6.2 The Natural Base e

Mini Lab: Determining the value of The Natural Base e

We have currently learned about compound interest $A = P\left(1+\frac{1}{n}\right)^{nt}$. Complete the chart below, round all answers two 4 decimal places.

Compounded ______ times	n=	$\left(1+\frac{1}{n}\right)^{n} =$
Annually	1	2
Biannually	2	2.25
Quarterly		
Monthly		
Weekly		
Daily		
Hourly		
Minutely		
Every Second		
Every Half a Second		

What happens as you increase the frequency of being compounded?

The Natural Base e

There are numbers, such as π *and* i, that mathematicians call special. Another special number is denoted by the letter e. We call this **The Natural Base e.**

Core Concept

The Natural Base e

The natural base e is irrational. It is defined as follows:

As x approaches $+\infty$, $\left(1+\frac{1}{x}\right)^{x}$ approaches $e \approx 2.71828182846$.

Example 1: Simplifying Natural Base Expressions

Simplify each expression.

a) $e^3 \cdot e^5$

b) $\frac{16e^5}{4e^4}$

c) $(3e^{-4x})^2$

Graphing Natural Base e

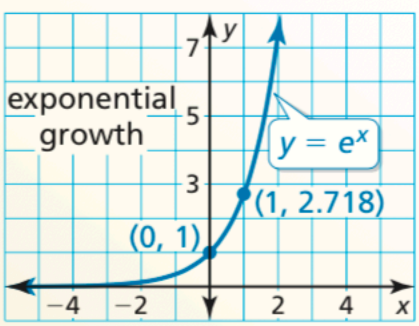
Core Concept

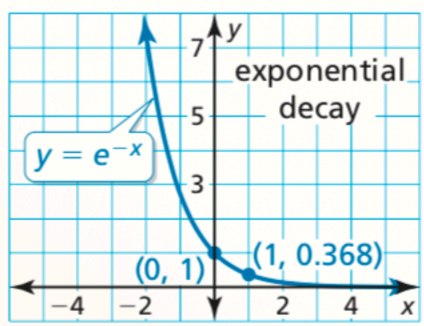
Natural Base Functions

A function of the form $y = ae^{rx}$ is called a *natural base exponential function*.

- When $a > 0$ and $r > 0$, the function is an exponential growth function.
- When $a > 0$ and $r < 0$, the function is an exponential decay function.

The graphs of the basic functions $y = e^x$ and $y = e^{-x}$ are shown.





Example 2: Graphing Natural Base Functions
Tell whether each function represents exponential growth or decay.

a) $y = 3e^x$

b) $f(x) = e^{-0.5x}$

Thinking back to the Mini Lab:

We have learned that $A = P\left(1 + \frac{1}{n}\right)^{nt}$ can be used to determine the balance of an account earning compound interest. If we start increasing the frequency of compounding to infinity, the compound interest formula approximates the following formula.

Core Concept

Continuously Compounded Interest

When interest is compounded *continuously*, the amount A in an account after t years is given by the formula

$$A = Pe^{rt}$$

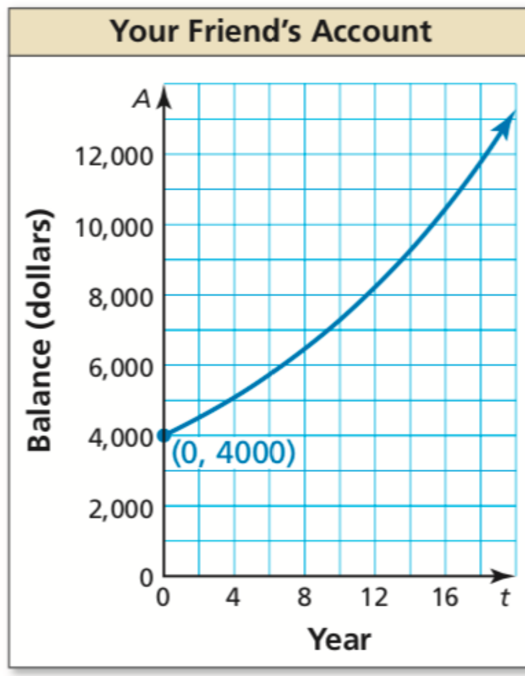
where P is the principal and r is the annual interest rate expressed as a decimal.

