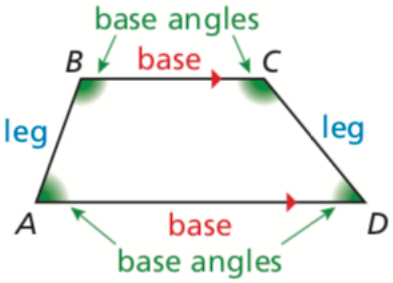
7.5 Properties of Trapezoids and Kites

Do Now: Sketch the following

1) Trapezoid 2) Kite 3) Isosceles Trapezoid

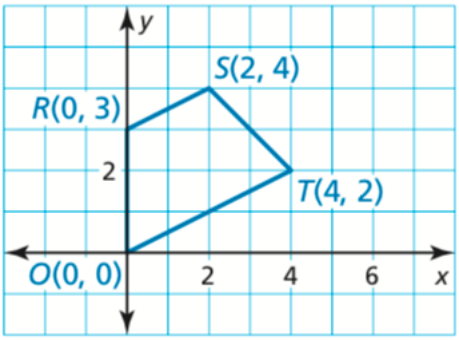
Parts of a Trapezoid

A quadrilateral with exactly one pair of parallel sides is called a **Trapezoid**



Example 1: Identifying a Trapezoid in the Coordinate Plane

Show that ORST is a trapezoid.
Is it isosceles?



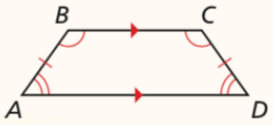
Theorems

Theorem 7.14 Isosceles Trapezoid Base Angles Theorem

If a trapezoid is isosceles, then each pair of base angles is congruent.

If trapezoid $ABCD$ is isosceles, then $\angle A \cong \angle D$ and $\angle B \cong \angle C$.

Proof Ex. 39, p. 405

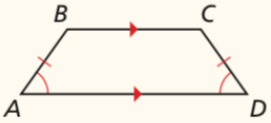


Theorem 7.15 Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.

If $\angle A \cong \angle D$ (or if $\angle B \cong \angle C$), then trapezoid $ABCD$ is isosceles.

Proof Ex. 40, p. 405

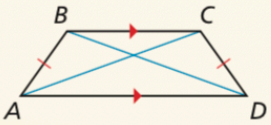


Theorem 7.16 Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

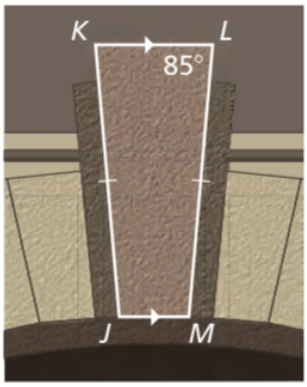
Trapezoid $ABCD$ is isosceles if and only if $\overline{AC} \cong \overline{BD}$.

Proof Ex. 51, p. 406



Example 2: Using properties of Isosceles

The stone above the arch in the diagram is an isosceles trapezoid. Determine the measures of angle K, M, & J.



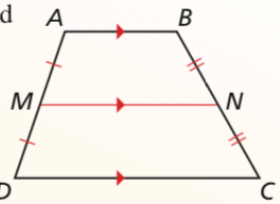
Theorem

Theorem 7.17 Trapezoid Midsegment Theorem

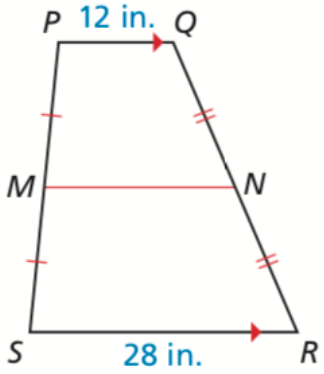
The midsegment of a trapezoid is parallel to each base, and its length is one-half the sum of the lengths of the bases.

If $\overline{MN}$ is the midsegment of trapezoid $ABCD$, then $\overline{MN} \parallel \overline{AB}, \overline{MN} \parallel \overline{DC}$, and $MN = \frac{1}{2}(AB + CD)$.

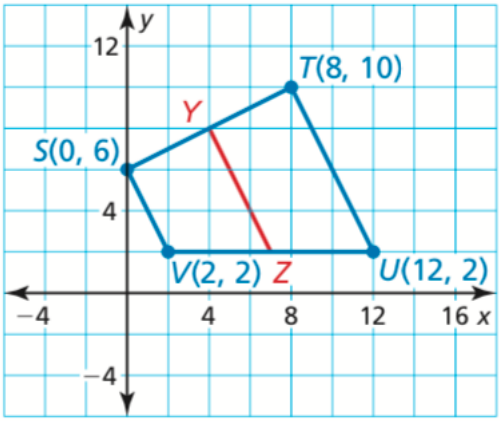
Proof Ex. 49, p. 406



Example 3: Using the Midsegment of a Trapezoid.
In the diagram, MN is the midsegment of trapezoid PQRS. Find MN.



