6.5 Properties of Logarithms

Properties of Logarithms

Core Concept

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property	$\log_b mn = \log_b m + \log_b n$
Quotient Property	$\log_b \frac{m}{n} = \log_b m - \log_b n$
Power Property	$\log_b m^n = n \log_b m$

Example 1: Using Properties of Logarithms

Use $\log_2 3 \approx 1.585$ and $\log_2 7 \approx 2.807$ to evaluate each logarithm.

a) $\log_2 \frac{3}{7}$ b) $\log_2 21$ c) $\log_2 49$

Example 2: Expanding a Logarithmic Expression

Expand $\ln \frac{5x^7}{y}$

Example 3: Condensing a Logarithmic Expression

Condense $\log 9 + 3 \log 2 - \log 3$

Core Concept

Change-of-Base Formula

If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}.$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Example 4: Changing a Base Using Common Logarithms
Evaluate $\log_3 8$ using common logarithms

Example 5: Changing a Base Using Common Logarithms
Evaluate $\log_6 24$ using natural logarithms

Example 6: Modeling with Mathematics

For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the intensity of the sound doubles. By how many decibels does the loudness increase?

Homework:
3, 5, 9-12, 14-30even, 33, 34, 43, 44

6.5 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** To condense the expression $\log_3 2x + \log_3 y$, you need to use the _____ Property of Logarithms.

2. **WRITING** Describe two ways to evaluate $\log_7 12$ using a calculator.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, use $\log_7 4 \approx 0.712$ and $\log_7 12 \approx 1.277$ to evaluate the logarithm. *(See Example 1.)*

3. $\log_7 3$
4. $\log_7 48$
5. $\log_7 16$
6. $\log_7 64$
7. $\log_7 \frac{1}{4}$
8. $\log_7 \frac{1}{3}$

In Exercises 9–12, match the expression with the logarithm that has the same value. Justify your answer.

9. $\log_3 6 - \log_3 2$
10. $2 \log_3 6$
11. $6 \log_3 2$
12. $\log_3 6 + \log_3 2$

A. $\log_3 64$
B. $\log_3 3$
C. $\log_3 12$
D. $\log_3 36$

In Exercises 13–20, expand the logarithmic expression. *(See Example 2.)*

13. $\log_3 4x$
14. $\log_8 3x$
15. $\log 10x^5$
16. $\ln 3x^4$
17. $\ln \frac{x}{3y}$
18. $\ln \frac{6x^2}{y^4}$
19. $\log_7 5\sqrt{x}$
20. $\log_5 \sqrt[3]{x^2 y}$

ERROR ANALYSIS In Exercises 21 and 22, describe and correct the error in expanding the logarithmic expression.

21. ✗ $\log_2 5x = (\log_2 5)(\log_2 x)$

22. ✗ $\ln 8x^3 = 3 \ln 8 + \ln x$

In Exercises 23–30, condense the logarithmic expression. *(See Example 3.)*

23. $\log_4 7 - \log_4 10$
24. $\ln 12 - \ln 4$
25. $6 \ln x + 4 \ln y$
26. $2 \log x + \log 11$
27. $\log_5 4 + \frac{1}{3} \log_5 x$
28. $6 \ln 2 - 4 \ln y$
29. $5 \ln 2 + 7 \ln x + 4 \ln y$
30. $\log_3 4 + 2 \log_3 \frac{1}{2} + \log_3 x$

31. **REASONING** Which of the following is *not* equivalent to $\log_5 \frac{y^4}{3x}$? Justify your answer.

   (A) $4 \log_5 y - \log_5 3x$
   (B) $4 \log_5 y - \log_5 3 + \log_5 x$
   (C) $4 \log_5 y - \log_5 3 - \log_5 x$
   (D) $\log_5 y^4 - \log_5 3 - \log_5 x$

32. **REASONING** Which of the following equations is correct? Justify your answer.

   (A) $\log_7 x + 2 \log_7 y = \log_7 (x + y^2)$
   (B) $9 \log x - 2 \log y = \log \frac{x^9}{y^2}$
   (C) $5 \log_4 x + 7 \log_2 y = \log_6 x^5 y^7$
   (D) $\log_9 x - 5 \log_9 y = \log_9 \frac{x}{5y}$

In Exercises 33–40, use the change-of-base formula to evaluate the logarithm. *(See Examples 4 and 5.)*

33. $\log_4 7$

34. $\log_5 13$

35. $\log_9 15$

36. $\log_8 22$

37. $\log_6 17$

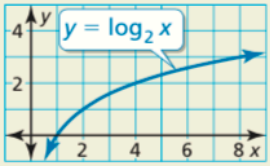
38. $\log_2 28$

39. $\log_7 \frac{3}{16}$

40. $\log_3 \frac{9}{40}$

41. MAKING AN ARGUMENT Your friend claims you can use the change-of-base formula to graph $y = \log_3 x$ using a graphing calculator. Is your friend correct? Explain your reasoning.

42. HOW DO YOU SEE IT? Use the graph to determine the value of $\frac{\log 8}{\log 2}$.



MODELING WITH MATHEMATICS In Exercises 43 and 44, use the function $L(I)$ given in Example 6.

43. The blue whale can produce sound with an intensity that is 1 million times greater than the intensity of the loudest sound a human can make. Find the difference in the decibel levels of the sounds made by a blue whale and a human. *(See Example 6.)*

44. The intensity of the sound of a certain television advertisement is 10 times greater than the intensity of the television program. By how many decibels does the loudness increase?

Intensity of Television Sound

During show:
Intensity = I

During ad:
Intensity = $10I$

45. REWRITING A FORMULA Under certain conditions, the wind speed s (in knots) at an altitude of h meters above a grassy plain can be modeled by the function

$$s(h) = 2 \ln 100h.$$

a. By what amount does the wind speed increase when the altitude doubles?

b. Show that the given function can be written in terms of common logarithms as

$$s(h) = \frac{2}{\log e}(\log h + 2).$$

46. THOUGHT PROVOKING Determine whether the formula

$$\log_b(M + N) = \log_b M + \log_b N$$

is true for all positive, real values of M, N, and b (with $b \neq 1$). Justify your answer.

47. USING STRUCTURE Use the properties of exponents to prove the change-of-base formula. (*Hint:* Let $x = \log_b a$, $y = \log_b c$, and $z = \log_c a$.)

48. CRITICAL THINKING Describe *three* ways to transform the graph of $f(x) = \log x$ to obtain the graph of $g(x) = \log 100x - 1$. Justify your answers.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality by graphing. *(Section 3.6)*

49. $x^2 - 4 > 0$

50. $2(x - 6)^2 - 5 \geq 37$

51. $x^2 + 13x + 42 < 0$

52. $-x^2 - 4x + 6 \leq -6$

Solve the equation by graphing the related system of equations. *(Section 3.5)*

53. $4x^2 - 3x - 6 = -x^2 + 5x + 3$

54. $-(x + 3)(x - 2) = x^2 - 6x$

55. $2x^2 - 4x - 5 = -(x + 3)^2 + 10$

56. $-(x + 7)^2 + 5 = (x + 10)^2 - 3$