Example 3: Modeling with Mathematics

You and your friend each have accounts that earn annual interest compounded continuously. The balance A (in dollars) of your account after t years can be modeled by $A = 4500e^{0.04t}$. The graph shows the balance of your friend's account over time.

a) Which account has the greater principal?

b) Which has a greater balance after 10 years?



Try on your own.

You deposit $4250 in an account that earns 5% annual interest compounded continuously. Compare the balance after 10 years with the accounts in Example 3.

Homework:

3-21odd, 27-33odd, 35, 40, 41

6.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** What is the natural base e?

2. **WRITING** Tell whether the function $f(x) = \frac{1}{3}e^{4x}$ represents exponential growth or exponential decay. Explain.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, simplify the expression. *(See Example 1.)*

3. $e^3 \cdot e^5$
4. $e^{-4} \cdot e^6$
5. $\dfrac{11e^9}{22e^{10}}$
6. $\dfrac{27e^7}{3e^4}$
7. $(5e^{7x})^4$
8. $(4e^{-2x})^3$
9. $\sqrt{9e^{6x}}$
10. $\sqrt[3]{8e^{12x}}$
11. $e^x \cdot e^{-6x} \cdot e^8$
12. $e^x \cdot e^4 \cdot e^{x+3}$

ERROR ANALYSIS **In Exercises 13 and 14, describe and correct the error in simplifying the expression.**

13. ✗
$$(4e^{3x})^2 = 4e^{(3x)(2)}$$
$$= 4e^{6x}$$

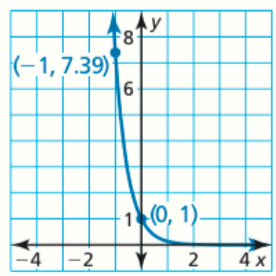
14. ✗
$$\frac{e^{5x}}{e^{-2x}} = e^{5x-2x}$$
$$= e^{3x}$$

In Exercises 15–22, tell whether the function represents *exponential growth* or *exponential decay*. Then graph the function. *(See Example 2.)*

15. $y = e^{3x}$
16. $y = e^{-2x}$
17. $y = 2e^{-x}$
18. $y = 3e^{2x}$
19. $y = 0.5e^x$
20. $y = 0.25e^{-3x}$
21. $y = 0.4e^{-0.25x}$
22. $y = 0.6e^{0.5x}$

ANALYZING EQUATIONS **In Exercises 23–26, match the function with its graph. Explain your reasoning.**

23. $y = e^{2x}$
24. $y = e^{-2x}$
25. $y = 4e^{-0.5x}$
26. $y = 0.75e^x$

A.


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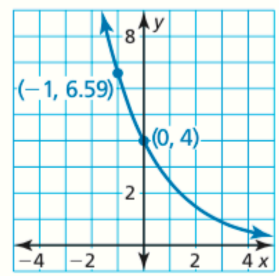
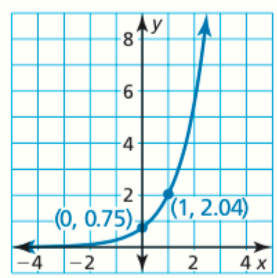
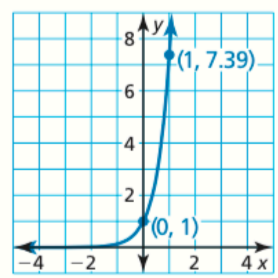


C.


D.