Example 4: Using midsegments in a coordinate plane.
Find the length of midsegment YZ in trapezoid STUV.



What is a kite?
A quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.

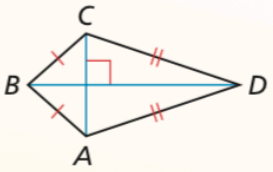
Theorems

Theorem 7.18 Kite Diagonals Theorem

If a quadrilateral is a kite, then its diagonals are perpendicular.

If quadrilateral $ABCD$ is a kite, then $\overline{AC} \perp \overline{BD}$.

Proof p. 401

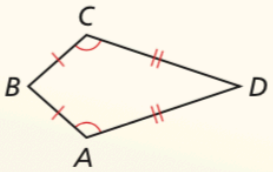


Theorem 7.19 Kite Opposite Angles Theorem

If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent.

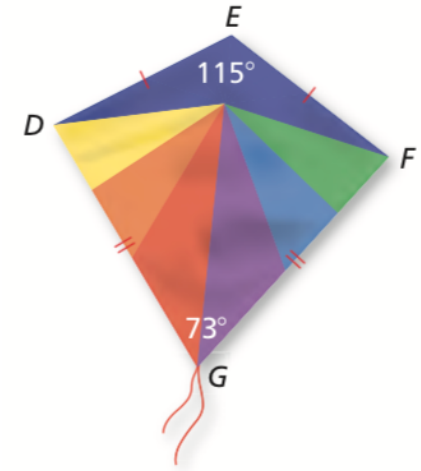
If quadrilateral $ABCD$ is a kite and $\overline{BC} \cong \overline{BA}$, then $\angle A \cong \angle C$ and $\angle B \ncong \angle D$.

Proof Ex. 47, p. 406

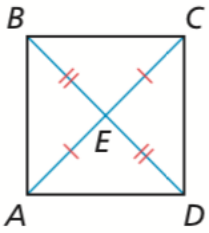


Example 5: Finding angle measures in a kite

Find the measure of angle D in the kite shown.

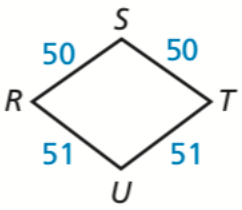


Example 6: Identifying a Quadrilateral
What is the most specific name for quadrilateral ABCD?

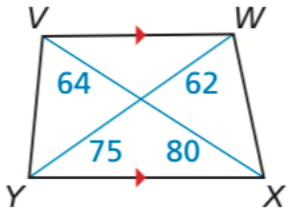


Try on your own.

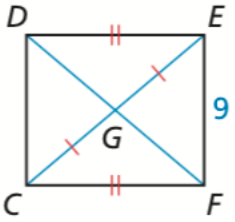
8.



9.



10.



Homework: 7-12, 13, 16, 17, 21-24, 30*, 35,

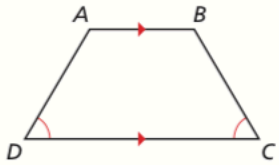
7.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Describe the differences between a trapezoid and a kite.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.



Is there enough information to prove that trapezoid $ABCD$ is isosceles?

Is there enough information to prove that $\overline{AB} \cong \overline{DC}$?

Is there enough information to prove that the non-parallel sides of trapezoid $ABCD$ are congruent?

Is there enough information to prove that the legs of trapezoid $ABCD$ are congruent?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, show that the quadrilateral with the given vertices is a trapezoid. Then decide whether it is isosceles. *(See Example 1.)*

3. $W(1, 4), X(1, 8), Y(-3, 9), Z(-3, 3)$

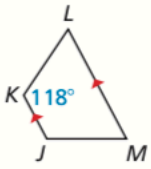
4. $D(-3, 3), E(-1, 1), F(1, -4), G(-3, 0)$

5. $M(-2, 0), N(0, 4), P(5, 4), Q(8, 0)$

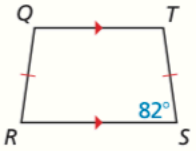
6. $H(1, 9), J(4, 2), K(5, 2), L(8, 9)$

In Exercises 7 and 8, find the measure of each angle in the isosceles trapezoid. *(See Example 2.)*

7.

