

## Chapter 8

### Graphing Quadratic Functions

#### 8.1 Graphing $f(x) = ax^2$

What is a quadratic function? How does it differ from a linear function?

##### Identifying a characteristics of a Quadratic Function:

A quadratic function is a nonlinear function that can be expressed as  $y = ax^2 + bx + c$  where  $a \neq 0$  is known as **standard form**. The shape of the graph is a “U-shape” which is known as a **parabola**.

**\*\*\* For this section we will be looking at quadratic functions when  $b = 0$  &  $c = 0$  \*\*\***

##### Example 1: Graphing $y = ax^2$

Graph  $y = x^2$

X	-2	-1	0	1	2
Y					

What is the A & B value?

Where is the vertex?

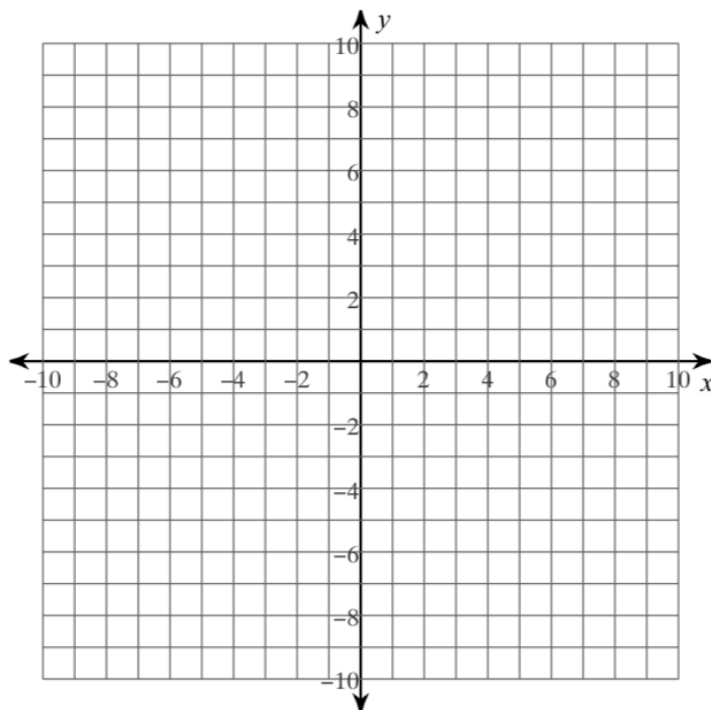
Where is the axis of symmetry?

What is the domain?

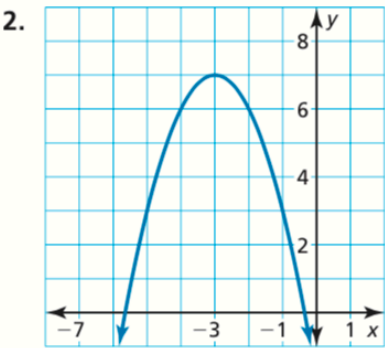
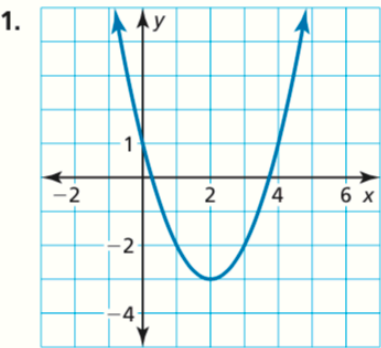
What is the range?

Where is the function decreasing?

Where is the function increasing?



Try on your own: Answer the same questions from example 1 for the following.



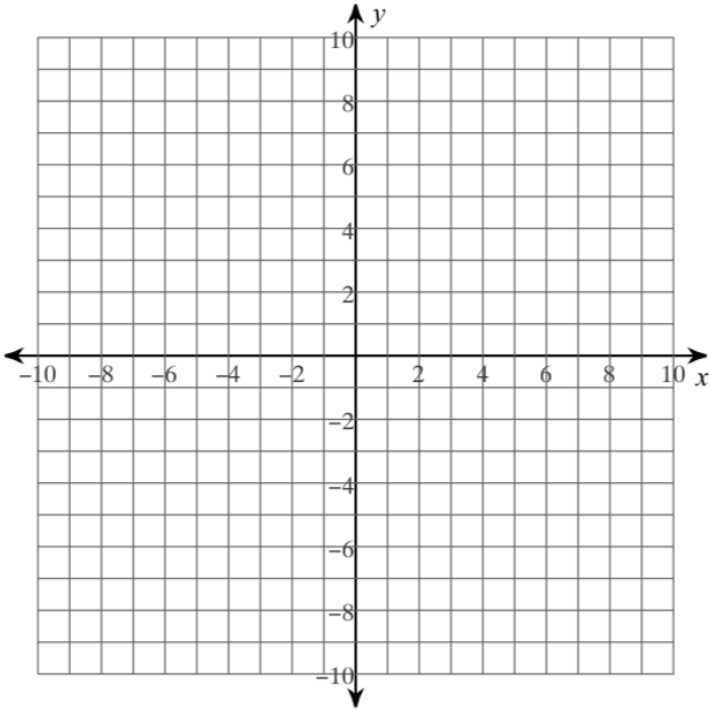
**Example 2:** Graphing  $y = ax^2$  when  $a > 0$   
Graph  $g(x) = 2x^2$ . Compare the graph to the graph of  $f(x) = x^2$

