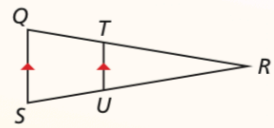
8.4 Proportionality Theorems

Theorems

Theorem 8.6 Triangle Proportionality Theorem

If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.

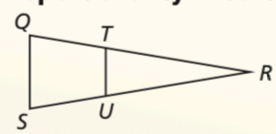


If $\overline{TU} \parallel \overline{QS}$, then $\frac{RT}{TQ} = \frac{RU}{US}$.

Proof Ex. 27, p. 451

Theorem 8.7 Converse of the Triangle Proportionality Theorem

If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

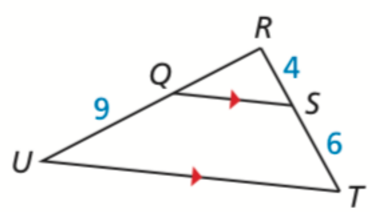


If $\frac{RT}{TQ} = \frac{RU}{US}$, then $\overline{TU} \parallel \overline{QS}$.

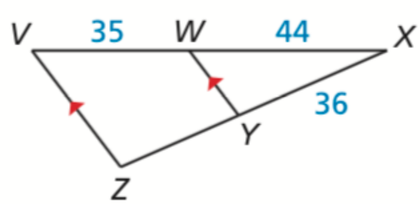
Proof Ex. 28, p. 451

Example 1: Finding the Lengths of a Segment

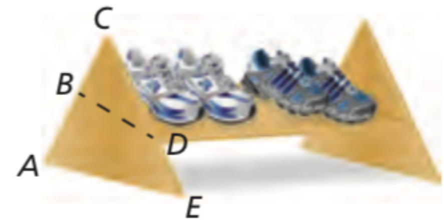
In the diagram, $QS || UT, RS = 4, ST = 6, and\ QU = 9$. What is the length of RQ?



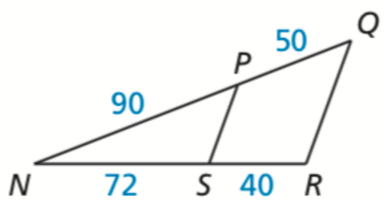
Try on your own: Find the length of YZ



Example 2: The shoe rack shown, BA=33cm, CB=27cm, CD=44cm, and DE=25cm. Determine if the shelf is parallel to the floor.



Try on your own: Determine whether $PS||QR$

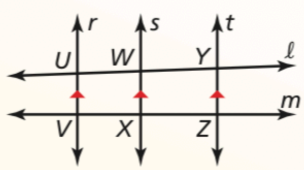


Using other proportionality Theorems

Theorem

Theorem 8.8 Three Parallel Lines Theorem

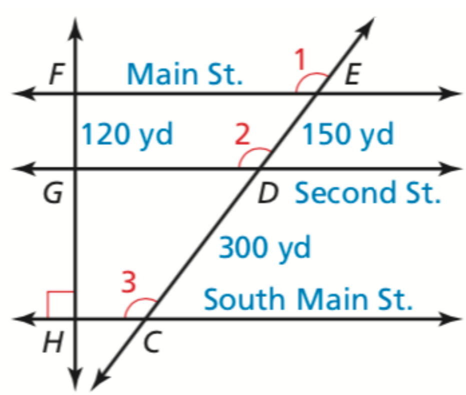
If three parallel lines intersect two transversals, then they divide the transversals proportionally.



$$\frac{UW}{WY} = \frac{VX}{XZ}$$

Proof Ex. 32, p. 451

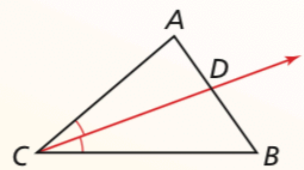
Example 3: Using the Three Parallel Lines Theorem
Find the distance HF between Main Street and South Main Street



Theorem

Theorem 8.9 Triangle Angle Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the other two sides.

