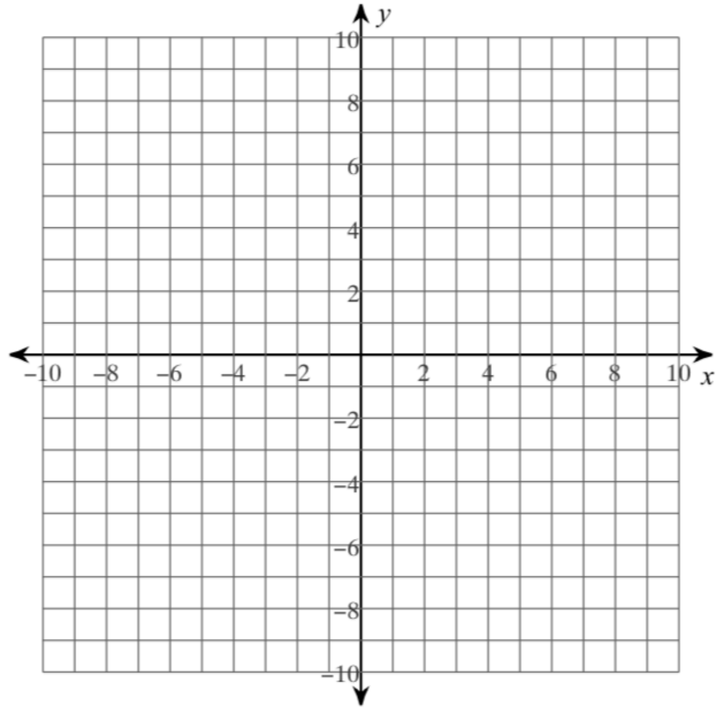
8.2 Graphing $f(x) = ax^2 + c$

Example 1: Graph the function $f(x) = ax^2 + c$

X	-2	-1	0	1	2
g(x)					

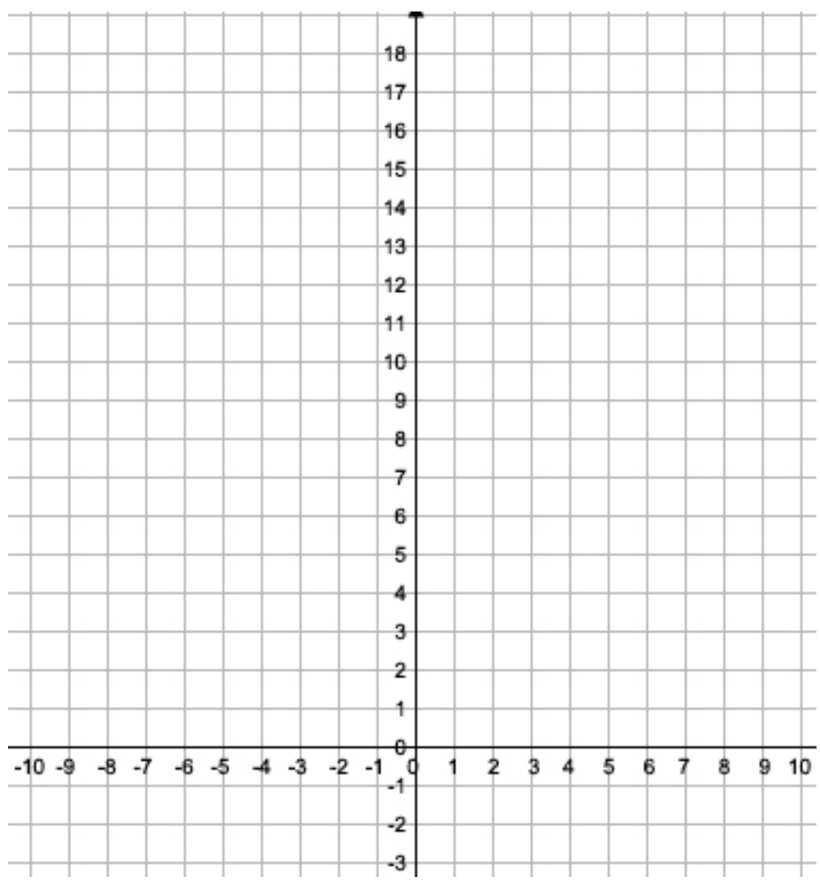
Compare $g(x) = x^2 - 2$ to $f(x) = x^2$.
What do you think c does to the function?



