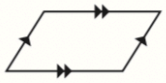
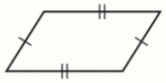
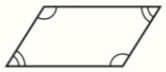
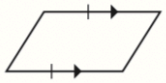
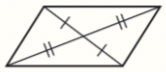
7.3 Proving that a Quadrilateral is a Parallelogram

Now that we know about parallelograms,
what are ways we can prove that a quadrilateral is a parallelogram?

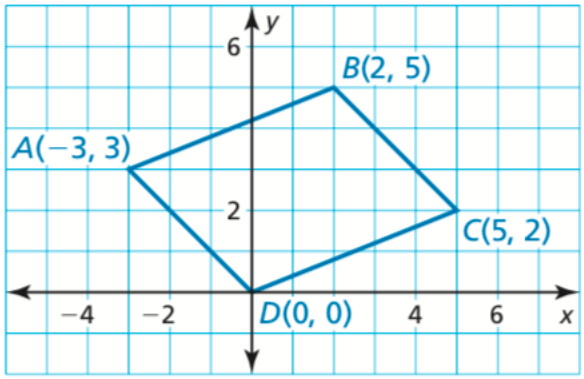
Concept Summary

Ways to Prove a Quadrilateral Is a Parallelogram

1. Show that both pairs of opposite sides are parallel. *(Definition)*	
2. Show that both pairs of opposite sides are congruent. *(Parallelogram Opposite Sides Converse)*	
3. Show that both pairs of opposite angles are congruent. *(Parallelogram Opposite Angles Converse)*	
4. Show that one pair of opposite sides are congruent and parallel. *(Opposite Sides Parallel and Congruent Theorem)*	
5. Show that the diagonals bisect each other. *(Parallelogram Diagonals Converse)*	

Example 1: Identifying a Parallelogram in the Coordinate Plane

Show 2 DIFFERENT ways that quad ABCD is a parallelogram:



Homework: 3-8, 11, 12, 17, 18, 23, 30, 35, 36, 37, 41

7.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** A quadrilateral has four congruent sides. Is the quadrilateral a parallelogram? Justify your answer.

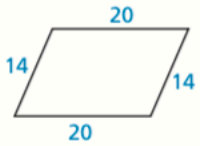
2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

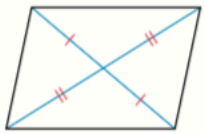
Construct a quadrilateral with opposite sides congruent.	Construct a quadrilateral with one pair of parallel sides.
Construct a quadrilateral with opposite angles congruent.	Construct a quadrilateral with one pair of opposite sides congruent and parallel.

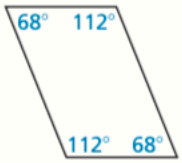
Monitoring Progress and Modeling with Mathematics

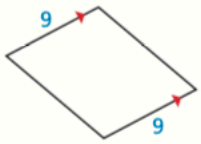
In Exercises 3–8, state which theorem you can use to show that the quadrilateral is a parallelogram. *(See Examples 1 and 3.)*

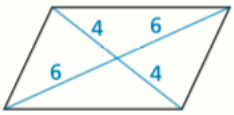
3. 

4. 

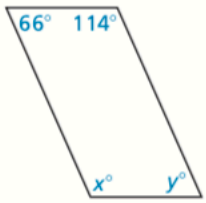
5. 

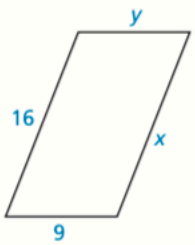
6. 

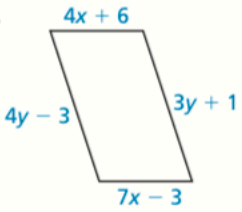
7. 

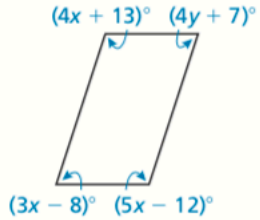
8. 

In Exercises 9–12, find the values of x and y that make the quadrilateral a parallelogram. *(See Example 2.)*

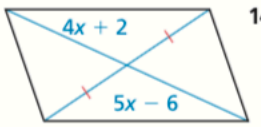
9. 

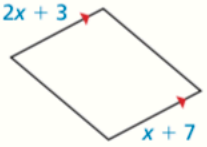
10. 

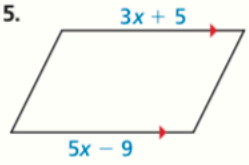
11. 

