Chapter 8
Similarity

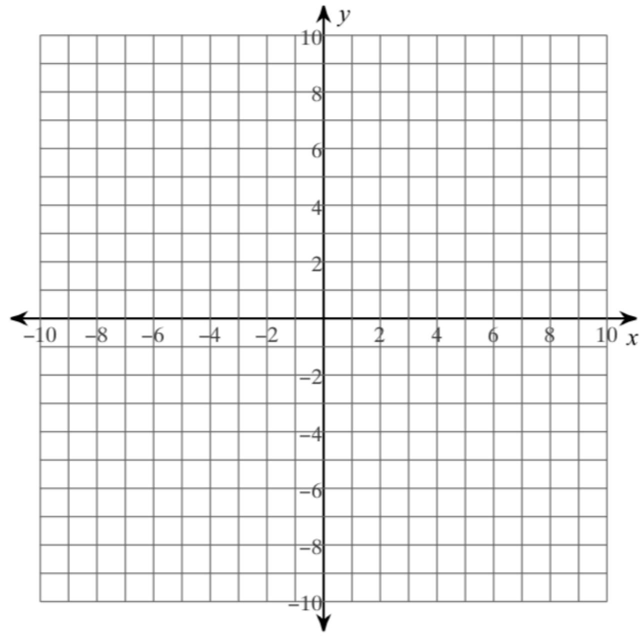
8.1 Similar Polygons

Recall: What does it mean for two or more objects to be similar?

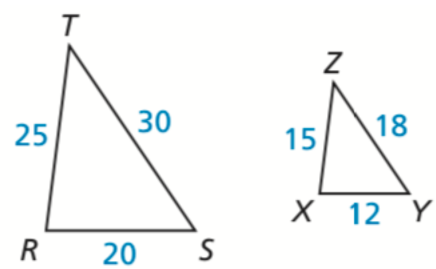
Spiral Review: Dilate triangle ABC, $A(0,0), B(0,3), C(4,0)$ by a scale factor or 2

What do you notice about the angles?

What do you notice about the sides?



Example 1: Using Similarity Statements
In the diagram$\Delta RST \sim \Delta XYZ$,.

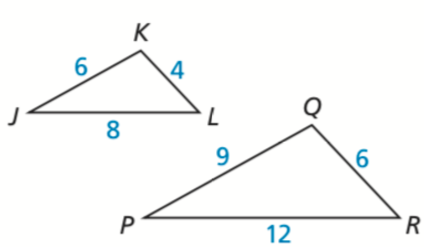


a) Find the scale factor from $\Delta RST \text{ to } \Delta XYZ$.

b) List all pairs of congruent angles.

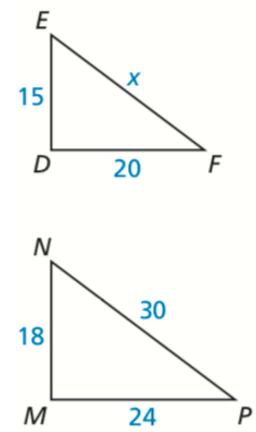
c) Write the ratios of the corresponding side lengths

Try on your own:



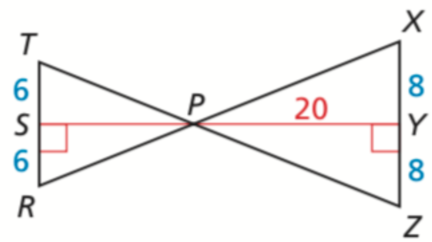
Example 2: Finding a Corresponding Length

In the diagram, $\Delta DEF \sim \Delta MNP$. Find the value of x



Example 3: Finding a Corresponding Length.

In the diagram, $\Delta TPR \sim \Delta XPZ$
Find the length of the altitude PS.



Area and Perimeter of Similar Figures

Think back to the Spiral Review problem.

What is the **perimeter** of the original figure?

What is the **perimeter** of the image?

What is the **area** of the original figure?

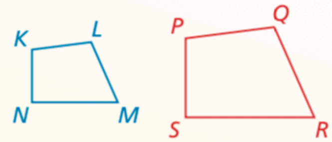
What is the **area** of the image?

What can you conclude?

Theorem

Theorem 8.1 Perimeters of Similar Polygons

If two polygons are similar, then the ratio of their perimeters is equal to the ratios of their corresponding side lengths.