Example 2: Graph the function $f(x) = ax^2 + c$

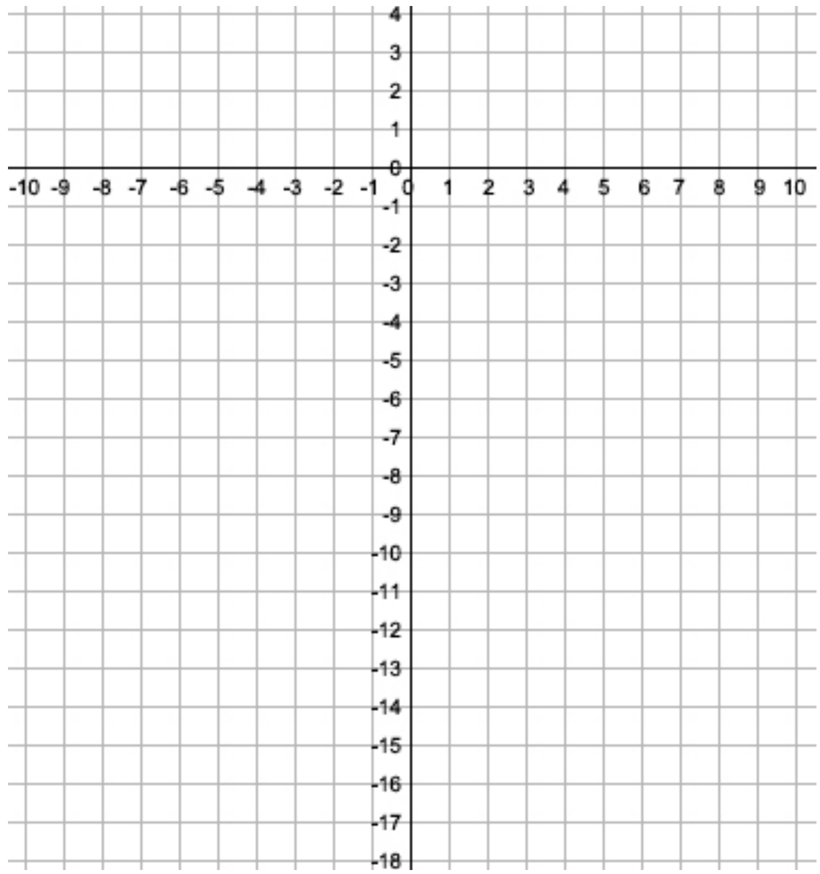
X	-2	-1	0	1	2
g(x)					

Compare $g(x) = 4x^2 + 1$ to $f(x) = x^2$.



Example 3: Translating the Graph of $f(x) = ax^2 + c$

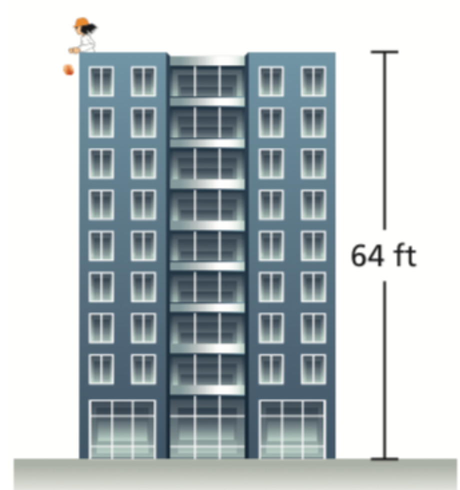
X	-4	-2	0	2	4
f(x)					
g(x)					



Compare $f(x) = -0.5x^2 + 2$ and
$g(x) = f(x) - 7$

Example 4: Solving a Real-Life Problem
The function $f(t) = -16t + s_o$ represents the approximate height (in feet) of a falling object t seconds after it is dropped from an initial height s_o (in feet). An egg is dropped from a height of 64ft.

a) After how many seconds does the egg hit the ground?



b) Suppose the initial height is adjusted by k feet.
How will this affect part (a)?