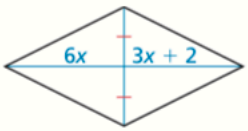
12. 

In Exercises 13–16, find the value of x that makes the quadrilateral a parallelogram. *(See Example 4.)*

13. 

14. 

15. 

16. 

In Exercises 17–20, graph the quadrilateral with the given vertices in a coordinate plane. Then show that the quadrilateral is a parallelogram. *(See Example 5.)*

17. $A(0, 1)$, $B(4, 4)$, $C(12, 4)$, $D(8, 1)$

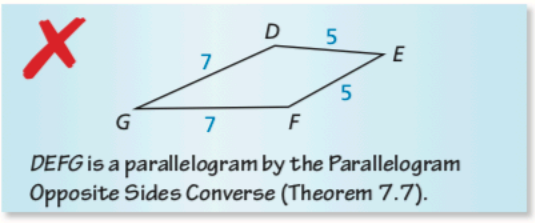
18. $E(-3, 0)$, $F(-3, 4)$, $G(3, -1)$, $H(3, -5)$

19. $J(-2, 3)$, $K(-5, 7)$, $L(3, 6)$, $M(6, 2)$

20. $N(-5, 0)$, $P(0, 4)$, $Q(3, 0)$, $R(-2, -4)$

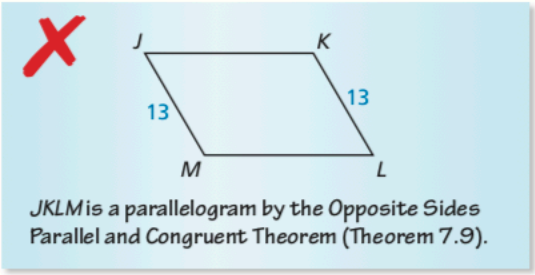
ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in identifying a parallelogram.**

21.



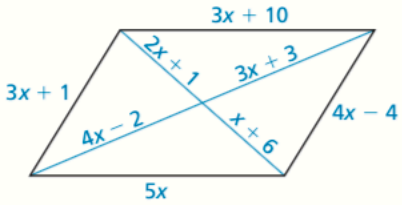
DEFG is a parallelogram by the Parallelogram Opposite Sides Converse (Theorem 7.7).

22.

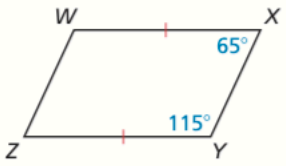


JKLM is a parallelogram by the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9).

23. **MATHEMATICAL CONNECTIONS** What value of x makes the quadrilateral a parallelogram? Explain how you found your answer.



24. **MAKING AN ARGUMENT** Your friend says you can show that quadrilateral $WXYZ$ is a parallelogram by using the Consecutive Interior Angles Converse (Theorem 3.8) and the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9). Is your friend correct? Explain your reasoning.



ANALYZING RELATIONSHIPS **In Exercises 25–27, write the indicated theorems as a biconditional statement.**

25. Parallelogram Opposite Sides Theorem (Theorem 7.3) and Parallelogram Opposite Sides Converse (Theorem 7.7)

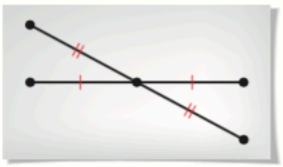
26. Parallelogram Opposite Angles Theorem (Theorem 7.4) and Parallelogram Opposite Angles Converse (Theorem 7.8)

27. Parallelogram Diagonals Theorem (Theorem 7.6) and Parallelogram Diagonals Converse (Theorem 7.10)

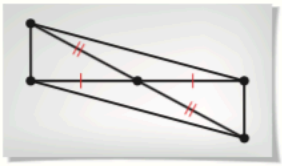
28. **CONSTRUCTION** Describe a method that uses the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9) to construct a parallelogram. Then construct a parallelogram using your method.

29. **REASONING** Follow the steps below to construct a parallelogram. Explain why this method works. State a theorem to support your answer.

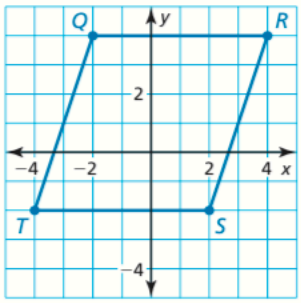
Step 1 Use a ruler to draw two segments that intersect at their midpoints.



