

## 8.4 Graphing $f(x) = a(x - h)^2 + k$

### Identifying Even and Odd Functions

A function is even when  $f(-x) = f(x)$ .

A function is odd when  $f(-x) = -f(x)$

#### Example 1: Identifying Even and Odd Functions

Determine whether each function is even, odd, or neither.

a)  $f(x) = 2x$

b)  $f(x) = x^2 - 2$

c)  $f(x) = 2x^2 + x - 2$

### Vertex Form: $f(x) = a(x - h)^2 + k$

#### Example 2: Graphing in vertex form.

Compare the graph to the graph of  $f(x) = x^2$

a)  $f(x) = (x + 2)^2$

b)  $f(x) = (x - 3)^2$

c)  $f(x) = (x + 5)^2$

Vertex: ( \_\_ , \_\_ )

Vertex: ( \_\_ , \_\_ )

Vertex: ( \_\_ , \_\_ )

Do you see a pattern?

#### Example 3: Graphing in vertex form.

Compare the graph to the graph of  $f(x) = x^2$

a)  $f(x) = (x + 4)^2 + 1$

b)  $f(x) = (x + 1)^2 - 5$

c)  $f(x) = (x + 3)^2 + 2$

Vertex: ( \_\_ , \_\_ )

Vertex: ( \_\_ , \_\_ )

Vertex: ( \_\_ , \_\_ )

Do you see a pattern?

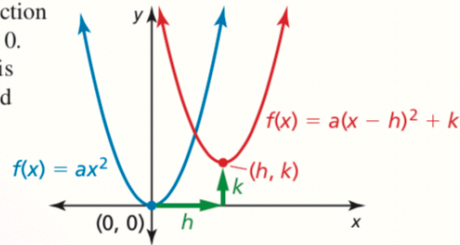
## Core Concept

### Graphing $f(x) = a(x - h)^2 + k$

The **vertex form** of a quadratic function is  $f(x) = a(x - h)^2 + k$ , where  $a \neq 0$ .

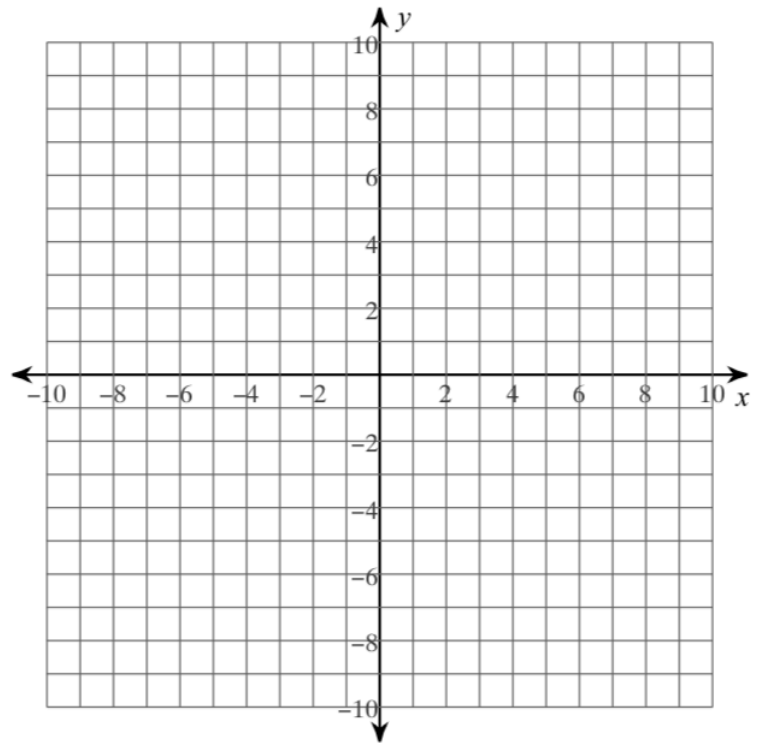
The graph of  $f(x) = a(x - h)^2 + k$  is a translation  $h$  units horizontally and  $k$  units vertically of the graph of  $f(x) = ax^2$ .

The vertex of the graph of  $f(x) = a(x - h)^2 + k$  is  $(h, k)$ , and the axis of symmetry is  $x = h$ .