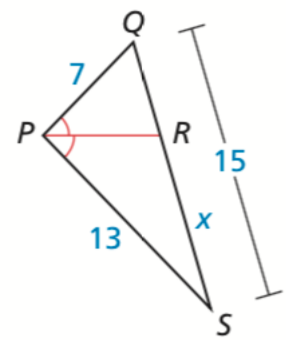


$$\frac{AD}{DB} = \frac{CA}{CB}$$

Proof Ex. 35, p. 452

Example 4: Using the Triangle Bisector Theorem

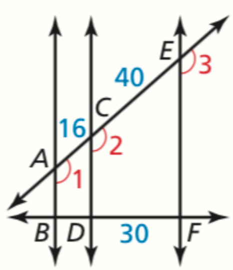
In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of RS



Try on your own:

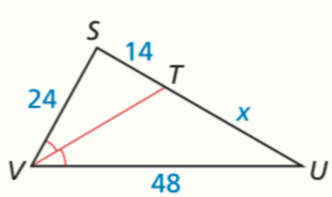
Find the length of the given line segment.

3. $\overline{BD}$



Find the value of the variable.

5.



Homework:

3-8, 13-17, 20, 22, 25*, 29, 30*, 38**

8.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

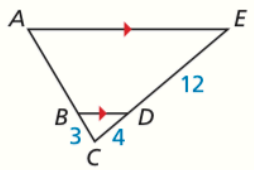
1. COMPLETE THE STATEMENT If a line divides two sides of a triangle proportionally, then it is _________ to the third side. This theorem is known as the ___________.

2. VOCABULARY In $\triangle ABC$, point R lies on $\overline{BC}$ and $\overrightarrow{AR}$ bisects $\angle CAB$. Write the proportionality statement for the triangle that is based on the Triangle Angle Bisector Theorem (Theorem 8.9).

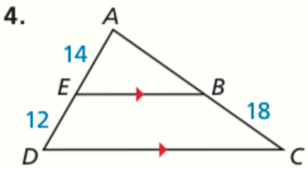
Monitoring Progress and Modeling with Mathematics

In Exercises 3 and 4, find the length of $\overline{AB}$.
(See Example 1.)

3.

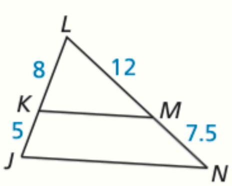


4.



In Exercises 5–8, determine whether $\overline{KM} \parallel \overline{JN}$.
(See Example 2.)

5.



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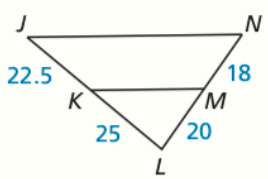

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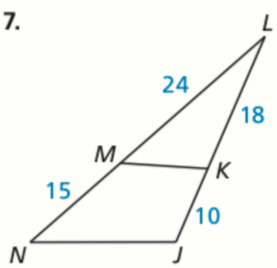
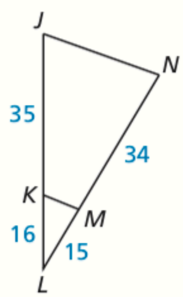


8.



CONSTRUCTION **In Exercises 9–12, draw a segment with the given length. Construct the point that divides the segment in the given ratio.**

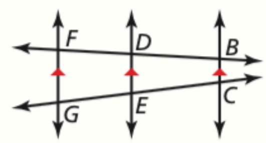
9. 3 in.; 1 to 4

10. 2 in.; 2 to 3

11. 12 cm; 1 to 3

12. 9 cm; 2 to 5

In Exercises 13–16, use the diagram to complete the proportion.



13. $\frac{BD}{BF} = \frac{\square}{CG}$

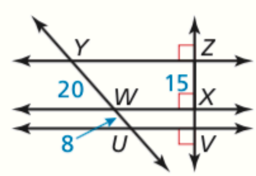
14. $\frac{CG}{\square} = \frac{BF}{DF}$

15. $\frac{EG}{CE} = \frac{DF}{\square}$

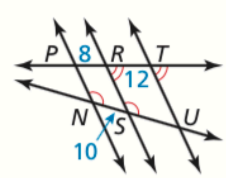
16. $\frac{\square}{BD} = \frac{CG}{CE}$

In Exercises 17 and 18, find the length of the indicated line segment. *(See Example 3.)*

17. $\overline{VX}$

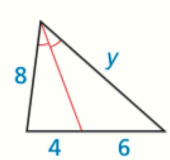


18. $\overline{SU}$

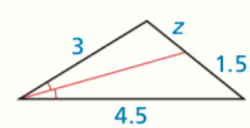


In Exercises 19–22, find the value of the variable.
(See Example 4.)

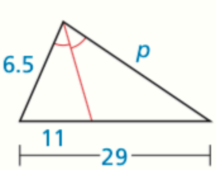
19.