Step 2 Connect the endpoints of the segments to form a parallelogram.

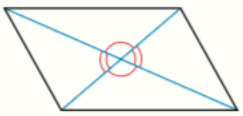


30. **MAKING AN ARGUMENT** Your brother says to show that quadrilateral $QRST$ is a parallelogram, you must show that $\overline{QR} \parallel \overline{TS}$ and $\overline{QT} \parallel \overline{RS}$. Your sister says that you must show that $\overline{QR} \cong \overline{TS}$ and $\overline{QT} \cong \overline{RS}$. Who is correct? Explain your reasoning.

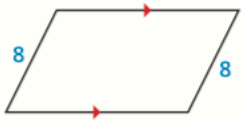


REASONING **In Exercises 31 and 32, your classmate incorrectly claims that the marked information can be used to show that the figure is a parallelogram. Draw a quadrilateral with the same marked properties that is clearly *not* a parallelogram.**

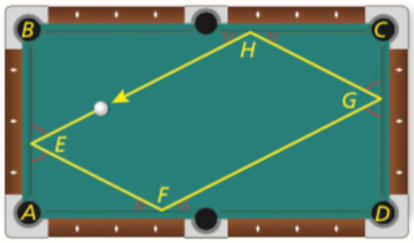
31.



32.



33. MODELING WITH MATHEMATICS You shoot a pool ball, and it rolls back to where it started, as shown in the diagram. The ball bounces off each wall at the same angle at which it hits the wall.



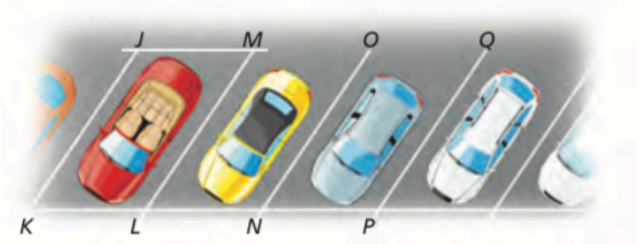
a. The ball hits the first wall at an angle of 63°. So $m\angle AEF = m\angle BEH = 63°$. What is $m\angle AFE$? Explain your reasoning.

b. Explain why $m\angle FGD = 63°$.

c. What is $m\angle GHC$? $m\angle EHB$?

d. Is quadrilateral $EFGH$ a parallelogram? Explain your reasoning.

34. MODELING WITH MATHEMATICS In the diagram of the parking lot shown, $m\angle JKL = 60°$, $JK = LM = 21$ feet, and $KL = JM = 9$ feet.



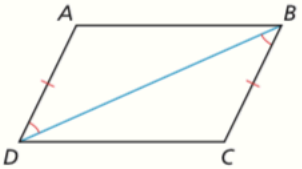
a. Explain how to show that parking space $JKLM$ is a parallelogram.

b. Find $m\angle JML$, $m\angle KJM$, and $m\angle KLM$.

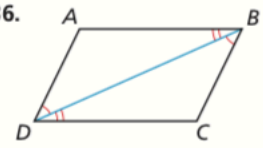
c. $\overline{LM} \parallel \overline{NO}$ and $\overline{NO} \parallel \overline{PQ}$. Which theorem could you use to show that $\overline{JK} \parallel \overline{PQ}$?

REASONING In Exercises 35–37, describe how to prove that $ABCD$ is a parallelogram.

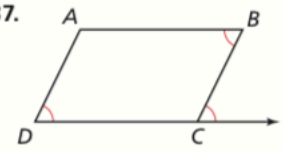
35.



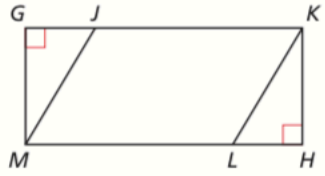
36.



37.



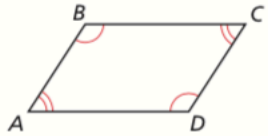
38. REASONING Quadrilateral $JKLM$ is a parallelogram. Describe how to prove that $\triangle MGJ \cong \triangle KHL$.



39. PROVING A THEOREM Prove the Parallelogram Opposite Angles Converse (Theorem 7.8). (*Hint*: Let $x°$ represent $m\angle A$ and $m\angle C$. Let $y°$ represent $m\angle B$ and $m\angle D$. Write and simplify an equation involving x and y.)