X	-2	-1	0	1	2
g(x)					

X	-2	-1	0	1	2
f(x)					

What do you notice?

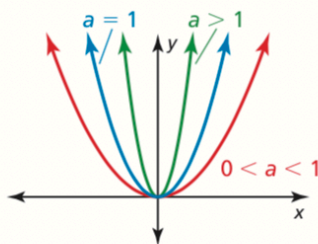
What do you think “a” does to the graph?



## Core Concept

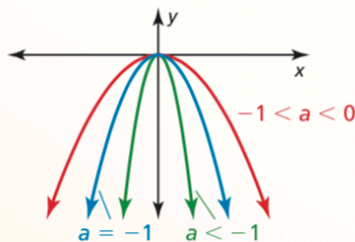
### > Graphing $f(x) = ax^2$ When $a > 0$

- When  $0 < a < 1$ , the graph of  $f(x) = ax^2$  is a vertical shrink of the graph of  $f(x) = x^2$ .
- When  $a > 1$ , the graph of  $f(x) = ax^2$  is a vertical stretch of the graph of  $f(x) = x^2$ .



### Graphing $f(x) = ax^2$ When $a < 0$

- When  $-1 < a < 0$ , the graph of  $f(x) = ax^2$  is a vertical shrink with a reflection in the  $x$ -axis of the graph of  $f(x) = x^2$ .
- When  $a < -1$ , the graph of  $f(x) = ax^2$  is a vertical stretch with a reflection in the  $x$ -axis of the graph of  $f(x) = x^2$ .



Example 3: Graphing  $y = ax^2$  when  $a < 0$

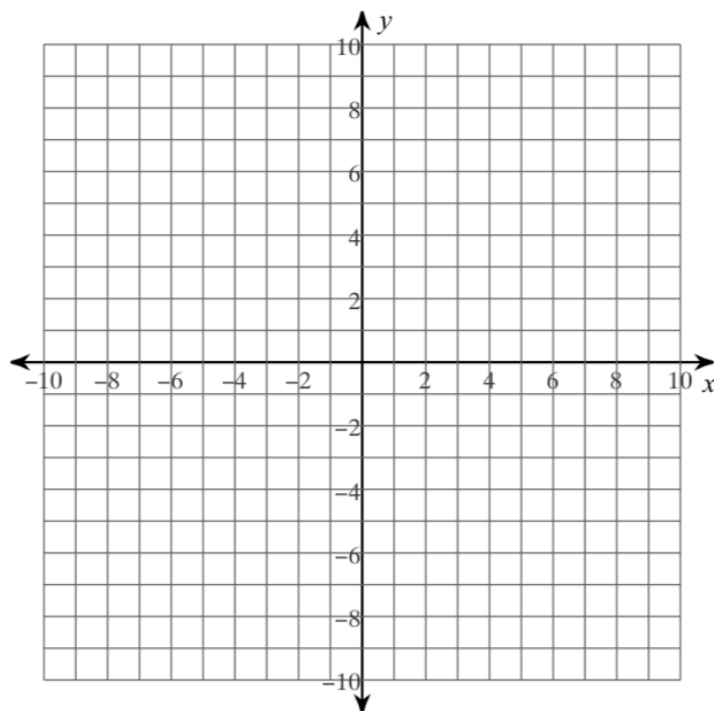
Graph  $h(x) = -\frac{1}{3}x^2$ . Compare the graph to the graph of  $f(x) = x^2$

X	-6	-3	0	3	6
$h(x)$					

X	-6	-3	0	3	6
$f(x)$					

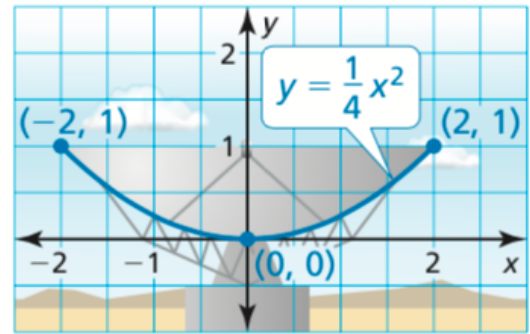
What do you notice?