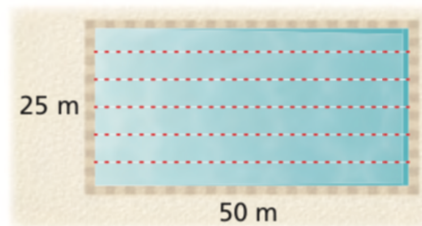


If $KLMN \sim PQRS$, then $\dfrac{PQ + QR + RS + SP}{KL + LM + MN + NK} = \dfrac{PQ}{KL} = \dfrac{QR}{LM} = \dfrac{RS}{MN} = \dfrac{SP}{NK}$.

Proof Ex. 52, p. 426; *BigIdeasMath.com*

Example 4: Modeling with Mathematics

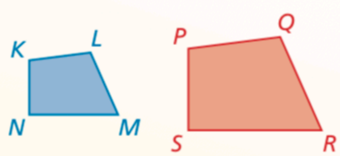
Bergen Community College plans to build a new swimming pool. An Olympic pool is rectangular with a length of 50m and a width of 25m. The new pool will be similar in shape to an Olympic pool but will have a length of 40m. Find the perimeters of an Olympic pool and the new pool.



Theorem

Theorem 8.2 Areas of Similar Polygons

If two polygons are similar, then the ratio of their areas is equal to the squares of the ratios of their corresponding side lengths.

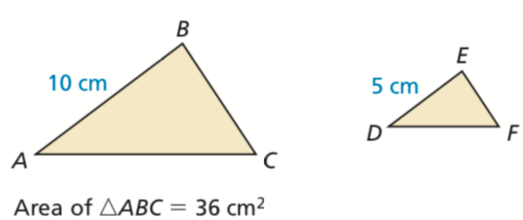


If $KLMN \sim PQRS$, then $\dfrac{\text{Area of } PQRS}{\text{Area of } KLMN} = \left(\dfrac{PQ}{KL}\right)^2 = \left(\dfrac{QR}{LM}\right)^2 = \left(\dfrac{RS}{MN}\right)^2 = \left(\dfrac{SP}{NK}\right)^2$.

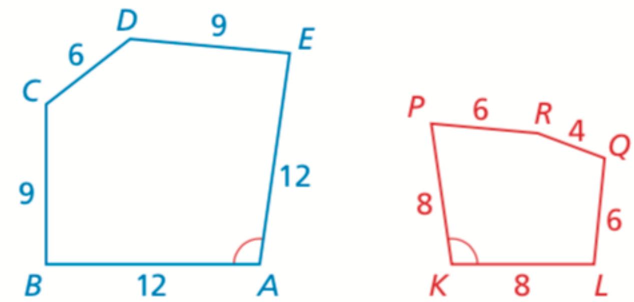
Proof Ex. 53, p. 426; *BigIdeasMath.com*

Example 5: Finding Areas of Similar Polygons

In the diagram, $\Delta ABC \sim \Delta DE$. Find the area of ΔDEF



Example 6: Deciding Whether Polygons Are Similar.
Decide whether ABCDE and KLQRP are similar. Explain your reasoning.



Homework:
3, 5, 7, 10, 12, 13, 16, 19, 20, 25, 26, 27, 28-34, 51

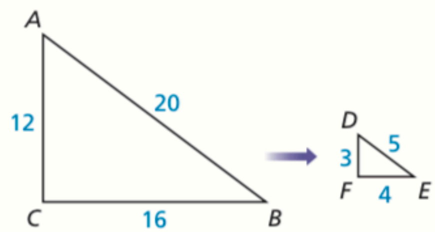
8.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** For two figures to be similar, the corresponding angles must be ____________, and the corresponding side lengths must be ________________.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.



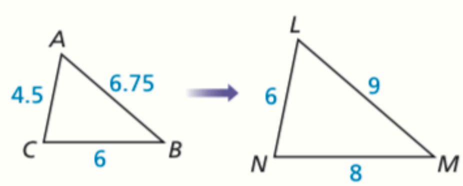
$\triangle ABC \sim \triangle DEF$

- What is the scale factor?
- What is the ratio of their areas?
- What is the ratio of their corresponding side lengths?
- What is the ratio of their perimeters?

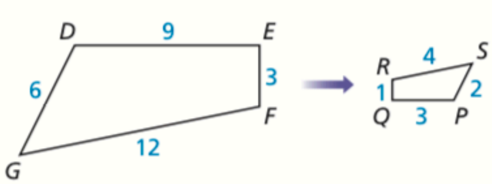
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the scale factor. Then list all pairs of congruent angles and write the ratios of the corresponding side lengths in a statement of proportionality. *(See Example 1.)*

3. $\triangle ABC \sim \triangle LMN$

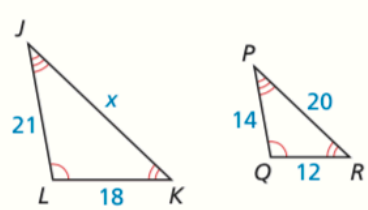


4. $DEFG \sim PQRS$



In Exercises 5–8, the polygons are similar. Find the value of x. *(See Example 2.)*

5.