Given $\angle A \cong \angle C$, $\angle B \cong \angle D$

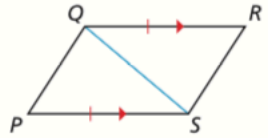
Prove $ABCD$ is a parallelogram.



40. PROVING A THEOREM Use the diagram of $PQRS$ with the auxiliary line segment drawn to prove the Opposite Sides Parallel and Congruent Theorem (Theorem 7.9).

Given $\overline{QR} \parallel \overline{PS}$, $\overline{QR} \cong \overline{PS}$

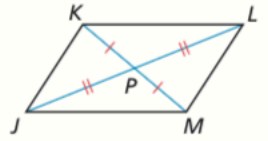
Prove $PQRS$ is a parallelogram.



41. PROVING A THEOREM Prove the Parallelogram Diagonals Converse (Theorem 7.10).

Given Diagonals $\overline{JL}$ and $\overline{KM}$ bisect each other.

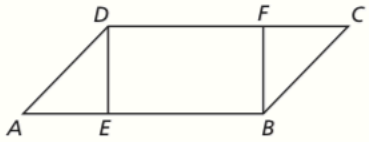
Prove $JKLM$ is a parallelogram.



42. PROOF Write a proof.

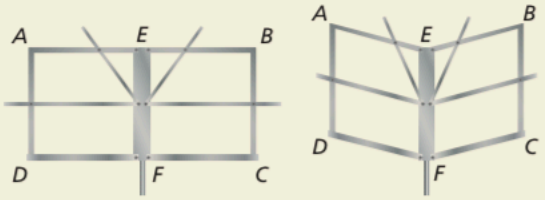
Given $DEBF$ is a parallelogram.
$AE = CF$

Prove $ABCD$ is a parallelogram.

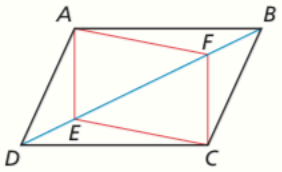


43. REASONING Three interior angle measures of a quadrilateral are 67°, 67°, and 113°. Is this enough information to conclude that the quadrilateral is a parallelogram? Explain your reasoning.

44. HOW DO YOU SEE IT? A music stand can be folded up, as shown. In the diagrams, $AEFD$ and $EBCF$ are parallelograms. Which labeled segments remain parallel as the stand is folded?



45. CRITICAL THINKING In the diagram, $ABCD$ is a parallelogram, $BF = DE = 12$, and $CF = 8$. Find AE. Explain your reasoning.



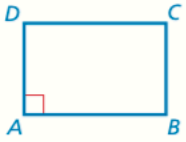
46. THOUGHT PROVOKING Create a regular hexagon using congruent parallelograms.

47. WRITING The Parallelogram Consecutive Angles Theorem (Theorem 7.5) says that if a quadrilateral is a parallelogram, then its consecutive angles are supplementary. Write the converse of this theorem. Then write a plan for proving the converse. Include a diagram.

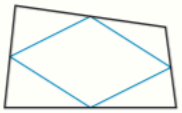
48. PROOF Write a proof.

Given $ABCD$ is a parallelogram.
$\angle A$ is a right angle.

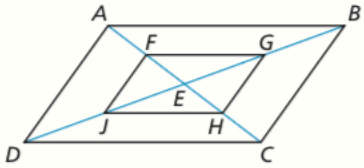
Prove $\angle B$, $\angle C$, and $\angle D$ are right angles.



49. ABSTRACT REASONING The midpoints of the sides of a quadrilateral have been joined to form what looks like a parallelogram. Show that a quadrilateral formed by connecting the midpoints of the sides of any quadrilateral is always a parallelogram. (*Hint*: Draw a diagram. Include a diagonal of the larger quadrilateral. Show how two sides of the smaller quadrilateral relate to the diagonal.)



50. CRITICAL THINKING Show that if $ABCD$ is a parallelogram with its diagonals intersecting at E, then you can connect the midpoints F, G, H, and J of $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, and $\overline{DE}$, respectively, to form another parallelogram, $FGHJ$.

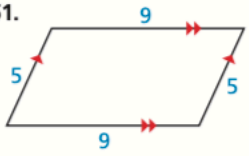


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

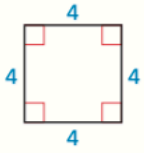
Classify the quadrilateral. *(Skills Review Handbook)*

51.



52.

53.



54.