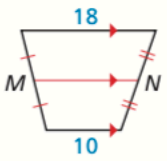


8.

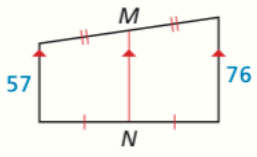


In Exercises 9 and 10, find the length of the midsegment of the trapezoid. *(See Example 3.)*

9.

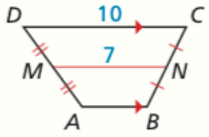


10.

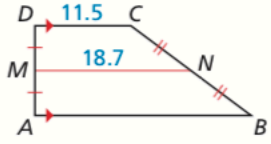


In Exercises 11 and 12, find AB.

11.



12.



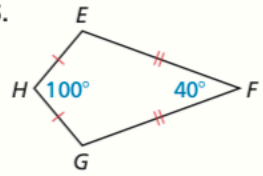
In Exercises 13 and 14, find the length of the midsegment of the trapezoid with the given vertices. *(See Example 4.)*

13. $A(2, 0), B(8, -4), C(12, 2), D(0, 10)$

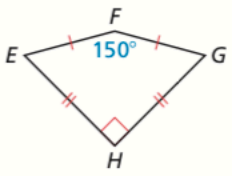
14. $S(-2, 4), T(-2, -4), U(3, -2), V(13, 10)$

In Exercises 15–18, find $m\angle G$. *(See Example 5.)*

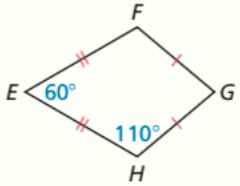
15.



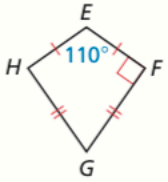
16.



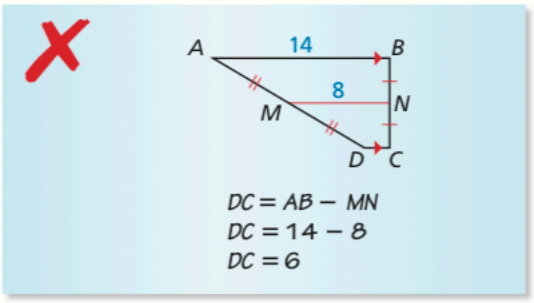
17.



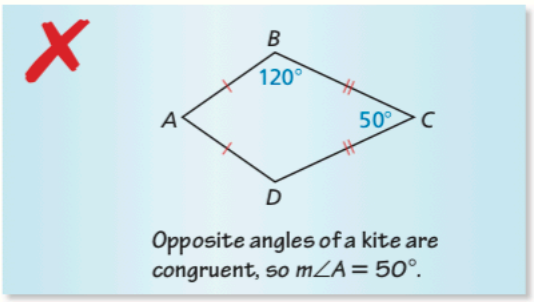
18.



19. ERROR ANALYSIS Describe and correct the error in finding DC.



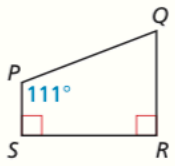
20. ERROR ANALYSIS Describe and correct the error in finding $m\angle A$.



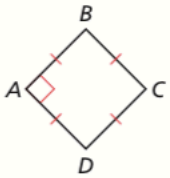
In Exercises 21–24, give the most specific name for the quadrilateral. Explain your reasoning. *(See Example 6.)*

21.

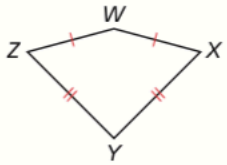
22.



23.

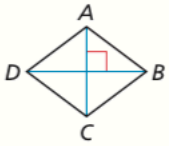


24.

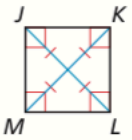


REASONING In Exercises 25 and 26, tell whether enough information is given in the diagram to classify the quadrilateral by the indicated name. Explain.

25. rhombus

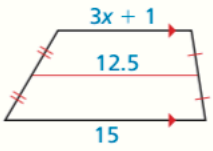


26. square

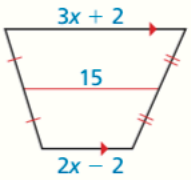


MATHEMATICAL CONNECTIONS In Exercises 27 and 28, find the value of x.

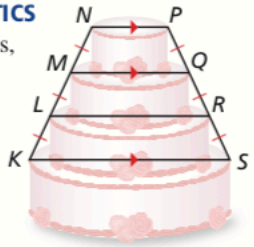
27.