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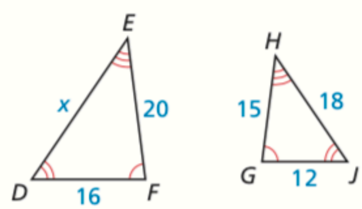

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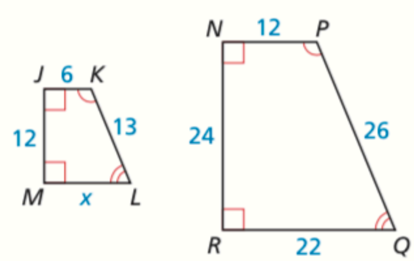

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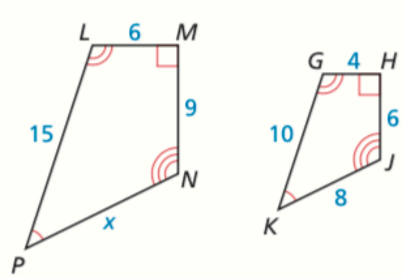
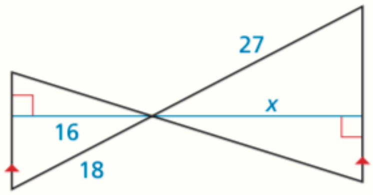
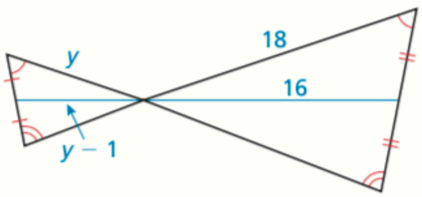


In Exercises 9 and 10, the black triangles are similar. Identify the type of segment shown in blue and find the value of the variable. *(See Example 3.)*

9.

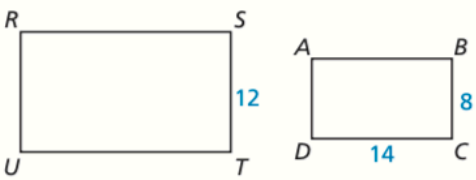


10.

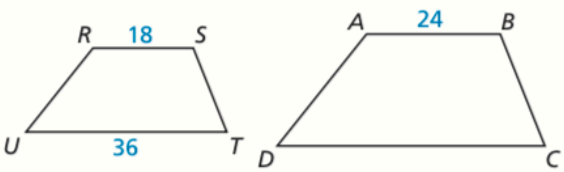


In Exercises 11 and 12, *RSTU* ~ *ABCD*. Find the ratio of their perimeters.

11.



12.



In Exercises 13–16, two polygons are similar. The perimeter of one polygon and the ratio of the corresponding side lengths are given. Find the perimeter of the other polygon.

13. perimeter of smaller polygon: 48 cm; ratio: $\frac{2}{3}$

14. perimeter of smaller polygon: 66 ft; ratio: $\frac{3}{4}$

15. perimeter of larger polygon: 120 yd; ratio: $\frac{1}{6}$

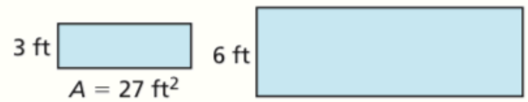
16. perimeter of larger polygon: 85 m; ratio: $\frac{2}{5}$

17. MODELING WITH MATHEMATICS A school gymnasium is being remodeled. The basketball court will be similar to an NCAA basketball court, which has a length of 94 feet and a width of 50 feet. The school plans to make the width of the new court 45 feet. Find the perimeters of an NCAA court and of the new court in the school. *(See Example 4.)*

18. MODELING WITH MATHEMATICS Your family has decided to put a rectangular patio in your backyard, similar to the shape of your backyard. Your backyard has a length of 45 feet and a width of 20 feet. The length of your new patio is 18 feet. Find the perimeters of your backyard and of the patio.

In Exercises 19–22, the polygons are similar. The area of one polygon is given. Find the area of the other polygon. *(See Example 5.)*

19.



