

5.5 Performing Function Operations

Do Now: Solve the Inequality

$$4\sqrt{x-2} > 20$$

So far we have learned how to add, subtract, multiple and divide polynomials. Now let us learn how to apply these rules to functions!

Core Concept

Operations on Functions

Let f and g be any two functions. A new function can be defined by performing any of the four basic operations on f and g .

| Operation | Definition | Example: $f(x) = 5x$, $g(x) = x + 2$ |
|----------------|---|--|
| Addition | $(f + g)(x) = f(x) + g(x)$ | $(f + g)(x) = 5x + (x + 2) = 6x + 2$ |
| Subtraction | $(f - g)(x) = f(x) - g(x)$ | $(f - g)(x) = 5x - (x + 2) = 4x - 2$ |
| Multiplication | $(fg)(x) = f(x) \cdot g(x)$ | $(fg)(x) = 5x(x + 2) = 5x^2 + 10x$ |
| Division | $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ | $\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$ |

The domains of the sum, difference, product, and quotient functions consist of the x -values that are in the domains of both f and g . Additionally, the domain of the quotient does not include x -values for which $g(x) = 0$.

Example 1: Adding Two Functions

Let $f(x) = 3\sqrt{x}$ and $g(x) = -10\sqrt{x}$. Find $(f + g)(x)$ and state the domain. Then evaluate $(f + g)(4)$.

Example 2: Subtracting Two Functions

Let $f(x) = 3x^3 - 2x^2 + 5$ and $g(x) = x^3 - 3x^2 + 4x - 2$. Find $(f - g)(x)$ and state the domain. Then evaluate $(f - g)(-2)$.

Example 3: Multiplying Two Functions

Let $f(x) = x^2$ and $g(x) = \sqrt{x}$. Find $(fg)(x)$ and state the domain. Then evaluate $(fg)(9)$

Example 4: Dividing Two Functions

Let $f(x) = 6x$ and $g(x) = x^{3/4}$. Find $\left(\frac{f}{g}\right)(x)$ and state the domain. Then evaluate $\left(\frac{f}{g}\right)(16)$

Example 5: Performing Function Operations Using Technology

Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{9 - x^2}$. Use a graphing calculator to evaluate

$(f + g)(x)$, $(f - g)(x)$, $(fg)(x)$, & $\left(\frac{f}{g}\right)(x)$ when $x = 2$. Round to the nearest hundredth.

Example 6: Performing Function Operations

Let $f(x) = x + 4$ and $g(x) = x^2 - 1$. Find $(f \circ g)(x)$ and state the domain. Then evaluate $(f \circ g)(2)$