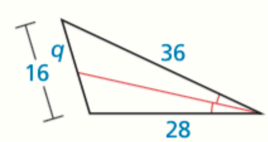
20.



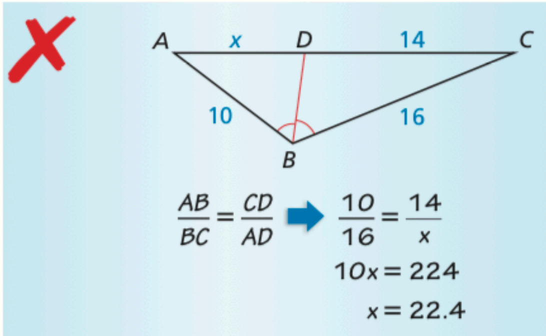
21.



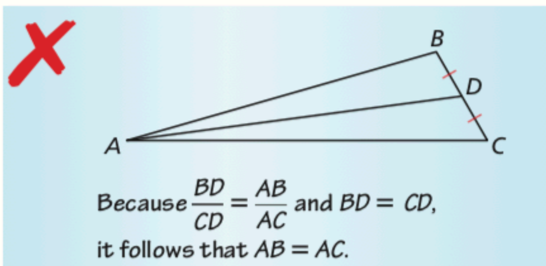
22.



23. **ERROR ANALYSIS** Describe and correct the error in solving for *x*.

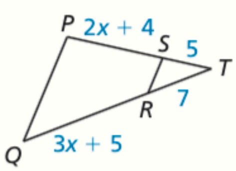


24. **ERROR ANALYSIS** Describe and correct the error in the student's reasoning.

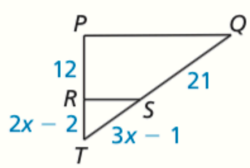


MATHEMATICAL CONNECTIONS In Exercises 25 and 26, find the value of *x* for which $\overline{PQ} \parallel \overline{RS}$.

25.



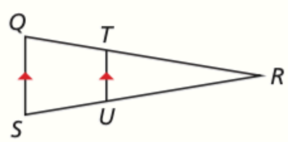
26.



27. **PROVING A THEOREM** Prove the Triangle Proportionality Theorem (Theorem 8.6).

Given $\overline{QS} \parallel \overline{TU}$

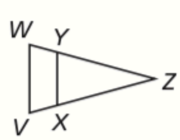
Prove $\frac{QT}{TR} = \frac{SU}{UR}$



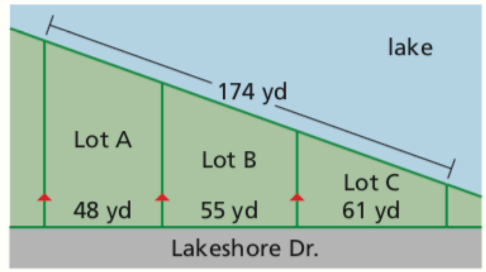
28. **PROVING A THEOREM** Prove the Converse of the Triangle Proportionality Theorem (Theorem 8.7).

Given $\frac{ZY}{YW} = \frac{ZX}{XV}$

Prove $\overline{YX} \parallel \overline{WV}$



29. **MODELING WITH MATHEMATICS** The real estate term *lake frontage* refers to the distance along the edge of a piece of property that touches a lake.

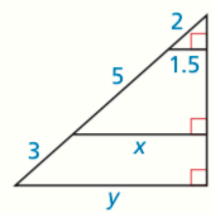


a. Find the lake frontage (to the nearest tenth) of each lot shown.

b. In general, the more lake frontage a lot has, the higher its selling price. Which lot(s) should be listed for the highest price?

c. Suppose that lot prices are in the same ratio as lake frontages. If the least expensive lot is $250,000, what are the prices of the other lots? Explain your reasoning.

30. **USING STRUCTURE** Use the diagram to find the values of *x* and *y*.

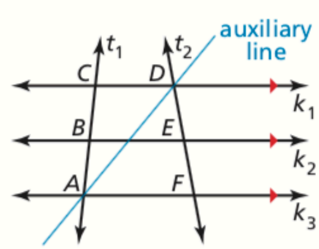


31. **REASONING** In the construction on page 447, explain why you can apply the Triangle Proportionality Theorem (Theorem 8.6) in Step 3.