### Example 4: Transforming the Graph of $y = a(x - h)^2 + k$

Graph  $f(x) = -2(x - 3)^2 + 3$ , then graph  $g(x) = f(x + 5)$  &  $h(x) = f(x) - 5$



### Example 5: Modeling with Mathematics

Water fountains are usually designed to give a specific visual effect. For example, the water fountain shown consists of streams of water that are shaped like parabolas. Notice how the streams are designed to land on the underwater spotlights. Write and graph a quadratic function that models the path of a stream of water with a maximum height of 5 feet, represented by a vertex of  $(3, 5)$ , landing on a spotlight 6 feet from the water jet, represented by  $(6, 0)$ .



## 8.4 Exercises

Dynamic Solutions available at [BigIdeasMath.com](http://BigIdeasMath.com)

### Vocabulary and Core Concept Check

- VOCABULARY** Compare the graph of an even function with the graph of an odd function.
- OPEN-ENDED** Write a quadratic function whose graph has a vertex of  $(1, 2)$ .
- WRITING** Describe the transformation from the graph of  $f(x) = ax^2$  to the graph of  $g(x) = a(x - h)^2 + k$ .
- WHICH ONE DOESN'T BELONG?** Which function does *not* belong with the other three? Explain your reasoning.

$$f(x) = 8(x + 4)^2$$

$$f(x) = (x - 2)^2 + 4$$

$$f(x) = 2(x + 0)^2$$

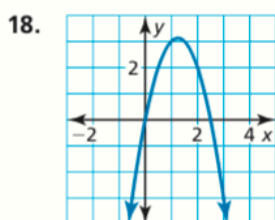
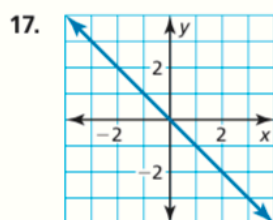
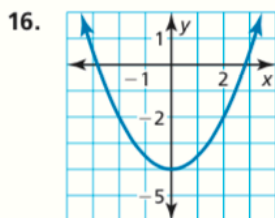
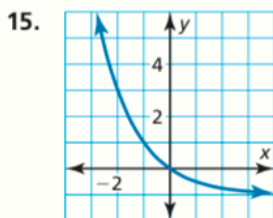
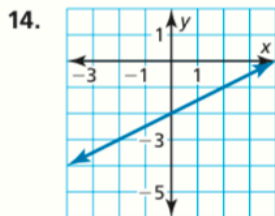
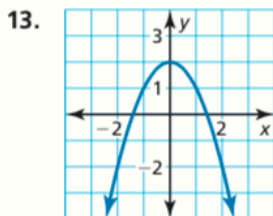
$$f(x) = 3(x + 1)^2 + 1$$

### Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, determine whether the function is *even*, *odd*, or *neither*. (See Example 1.)

- $f(x) = 4x + 3$
- $g(x) = 3x^2$
- $h(x) = 5^x + 2$
- $m(x) = 2x^2 - 7x$
- $p(x) = -x^2 + 8$
- $f(x) = -\frac{1}{2}x$
- $n(x) = 2x^2 - 7x + 3$
- $r(x) = -6x^2 + 5$

In Exercises 13–18, determine whether the function represented by the graph is *even*, *odd*, or *neither*.



In Exercises 19–22, find the vertex and the axis of symmetry of the graph of the function.

- $f(x) = 3(x + 1)^2$
- $f(x) = \frac{1}{4}(x - 6)^2$
- $y = -\frac{1}{8}(x - 4)^2$
- $y = -5(x + 9)^2$

In Exercises 23–28, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Example 2.)

- $g(x) = 2(x + 3)^2$
- $p(x) = 3(x - 1)^2$
- $r(x) = \frac{1}{4}(x + 10)^2$
- $n(x) = \frac{1}{3}(x - 6)^2$
- $d(x) = \frac{1}{5}(x - 5)^2$
- $q(x) = 6(x + 2)^2$

29. **ERROR ANALYSIS** Describe and correct the error in determining whether the function  $f(x) = x^2 + 3$  is even, odd, or neither.



$$\begin{aligned} f(x) &= x^2 + 3 \\ f(-x) &= (-x)^2 + 3 \\ &= x^2 + 3 \\ &= f(x) \end{aligned}$$

So,  $f(x)$  is an odd function.

30. **ERROR ANALYSIS** Describe and correct the error in finding the vertex of the graph of the function.



$$y = -(x + 8)^2$$

Because  $h = -8$ , the vertex is  $(0, -8)$ .

In Exercises 31–34, find the vertex and the axis of symmetry of the graph of the function.