Homework:
3-15odd(use calc), 19-27odd, 30, 31, 33, 37, 41

8.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** State the vertex and axis of symmetry of the graph of $y = ax^2 + c$.

2. **WRITING** How does the graph of $y = ax^2 + c$ compare to the graph of $y = ax^2$?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 1.)*

3. $g(x) = x^2 + 6$
4. $h(x) = x^2 + 8$
5. $p(x) = x^2 - 3$
6. $q(x) = x^2 - 1$

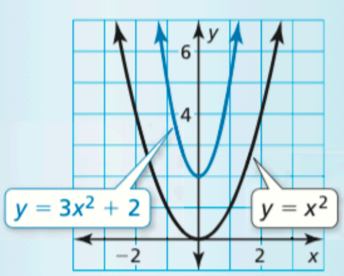
In Exercises 7–12, graph the function. Compare the graph to the graph of $f(x) = x^2$. *(See Example 2.)*

7. $g(x) = -x^2 + 3$
8. $h(x) = -x^2 - 7$
9. $s(x) = 2x^2 - 4$
10. $t(x) = -3x^2 + 1$
11. $p(x) = -\frac{1}{3}x^2 - 2$
12. $q(x) = \frac{1}{2}x^2 + 6$

In Exercises 13–16, describe the transformation from the graph of f to the graph of g. Then graph f and g in the same coordinate plane. Write an equation that represents g in terms of x. *(See Example 3.)*

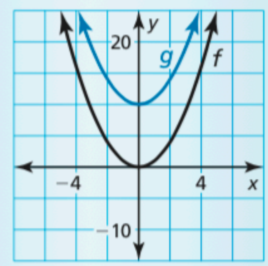
13. $f(x) = 3x^2 + 4$
    $g(x) = f(x) + 2$
14. $f(x) = \frac{1}{2}x^2 + 1$
    $g(x) = f(x) - 4$
15. $f(x) = -\frac{1}{4}x^2 - 6$
    $g(x) = f(x) - 3$
16. $f(x) = 4x^2 - 5$
    $g(x) = f(x) + 7$

17. **ERROR ANALYSIS** Describe and correct the error in comparing the graphs.



The graph of $y = 3x^2 + 2$ is a vertical shrink by a factor of 3 and a translation 2 units up of the graph of $y = x^2$.

18. **ERROR ANALYSIS** Describe and correct the error in graphing and comparing $f(x) = x^2$ and $g(x) = x^2 - 10$.



Both graphs open up and have the same axis of symmetry. However, the vertex of the graph of g, (0, 10), is 10 units above the vertex of the graph of f, (0, 0).

In Exercises 19–26, find the zeros of the function.

19. $y = x^2 - 1$
20. $y = x^2 - 36$
21. $f(x) = -x^2 + 25$
22. $f(x) = -x^2 + 49$
23. $f(x) = 4x^2 - 16$
24. $f(x) = 3x^2 - 27$
25. $f(x) = -12x^2 + 3$
26. $f(x) = -8x^2 + 98$

27. **MODELING WITH MATHEMATICS** A water balloon is dropped from a height of 144 feet. *(See Example 4.)*
    a. After how many seconds does the water balloon hit the ground?
    b. Suppose the initial height is adjusted by k feet. How does this affect part (a)?

28. **MODELING WITH MATHEMATICS** The function $y = -16x^2 + 36$ represents the height y (in feet) of an apple x seconds after falling from a tree. Find and interpret the x- and y-intercepts.