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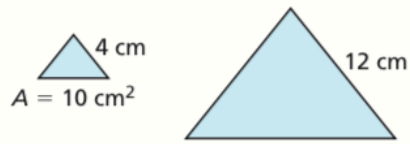

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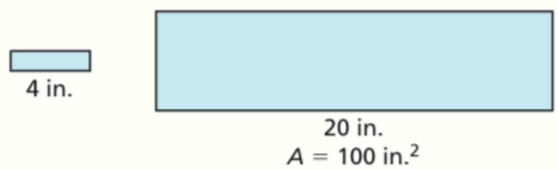
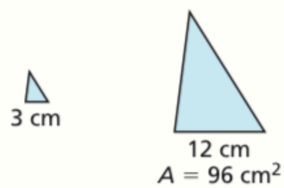
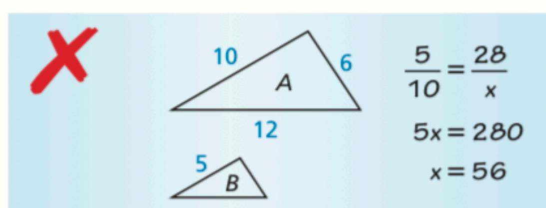


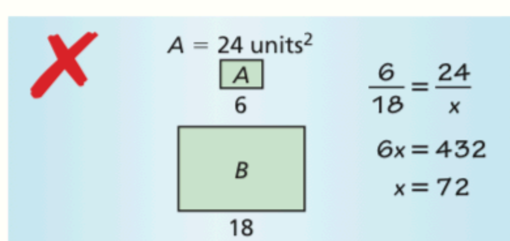
22.



23. ERROR ANALYSIS Describe and correct the error in finding the perimeter of triangle B. The triangles are similar.

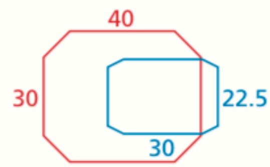


24. ERROR ANALYSIS Describe and correct the error in finding the area of rectangle B. The rectangles are similar.

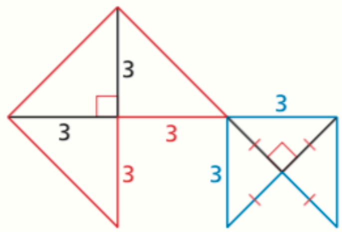


In Exercises 25 and 26, decide whether the red and blue polygons are similar. *(See Example 6.)*

25.



26.

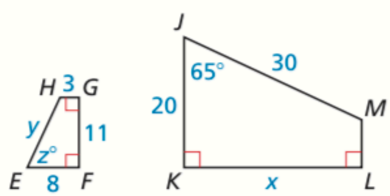


27. REASONING Triangles *ABC* and *DEF* are similar. Which statement is correct? Select all that apply.

Ⓐ $\frac{BC}{EF} = \frac{AC}{DF}$ Ⓑ $\frac{AB}{DE} = \frac{CA}{FE}$

Ⓒ $\frac{AB}{EF} = \frac{BC}{DE}$ Ⓓ $\frac{CA}{FD} = \frac{BC}{EF}$

ANALYZING RELATIONSHIPS In Exercises 28–34, *JKLM* ~ *EFGH*.



28. Find the scale factor of *JKLM* to *EFGH*.

29. Find the scale factor of *EFGH* to *JKLM*.

30. Find the values of *x*, *y*, and *z*.

31. Find the perimeter of each polygon.

32. Find the ratio of the perimeters of *JKLM* to *EFGH*.

33. Find the area of each polygon.

34. Find the ratio of the areas of *JKLM* to *EFGH*.

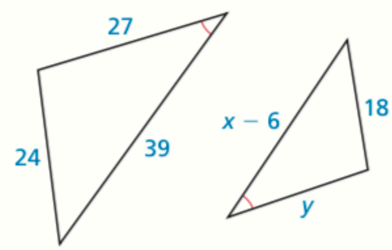
35. USING STRUCTURE Rectangle A is similar to rectangle B. Rectangle A has side lengths of 6 and 12. Rectangle B has a side length of 18. What are the possible values for the length of the other side of rectangle B? Select all that apply.

Ⓐ 6 Ⓑ 9 Ⓒ 24 Ⓓ 36

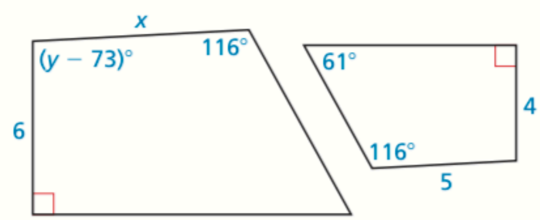
36. DRAWING CONCLUSIONS In table tennis, the table is a rectangle 9 feet long and 5 feet wide. A tennis court is a rectangle 78 feet long and 36 feet wide. Are the two surfaces similar? Explain. If so, find the scale factor of the tennis court to the table.