What do you think “a” does to the graph?



**Example 4:** Solving a Real-Life Problem

The diagram shows the cross section of a satellite dish, where  $x$  and  $y$  are measured in meters. Find the width and depth of the dish.



Homework:

3, 4, 5-15odd, 18, 20, 24, 25

# 8.1 Exercises

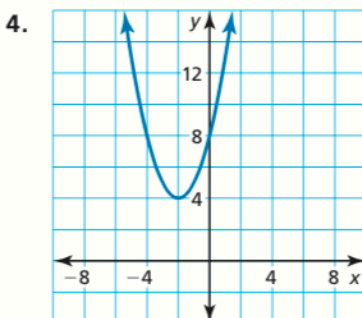
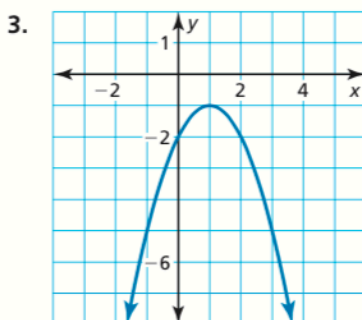
Dynamic Solutions available at [BigIdeasMath.com](http://BigIdeasMath.com)

## Vocabulary and Core Concept Check

- VOCABULARY** What is the U-shaped graph of a quadratic function called?
- WRITING** When does the graph of a quadratic function open up? open down?

## Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, identify characteristics of the quadratic function and its graph. (See Example 1.)



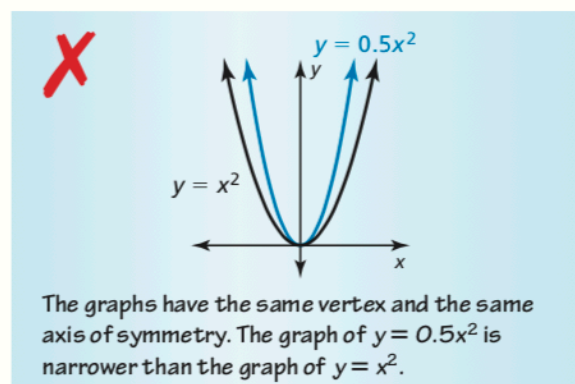
In Exercises 5–12, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Examples 2 and 3.)

- |                            |                              |
|----------------------------|------------------------------|
| 5. $g(x) = 6x^2$           | 6. $b(x) = 2.5x^2$           |
| 7. $h(x) = \frac{1}{4}x^2$ | 8. $j(x) = 0.75x^2$          |
| 9. $m(x) = -2x^2$          | 10. $q(x) = -\frac{9}{2}x^2$ |
| 11. $k(x) = -0.2x^2$       | 12. $p(x) = -\frac{2}{3}x^2$ |

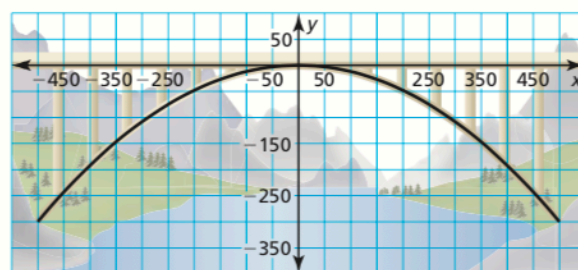
In Exercises 13–16, use a graphing calculator to graph the function. Compare the graph to the graph of  $y = -4x^2$ .

- |                    |                     |
|--------------------|---------------------|
| 13. $y = 4x^2$     | 14. $y = -0.4x^2$   |
| 15. $y = -0.04x^2$ | 16. $y = -0.004x^2$ |

17. **ERROR ANALYSIS** Describe and correct the error in graphing and comparing  $y = x^2$  and  $y = 0.5x^2$ .



18. **MODELING WITH MATHEMATICS** The arch support of a bridge can be modeled by  $y = -0.0012x^2$ , where  $x$  and  $y$  are measured in feet. Find the height and width of the arch. (See Example 4.)



19. **PROBLEM SOLVING** The breaking strength  $z$  (in pounds) of a manila rope can be modeled by  $z = 8900d^2$ , where  $d$  is the diameter (in inches) of the rope.