In Exercises 29–32, sketch a parabola with the given characteristics.

29. The parabola opens up, and the vertex is $(0, 3)$.

30. The vertex is $(0, 4)$, and one of the x-intercepts is 2.

31. The related function is increasing when $x < 0$, and the zeros are -1 and 1.

32. The highest point on the parabola is $(0, -5)$.

33. DRAWING CONCLUSIONS You and your friend both drop a ball at the same time. The function $h(x) = -16x^2 + 256$ represents the height (in feet) of your ball after x seconds. The function $g(x) = -16x^2 + 300$ represents the height (in feet) of your friend's ball after x seconds.

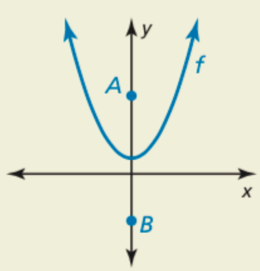
a. Write the function $T(x) = h(x) - g(x)$. What does $T(x)$ represent?

b. When your ball hits the ground, what is the height of your friend's ball? Use a graph to justify your answer.

34. MAKING AN ARGUMENT Your friend claims that in the equation $y = ax^2 + c$, the vertex changes when the value of a changes. Is your friend correct? Explain your reasoning.

35. MATHEMATICAL CONNECTIONS The area A (in square feet) of a square patio is represented by $A = x^2$, where x is the length of one side of the patio. You add 48 square feet to the patio, resulting in a total area of 192 square feet. What are the dimensions of the original patio? Use a graph to justify your answer.

36. HOW DO YOU SEE IT? The graph of $f(x) = ax^2 + c$ is shown. Points A and B are the same distance from the vertex of the graph of f. Which point is closer to the vertex of the graph of f as c increases?



37. REASONING Describe two methods you can use to find the zeros of the function $f(t) = -16t^2 + 400$. Check your answer by graphing.

38. PROBLEM SOLVING The paths of water from three different garden waterfalls are given below. Each function gives the height h (in feet) and the horizontal distance d (in feet) of the water.

Waterfall 1 $h = -3.1d^2 + 4.8$

Waterfall 2 $h = -3.5d^2 + 1.9$

Waterfall 3 $h = -1.1d^2 + 1.6$

a. Which waterfall drops water from the highest point?

b. Which waterfall follows the narrowest path?

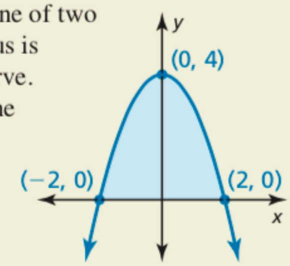
c. Which waterfall sends water the farthest?

39. WRITING EQUATIONS Two acorns fall to the ground from an oak tree. One falls 45 feet, while the other falls 32 feet.

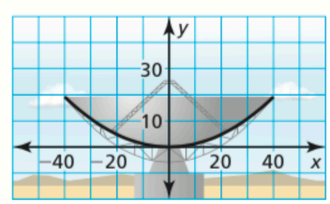
a. For each acorn, write an equation that represents the height h (in feet) as a function of the time t (in seconds).

b. Describe how the graphs of the two equations are related.

40. THOUGHT PROVOKING One of two classic problems in calculus is to find the area under a curve. Approximate the area of the region bounded by the parabola and the x-axis. Show your work.



41. CRITICAL THINKING A cross section of the parabolic surface of the antenna shown can be modeled by $y = 0.012x^2$, where x and y are measured in feet. The antenna is moved up so that the outer edges of the dish are 25 feet above the x-axis. Where is the vertex of the cross section located? Explain.



Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Evaluate the expression when $a = 4$ and $b = -3$. *(Skills Review Handbook)*

42. $\frac{a}{4b}$ **43.** $-\frac{b}{2a}$ **44.** $\frac{a - b}{3a + b}$ **45.** $-\frac{b + 2a}{ab}$