Exercises 37–44, solve the inequality. *(See Example 7.)*

37. $2\sqrt[3]{x} - 5 \geq 3$

38. $\sqrt[3]{x - 4} \leq 5$

39. $4\sqrt{x - 2} > 20$

40. $7\sqrt{x} + 1 < 9$

41. $2\sqrt{x} + 3 \leq 8$

42. $\sqrt[3]{x + 7} \geq 3$

43. $-2\sqrt[3]{x + 4} < 12$

44. $-0.25\sqrt{x} - 6 \leq -3$

45. MODELING WITH MATHEMATICS The length ℓ (in inches) of a standard nail can be modeled by $\ell = 54d^{3/2}$, where d is the diameter (in inches) of the nail. What is the diameter of a standard nail that is 3 inches long?

46. DRAWING CONCLUSIONS "Hang time" is the time you are suspended in the air during a jump. Your hang time t (in seconds) is given by the function $t = 0.5\sqrt{h}$, where h is the height (in feet) of the jump. Suppose a kangaroo and a snowboarder jump with the hang times shown.

a. Find the heights that the snowboarder and the kangaroo jump.

b. Double the hang times of the snowboarder and the kangaroo and calculate the corresponding heights of each jump.

c. When the hang time doubles, does the height of the jump double? Explain.

USING TOOLS In Exercises 47–52, solve the nonlinear system. Justify your answer with a graph.

47. $y^2 = x - 3$
$y = x - 3$

48. $y^2 = 4x + 17$
$y = x + 5$

49. $x^2 + y^2 = 4$
$y = x - 2$

50. $x^2 + y^2 = 25$
$y = -\frac{3}{4}x + \frac{25}{4}$

51. $x^2 + y^2 = 1$
$y = \frac{1}{2}x^2 - 1$

52. $x^2 + y^2 = 4$
$y^2 = x + 2$

53. PROBLEM SOLVING The speed s (in miles per hour) of a car can be given by $s = \sqrt{30fd}$, where f is the coefficient of friction and d is the stopping distance (in feet). The table shows the coefficient of friction for different surfaces.

Surface	Coefficient of friction, f
dry asphalt	0.75
wet asphalt	0.30
snow	0.30
ice	0.15

a. Compare the stopping distances of a car traveling 45 miles per hour on the surfaces given in the table.

b. You are driving 35 miles per hour on an icy road when a deer jumps in front of your car. How far away must you begin to brake to avoid hitting the deer? Justify your answer.

54. MODELING WITH MATHEMATICS The Beaufort wind scale was devised to measure wind speed. The Beaufort numbers B, which range from 0 to 12, can be modeled by $B = 1.69\sqrt{s + 4.25} - 3.55$, where s is the wind speed (in miles per hour).

Beaufort number	Force of wind
0	calm
3	gentle breeze
6	strong breeze
9	strong gale
12	hurricane

a. What is the wind speed for $B = 0$? $B = 3$?

b. Write an inequality that describes the range of wind speeds represented by the Beaufort model.

55. USING TOOLS Solve the equation $x - 4 = \sqrt{2x}$. Then solve the equation $x - 4 = -\sqrt{2x}$.

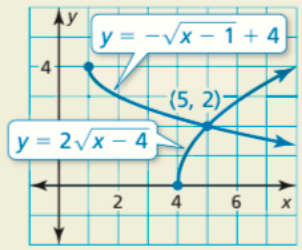
a. How does changing $\sqrt{2x}$ to $-\sqrt{2x}$ change the solution(s) of the equation?

b. Justify your answer in part (a) using graphs.

56. MAKING AN ARGUMENT Your friend says it is impossible for a radical equation to have two extraneous solutions. Is your friend correct? Explain your reasoning.

57. USING STRUCTURE Explain how you know the radical equation $\sqrt{x+4} = -5$ has no real solution without solving it.

58. HOW DO YOU SEE IT? Use the graph to find the solution of the equation $2\sqrt{x-4} = -\sqrt{x-1} + 4$. Explain your reasoning.



59. WRITING A company determines that the price p of a product can be modeled by $p = 70 - \sqrt{0.02x + 1}$, where x is the number of units of the product demanded per day. Describe the effect that raising the price has on the number of units demanded.

60. THOUGHT PROVOKING City officials rope off a circular area to prepare for a concert in the park. They estimate that each person occupies 6 square feet. Describe how you can use a radical inequality to determine the possible radius of the region when P people are expected to attend the concert.

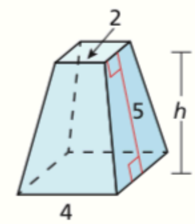
61. MATHEMATICAL CONNECTIONS The Moeraki Boulders along the coast of New Zealand are stone spheres with radii of approximately 3 feet. A formula for the radius of a sphere is

$$r = \frac{1}{2}\sqrt{\frac{S}{\pi}}$$

where S is the surface area of the sphere. Find the surface area of a Moeraki Boulder.

62. PROBLEM SOLVING You are trying to determine the height of a truncated pyramid, which cannot be measured directly. The height h and slant height ℓ of the truncated pyramid are related by the formula below.

$$\ell = \sqrt{h^2 + \frac{1}{4}(b_2 - b_1)^2}$$



In the given formula, b_1 and b_2 are the side lengths of the upper and lower bases of the pyramid, respectively. When $\ell = 5$, $b_1 = 2$, and $b_2 = 4$, what is the height of the pyramid?

63. REWRITING A FORMULA A burning candle has a radius of r inches and was initially h_0 inches tall. After t minutes, the height of the candle has been reduced to h inches. These quantities are related by the formula

$$r = \sqrt{\frac{kt}{\pi(h_0 - h)}}$$

where k is a constant. Suppose the radius of a candle is 0.875 inch, its initial height is 6.5 inches, and $k = 0.04$.

a. Rewrite the formula, solving for h in terms of t.

b. Use your formula in part (a) to determine the height of the candle after burning 45 minutes.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Perform the indicated operation. *(Section 4.2 and Section 4.3)*

64. $(x^3 - 2x^2 + 3x + 1) + (x^4 - 7x)$

65. $(2x^5 + x^4 - 4x^2) - (x^5 - 3)$

66. $(x^3 + 2x^2 + 1)(x^2 + 5)$

67. $(x^4 + 2x^3 + 11x^2 + 14x - 16) \div (x + 2)$

Let $f(x) = x^3 - 4x^2 + 6$. Write a rule for g. Describe the graph of g as a transformation of the graph of f. *(Section 4.7)*

68. $g(x) = f(-x) + 4$

69. $g(x) = \frac{1}{2}f(x) - 3$

70. $g(x) = -f(x - 1) + 6$