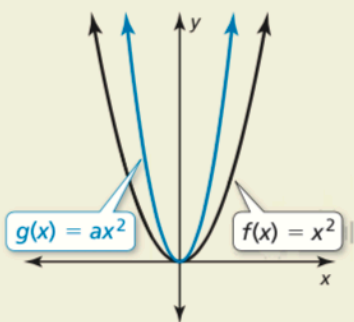
- Describe the domain and range of the function.
- Graph the function using the domain in part (a).



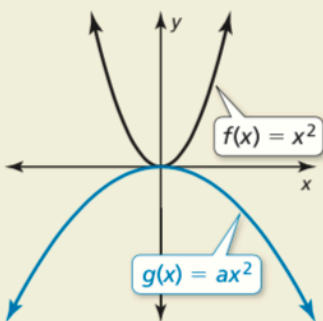
- A manila rope has four times the breaking strength of another manila rope. Does the stronger rope have four times the diameter? Explain.

20. **HOW DO YOU SEE IT?** Describe the possible values of  $a$ .

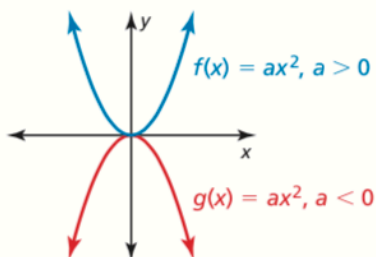
a.



b.



**ANALYZING GRAPHS** In Exercises 21–23, use the graph.

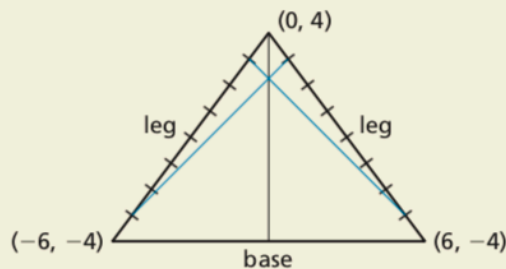


21. When is each function increasing?
22. When is each function decreasing?
23. Which function could include the point  $(-2, 3)$ ? Find the value of  $a$  when the graph passes through  $(-2, 3)$ .
24. **REASONING** Is the  $x$ -intercept of the graph of  $y = ax^2$  always 0? Justify your answer.
25. **REASONING** A parabola opens up and passes through  $(-4, 2)$  and  $(6, -3)$ . How do you know that  $(-4, 2)$  is not the vertex?

**ABSTRACT REASONING** In Exercises 26–29, determine whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

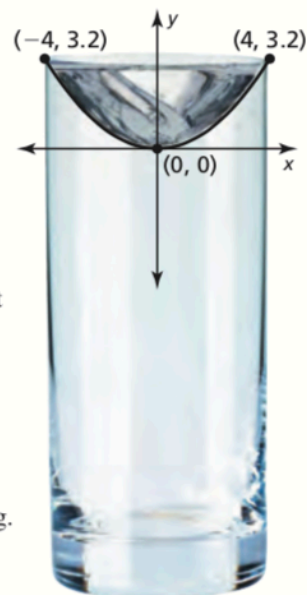
26. The graph of  $f(x) = ax^2$  is narrower than the graph of  $g(x) = x^2$  when  $a > 0$ .
27. The graph of  $f(x) = ax^2$  is narrower than the graph of  $g(x) = x^2$  when  $|a| > 1$ .
28. The graph of  $f(x) = ax^2$  is wider than the graph of  $g(x) = x^2$  when  $0 < |a| < 1$ .
29. The graph of  $f(x) = ax^2$  is wider than the graph of  $g(x) = dx^2$  when  $|a| > |d|$ .

30. **THOUGHT PROVOKING** Draw the isosceles triangle shown. Divide each leg into eight congruent segments. Connect the highest point of one leg with the lowest point of the other leg. Then connect the second highest point of one leg to the second lowest point of the other leg. Continue this process. Write a quadratic equation whose graph models the shape that appears.



31. **MAKING AN ARGUMENT**

The diagram shows the parabolic cross section of a swirling glass of water, where  $x$  and  $y$  are measured in centimeters.



- a. About how wide is the mouth of the glass?
- b. Your friend claims that the rotational speed of the water would have to increase for the cross section to be modeled by  $y = 0.1x^2$ . Is your friend correct? Explain your reasoning.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when  $n = 3$  and  $x = -2$ . (*Skills Review Handbook*)

32.  $n^2 + 5$

33.  $3x^2 - 9$

34.  $-4n^2 + 11$

35.  $n + 2x^2$