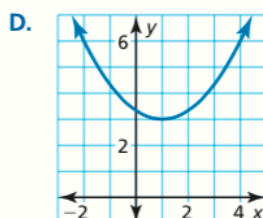
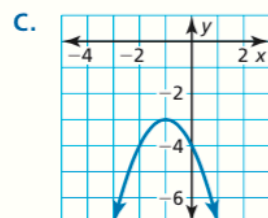
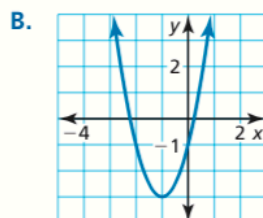
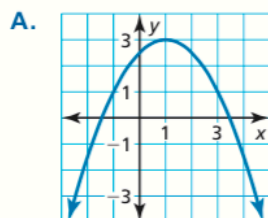
31.  $y = -6(x + 4)^2 - 3$     32.  $f(x) = 3(x - 3)^2 + 6$

33.  $f(x) = -4(x + 3)^2 + 1$     34.  $y = -(x - 6)^2 - 5$

In Exercises 35–38, match the function with its graph.

35.  $y = -(x + 1)^2 - 3$     36.  $y = -\frac{1}{2}(x - 1)^2 + 3$

37.  $y = \frac{1}{3}(x - 1)^2 + 3$     38.  $y = 2(x + 1)^2 - 3$



In Exercises 39–44, graph the function. Compare the graph to the graph of  $f(x) = x^2$ . (See Example 3.)

39.  $h(x) = (x - 2)^2 + 4$     40.  $g(x) = (x + 1)^2 - 7$

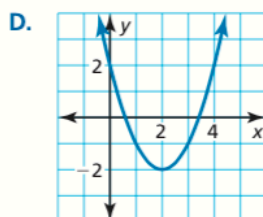
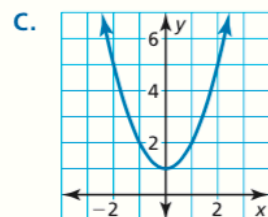
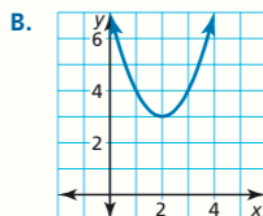
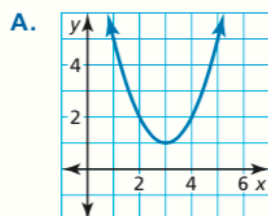
41.  $r(x) = 4(x - 1)^2 - 5$     42.  $n(x) = -(x + 4)^2 + 2$

43.  $g(x) = -\frac{1}{3}(x + 3)^2 - 2$     44.  $r(x) = \frac{1}{2}(x - 2)^2 - 4$

In Exercises 45–48, let  $f(x) = (x - 2)^2 + 1$ . Match the function with its graph.

45.  $g(x) = f(x - 1)$     46.  $r(x) = f(x + 2)$

47.  $h(x) = f(x) + 2$     48.  $p(x) = f(x) - 3$



In Exercises 49–54, graph  $g$ . (See Example 4.)

49.  $f(x) = 2(x - 1)^2 + 1$ ;  $g(x) = f(x + 3)$

50.  $f(x) = -(x + 1)^2 + 2$ ;  $g(x) = \frac{1}{2}f(x)$

51.  $f(x) = -3(x + 5)^2 - 6$ ;  $g(x) = 2f(x)$

52.  $f(x) = 5(x - 3)^2 - 1$ ;  $g(x) = f(x) - 6$

53.  $f(x) = (x + 3)^2 + 5$ ;  $g(x) = f(x - 4)$

54.  $f(x) = -2(x - 4)^2 - 8$ ;  $g(x) = -f(x)$

55. **MODELING WITH MATHEMATICS** The height (in meters) of a bird diving to catch a fish is represented by  $h(t) = 5(t - 2.5)^2$ , where  $t$  is the number of seconds after beginning the dive.

- Graph  $h$ .
- Another bird's dive is represented by  $r(t) = 2h(t)$ . Graph  $r$ .
- Compare the graphs. Which bird starts its dive from a greater height? Explain.



56. **MODELING WITH MATHEMATICS** A kicker punts a football. The height (in yards) of the football is represented by  $f(x) = -\frac{1}{9}(x - 30)^2 + 25$ , where  $x$  is the horizontal distance (in yards) from the kicker's goal line.