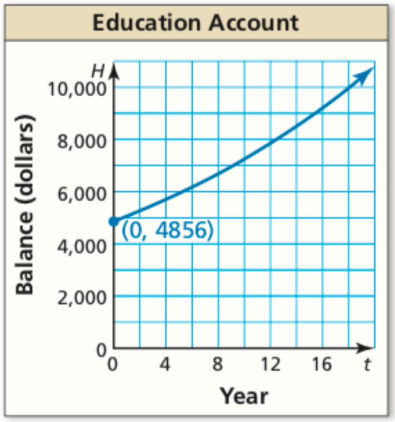
USING STRUCTURE **In Exercises 27–30, use the properties of exponents to rewrite the function in the form $y = a(1 + r)^t$ or $y = a(1 - r)^t$. Then find the percent rate of change.**

27. $y = e^{-0.25t}$
28. $y = e^{-0.75t}$
29. $y = 2e^{0.4t}$
30. $y = 0.5e^{0.8t}$

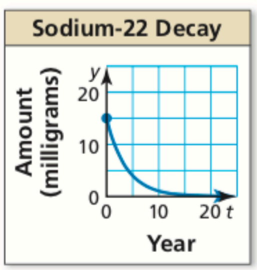
USING TOOLS **In Exercises 31–34, use a table of values or a graphing calculator to graph the function. Then identify the domain and range.**

31. $y = e^{x-2}$
32. $y = e^{x+1}$
33. $y = 2e^x + 1$
34. $y = 3e^x - 5$

35. **MODELING WITH MATHEMATICS** Investment accounts for a house and education earn annual interest compounded continuously. The balance H (in dollars) of the house fund after t years can be modeled by $H = 3224e^{0.05t}$. The graph shows the balance in the education fund over time. Which account has the greater principal? Which account has a greater balance after 10 years? *(See Example 3.)*



36. **MODELING WITH MATHEMATICS** Tritium and sodium-22 decay over time. In a sample of tritium, the amount y (in milligrams) remaining after t years is given by $y = 10e^{-0.0562t}$. The graph shows the amount of sodium-22 in a sample over time. Which sample started with a greater amount? Which has a greater amount after 10 years?



37. **OPEN-ENDED** Find values of a, b, r, and q such that $f(x) = ae^{rx}$ and $g(x) = be^{qx}$ are exponential decay functions, but $\frac{f(x)}{g(x)}$ represents exponential growth.

38. **THOUGHT PROVOKING** Explain why $A = P\left(1 + \frac{r}{n}\right)^{nt}$ approximates $A = Pe^{rt}$ as n approaches positive infinity.

39. **WRITING** Can the natural base e be written as a ratio of two integers? Explain.

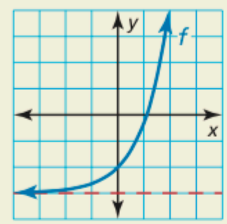
40. **MAKING AN ARGUMENT** Your friend evaluates $f(x) = e^{-x}$ when $x = 1000$ and concludes that the graph of $y = f(x)$ has an x-intercept at (1000, 0). Is your friend correct? Explain your reasoning.

41. **DRAWING CONCLUSIONS** You invest \$2500 in an account to save for college. Account 1 pays 6% annual interest compounded quarterly. Account 2 pays 4% annual interest compounded continuously. Which account should you choose to obtain the greater amount in 10 years? Justify your answer.

42. **HOW DO YOU SEE IT?** Use the graph to complete each statement.

a. $f(x)$ approaches ____ as x approaches $+\infty$.

b. $f(x)$ approaches ____ as x approaches $-\infty$.



43. **PROBLEM SOLVING** The growth of *Mycobacterium tuberculosis* bacteria can be modeled by the function $N(t) = ae^{0.166t}$, where N is the number of cells after t hours and a is the number of cells when $t = 0$.

a. At 1:00 P.M., there are 30 *M. tuberculosis* bacteria in a sample. Write a function that gives the number of bacteria after 1:00 P.M.

b. Use a graphing calculator to graph the function in part (a).

c. Describe how to find the number of cells in the sample at 3:45 P.M.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Write the number in scientific notation. *(Skills Review Handbook)*

44. 0.006   45. 5000   46. 26,000,000   47. 0.000000047

Find the inverse of the function. Then graph the function and its inverse. *(Section 5.6)*

48. $y = 3x + 5$

49. $y = x^2 - 1, x \leq 0$

50. $y = \sqrt{x + 6}$

51. $y = x^3 - 2$