MATHEMATICAL CONNECTIONS In Exercises 37 and 38, the two polygons are similar. Find the values of *x* and *y*.

37.



38.



ATTENDING TO PRECISION In Exercises 39–42, the figures are similar. Find the missing corresponding side length.

39. Figure A has a perimeter of 72 meters and one of the side lengths is 18 meters. Figure B has a perimeter of 120 meters.

40. Figure A has a perimeter of 24 inches. Figure B has a perimeter of 36 inches and one of the side lengths is 12 inches.

41. Figure A has an area of 48 square feet and one of the side lengths is 6 feet. Figure B has an area of 75 square feet.

42. Figure A has an area of 18 square feet. Figure B has an area of 98 square feet and one of the side lengths is 14 feet.

CRITICAL THINKING In Exercises 43–48, tell whether the polygons are *always, sometimes,* or *never* similar.

43. two isosceles triangles

44. two isosceles trapezoids

45. two rhombuses

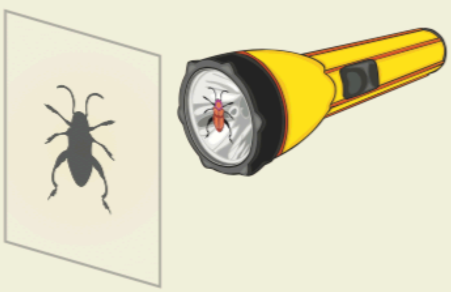
46. two squares

47. two regular polygons

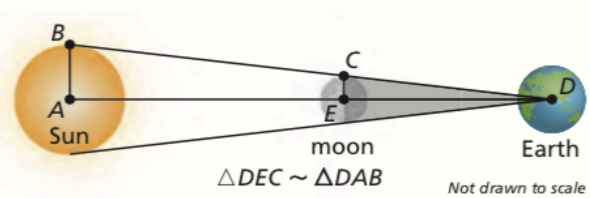
48. a right triangle and an equilateral triangle

49. MAKING AN ARGUMENT Your sister claims that when the side lengths of two rectangles are proportional, the two rectangles must be similar. Is she correct? Explain your reasoning.

50. HOW DO YOU SEE IT? You shine a flashlight directly on an object to project its image onto a parallel screen. Will the object and the image be similar? Explain your reasoning.



51. MODELING WITH MATHEMATICS During a total eclipse of the Sun, the moon is directly in line with the Sun and blocks the Sun's rays. The distance *DA* between Earth and the Sun is 93,000,000 miles, the distance *DE* between Earth and the moon is 240,000 miles, and the radius *AB* of the Sun is 432,500 miles. Use the diagram and the given measurements to estimate the radius *EC* of the moon.

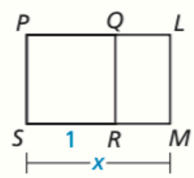


52. PROVING A THEOREM Prove the Perimeters of Similar Polygons Theorem (Theorem 8.1) for similar rectangles. Include a diagram in your proof.

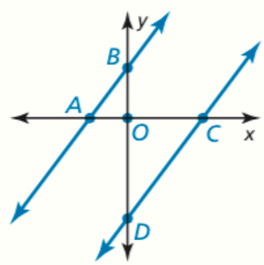
53. PROVING A THEOREM Prove the Areas of Similar Polygons Theorem (Theorem 8.2) for similar rectangles. Include a diagram in your proof.

54. THOUGHT PROVOKING The postulates and theorems in this book represent Euclidean geometry. In spherical geometry, all points are points on the surface of a sphere. A line is a circle on the sphere whose diameter is equal to the diameter of the sphere. A plane is the surface of the sphere. In spherical geometry, is it possible that two triangles are similar but not congruent? Explain your reasoning.

55. CRITICAL THINKING In the diagram, *PQRS* is a square, and *PLMS* ~ *LMRQ*. Find the exact value of *x*. This value is called the *golden ratio*. Golden rectangles have their length and width in this ratio. Show that the similar rectangles in the diagram are golden rectangles.



56. MATHEMATICAL CONNECTIONS The equations of the lines shown are $y = \frac{4}{3}x + 4$ and $y = \frac{4}{3}x - 8$. Show that $\triangle AOB \sim \triangle COD$.

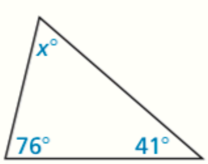


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of *x*. *(Section 5.1)*

57.



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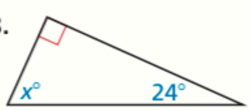
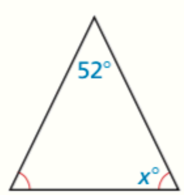


59.



60.