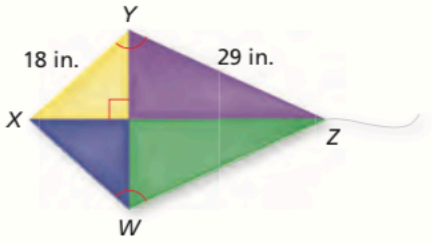
28.



29. MODELING WITH MATHEMATICS In the diagram, $NP = 8$ inches, and $LR = 20$ inches. What is the diameter of the bottom layer of the cake?

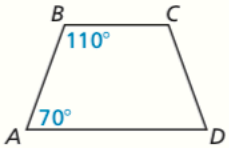


30. PROBLEM SOLVING You and a friend are building a kite. You need a stick to place from X to W and a stick to place from W to Z to finish constructing the frame. You want the kite to have the geometric shape of a kite. How long does each stick need to be? Explain your reasoning.

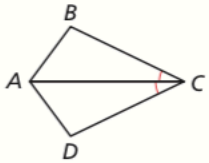


REASONING In Exercises 31–34, determine which pairs of segments or angles must be congruent so that you can prove that $ABCD$ is the indicated quadrilateral. Explain your reasoning. (There may be more than one right answer.)

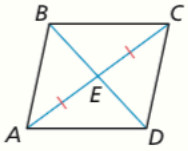
31. isosceles trapezoid



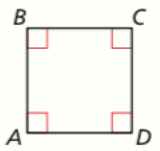
32. kite



33. parallelogram



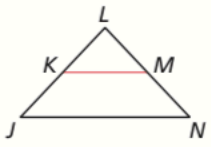
34. square



35. PROOF Write a proof.

Given $\overline{JL} \cong \overline{LN}, \overline{KM}$ is a midsegment of $\triangle JLN$.

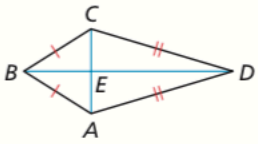
Prove Quadrilateral $JKMN$ is an isosceles trapezoid.



36. PROOF Write a proof.

Given $ABCD$ is a kite.
$\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

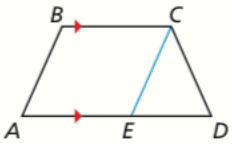
Prove $\overline{CE} \cong \overline{AE}$



37. ABSTRACT REASONING Point U lies on the perpendicular bisector of $\overline{RT}$. Describe the set of points S for which $RSTU$ is a kite.

38. REASONING Determine whether the points $A(4, 5)$, $B(-3, 3)$, $C(-6, -13)$, and $D(6, -2)$ are the vertices of a kite. Explain your reasoning,

PROVING A THEOREM In Exercises 39 and 40, use the diagram to prove the given theorem. In the diagram, $\overline{EC}$ is drawn parallel to $\overline{AB}$.



39. Isosceles Trapezoid Base Angles Theorem (Theorem 7.14)

Given $ABCD$ is an isosceles trapezoid.
$\overline{BC} \parallel \overline{AD}$

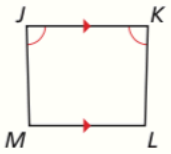
Prove $\angle A \cong \angle D$, $\angle B \cong \angle BCD$

40. Isosceles Trapezoid Base Angles Converse (Theorem 7.15)

Given $ABCD$ is a trapezoid.
$\angle A \cong \angle D$, $\overline{BC} \parallel \overline{AD}$

Prove $ABCD$ is an isosceles trapezoid.

41. MAKING AN ARGUMENT Your cousin claims there is enough information to prove that $JKLM$ is an isosceles trapezoid. Is your cousin correct? Explain.



42. MATHEMATICAL CONNECTIONS The bases of a trapezoid lie on the lines $y = 2x + 7$ and $y = 2x - 5$. Write the equation of the line that contains the midsegment of the trapezoid.

43. CONSTRUCTION $\overline{AC}$ and $\overline{BD}$ bisect each other.

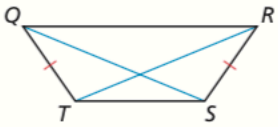
a. Construct quadrilateral $ABCD$ so that $\overline{AC}$ and $\overline{BD}$ are congruent, but not perpendicular. Classify the quadrilateral. Justify your answer.

b. Construct quadrilateral $ABCD$ so that $\overline{AC}$ and $\overline{BD}$ are perpendicular, but not congruent. Classify the quadrilateral. Justify your answer.

44. PROOF Write a proof.

Given $QRST$ is an isosceles trapezoid.