32. **PROVING A THEOREM** Use the diagram with the auxiliary line drawn to write a paragraph proof of the Three Parallel Lines Theorem (Theorem 8.8).

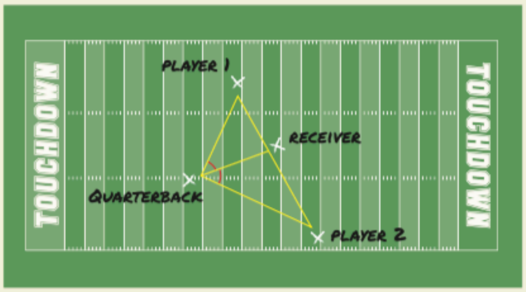
Given $k_1 \parallel k_2 \parallel k_3$

Prove $\frac{CB}{BA} = \frac{DE}{EF}$



33. **CRITICAL THINKING** In $\triangle LMN$, the angle bisector of $\angle M$ also bisects $\overline{LN}$. Classify $\triangle LMN$ as specifically as possible. Justify your answer.

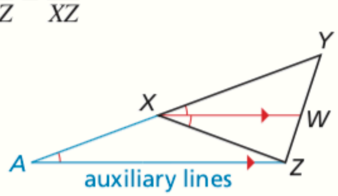
34. **HOW DO YOU SEE IT?** During a football game, the quarterback throws the ball to the receiver. The receiver is between two defensive players, as shown. If Player 1 is closer to the quarterback when the ball is thrown and both defensive players move at the same speed, which player will reach the receiver first? Explain your reasoning.



35. **PROVING A THEOREM** Use the diagram with the auxiliary lines drawn to write a paragraph proof of the Triangle Angle Bisector Theorem (Theorem 8.9).

Given $\angle YXW \cong \angle WXZ$

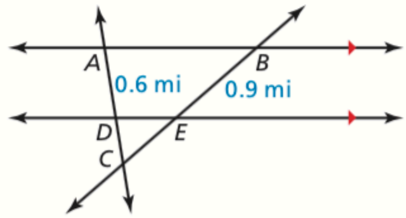
Prove $\dfrac{YW}{WZ} = \dfrac{XY}{XZ}$



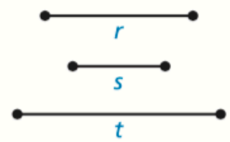
36. **THOUGHT PROVOKING** Write the converse of the Triangle Angle Bisector Theorem (Theorem 8.9). Is the converse true? Justify your answer.

37. **REASONING** How is the Triangle Midsegment Theorem (Theorem 6.8) related to the Triangle Proportionality Theorem (Theorem 8.6)? Explain your reasoning.

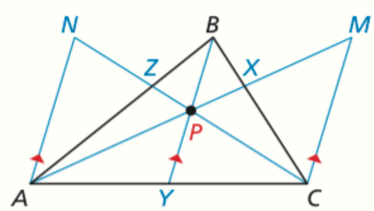
38. **MAKING AN ARGUMENT** Two people leave points A and B at the same time. They intend to meet at point C at the same time. The person who leaves point A walks at a speed of 3 miles per hour. You and a friend are trying to determine how fast the person who leaves point B must walk. Your friend claims you need to know the length of $\overline{AC}$. Is your friend correct? Explain your reasoning.



39. **CONSTRUCTION** Given segments with lengths r, s, and t, construct a segment of length x, such that $\dfrac{r}{s} = \dfrac{t}{x}$.



40. **PROOF** Prove *Ceva's Theorem*: If P is any point inside $\triangle ABC$, then $\dfrac{AY}{YC} \cdot \dfrac{CX}{XB} \cdot \dfrac{BZ}{ZA} = 1$.



(*Hint*: Draw segments parallel to $\overline{BY}$ through A and C, as shown. Apply the Triangle Proportionality Theorem (Theorem 8.6) to $\triangle ACM$. Show that $\triangle APN \sim \triangle MPC$, $\triangle CXM \sim \triangle BXP$, and $\triangle BZP \sim \triangle AZN$.)

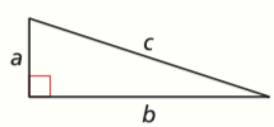
Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Use the triangle. *(Section 5.5)*

41. Which sides are the legs?

42. Which side is the hypotenuse?



Solve the equation. *(Skills Review Handbook)*

43. $x^2 = 121$

44. $x^2 + 16 = 25$

45. $36 + x^2 = 85$