- Graph  $f$ . Describe the domain and range.
- On the next possession, the kicker punts the football. The height of the football is represented by  $g(x) = f(x + 5)$ . Graph  $g$ . Describe the domain and range.
- Compare the graphs. On which possession does the kicker punt closer to his goal line? Explain.

In Exercises 57–62, write a quadratic function in vertex form whose graph has the given vertex and passes through the given point.

57. vertex:  $(1, 2)$ ; passes through  $(3, 10)$

58. vertex:  $(-3, 5)$ ; passes through  $(0, -14)$

59. vertex:  $(-2, -4)$ ; passes through  $(-1, -6)$

60. vertex:  $(1, 8)$ ; passes through  $(3, 12)$

61. vertex:  $(5, -2)$ ; passes through  $(7, 0)$

62. vertex:  $(-5, -1)$ ; passes through  $(-2, 2)$

- 63. MODELING WITH MATHEMATICS** A portion of a roller coaster track is in the shape of a parabola. Write and graph a quadratic function that models this portion of the roller coaster with a maximum height of 90 feet, represented by a vertex of (25, 90), passing through the point (50, 0). (See Example 5.)



- 64. MODELING WITH MATHEMATICS** A flare is launched from a boat and travels in a parabolic path until reaching the water. Write and graph a quadratic function that models the path of the flare with a maximum height of 300 meters, represented by a vertex of (59, 300), landing in the water at the point (119, 0).

**In Exercises 65–68, rewrite the quadratic function in vertex form.**

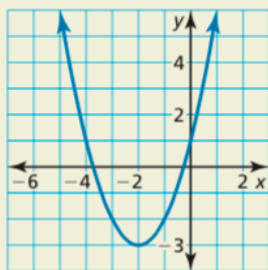
**65.**  $y = 2x^2 - 8x + 4$       **66.**  $y = 3x^2 + 6x - 1$

**67.**  $f(x) = -5x^2 + 10x + 3$

**68.**  $f(x) = -x^2 - 4x + 2$

- 69. REASONING** Can a function be symmetric about the  $x$ -axis? Explain.

- 70. HOW DO YOU SEE IT?** The graph of a quadratic function is shown. Determine which symbols to use to complete the vertex form of the quadratic function. Explain your reasoning.



$y = a(x \quad 2)^2 \quad 3$

**In Exercises 71–74, describe the transformation from the graph of  $f$  to the graph of  $h$ . Write an equation that represents  $h$  in terms of  $x$ .**

**71.**  $f(x) = -(x + 1)^2 - 2$       **72.**  $f(x) = 2(x - 1)^2 + 1$   
 $h(x) = f(x) + 4$        $h(x) = f(x - 5)$

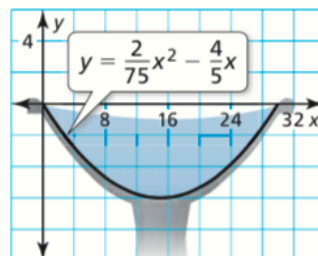
**73.**  $f(x) = 4(x - 2)^2 + 3$       **74.**  $f(x) = -(x + 5)^2 - 6$   
 $h(x) = 2f(x)$        $h(x) = \frac{1}{3}f(x)$

- 75. REASONING** The graph of  $y = x^2$  is translated 2 units right and 5 units down. Write an equation for the function in vertex form and in standard form. Describe advantages of writing the function in each form.

- 76. THOUGHT PROVOKING** Which of the following are true? Justify your answers.

- Any constant multiple of an even function is even.
- Any constant multiple of an odd function is odd.
- The sum or difference of two even functions is even.
- The sum or difference of two odd functions is odd.
- The sum or difference of an even function and an odd function is odd.

- 77. COMPARING FUNCTIONS** A cross section of a birdbath can be modeled by  $y = \frac{1}{81}(x - 18)^2 - 4$ , where  $x$  and  $y$  are measured in inches. The graph shows the cross section of another birdbath.



- Which birdbath is deeper? Explain.
  - Which birdbath is wider? Explain.
- 78. REASONING** Compare the graphs of  $y = 2x^2 + 8x + 8$  and  $y = x^2$  without graphing the functions. How can factoring help you compare the parabolas? Explain.
- 79. MAKING AN ARGUMENT** Your friend says all absolute value functions are even because of their symmetry. Is your friend correct? Explain.

## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Solve the equation.** (Section 7.4)

**80.**  $x(x - 1) = 0$

**81.**  $(x + 3)(x - 8) = 0$

**82.**  $(3x - 9)(4x + 12) = 0$