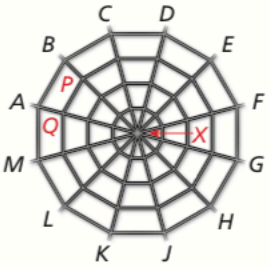
Prove $\angle TQS \cong \angle SRT$



45. MODELING WITH MATHEMATICS A plastic spiderweb is made in the shape of a regular dodecagon (12-sided polygon). $\overline{AB} \parallel \overline{PQ}$, and X is equidistant from the vertices of the dodecagon.

a. Are you given enough information to prove that $ABPQ$ is an isosceles trapezoid?

b. What is the measure of each interior angle of $ABPQ$?



46. ATTENDING TO PRECISION In trapezoid $PQRS$, $\overline{PQ} \parallel \overline{RS}$ and $\overline{MN}$ is the midsegment of $PQRS$. If $RS = 5 \cdot PQ$, what is the ratio of MN to RS?

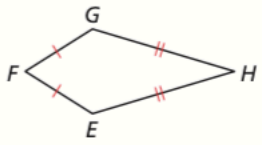
Ⓐ 3 : 5 Ⓑ 5 : 3

Ⓒ 1 : 2 Ⓓ 3 : 1

47. PROVING A THEOREM Use the plan for proof below to write a paragraph proof of the Kite Opposite Angles Theorem (Theorem 7.19).

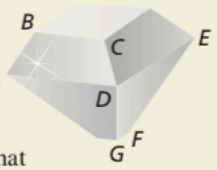
Given $EFGH$ is a kite.
$\overline{EF} \cong \overline{FG}, \overline{EH} \cong \overline{GH}$

Prove $\angle E \cong \angle G, \angle F \ncong \angle H$



Plan for Proof First show that $\angle E \cong \angle G$. Then use an indirect argument to show that $\angle F \ncong \angle H$.

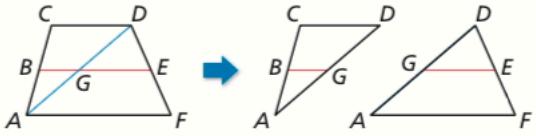
48. HOW DO YOU SEE IT? One of the earliest shapes used for cut diamonds is called the *table cut*, as shown in the figure. Each face of a cut gem is called a *facet*.



a. $\overline{BC} \parallel \overline{AD}$, and $\overline{AB}$ and $\overline{DC}$ are not parallel. What shape is the facet labeled $ABCD$?

b. $\overline{DE} \parallel \overline{GF}$, and $\overline{DG}$ and $\overline{EF}$ are congruent but not parallel. What shape is the facet labeled $DEFG$?

49. PROVING A THEOREM In the diagram below, $\overline{BG}$ is the midsegment of $\triangle ACD$, and $\overline{GE}$ is the midsegment of $\triangle ADF$. Use the diagram to prove the Trapezoid Midsegment Theorem (Theorem 7.17).



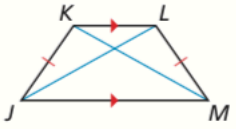
50. THOUGHT PROVOKING Is SSASS a valid congruence theorem for kites? Justify your answer.

51. PROVING A THEOREM To prove the biconditional statement in the Isosceles Trapezoid Diagonals Theorem (Theorem 7.16), you must prove both parts separately.

a. Prove part of the Isosceles Trapezoid Diagonals Theorem (Theorem 7.16).

Given $JKLM$ is an isosceles trapezoid.
$\overline{KL} \parallel \overline{JM}, \overline{JK} \cong \overline{LM}$

Prove $\overline{JL} \cong \overline{KM}$

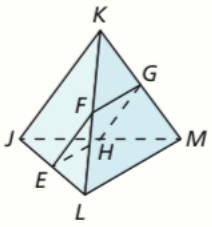


b. Write the other part of the Isosceles Trapezoid Diagonals Theorem (Theorem 7.16) as a conditional. Then prove the statement is true.

52. PROOF What special type of quadrilateral is $EFGH$? Write a proof to show that your answer is correct.

Given In the three-dimensional figure, $\overline{JK} \cong \overline{LM}$. E, F, G, and H are the midpoints of $\overline{JL}$, $\overline{KL}$, $\overline{KM}$, and $\overline{JM}$, respectively.

Prove $EFGH$ is a _____.

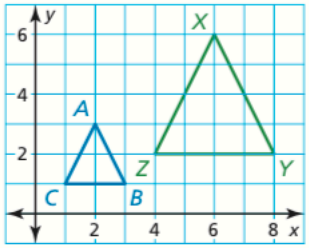


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Describe a similarity transformation that maps the blue preimage to the green image. *(Section 4.6)*

53.



54.