9.2 Special Right Triangles

There are two types of special right triangles; **45-45-90** Triangles and **30-60-90** Triangles

45-45-90 Triangles

If a triangle has two 45° angles and a 90° angle what type of triangle would it be?

Theorem

Theorem 9.4 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as each leg.

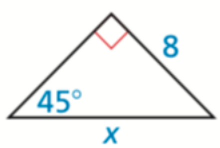
hypotenuse = leg $\cdot \sqrt{2}$

Proof Ex. 19, p. 476

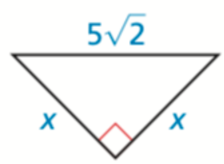
Example 1: Finding Lengths in a 45-45-90 Triangle

Find the value of x. Write your answer in simplest form.

a.



b.

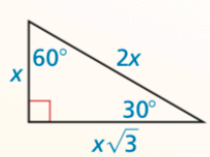


30-60-90 Triangles

Theorem

Theorem 9.5 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

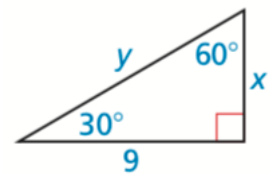


hypotenuse = shorter leg $\cdot$ 2

longer leg = shorter leg $\cdot \sqrt{3}$

Proof Ex. 21, p. 476

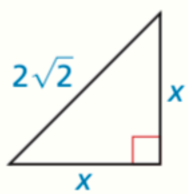
Example 2: Find the side lengths in a 30-60-90 Triangle.
Find the values of x and y. Write your answer in simplest form.



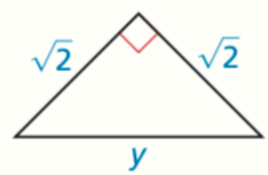
Try on your own.

Find the value of the variable. Write your answer in simplest form.

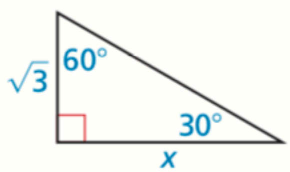
1.



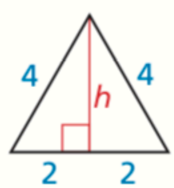
2.



3.

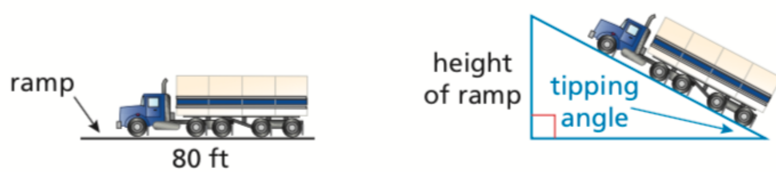


4.



Example 3: Modeling with Mathematics
The yield sign is shaped like an equilateral triangle. Estimate the area of the sign.

Example 4: Finding the Height of a Ramp
A tipping platform is a ramp used to unload trucks. How high is the end of an 80ft ramp when the tipping angle is 30°? 45°?



Homework: 3-10, 13, 14, 17, 22, 24, 25*

9.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

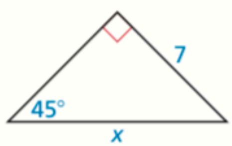
Vocabulary and Core Concept Check

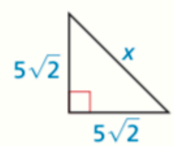
1. **VOCABULARY** Name two special right triangles by their angle measures.

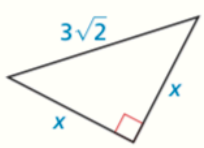
2. **WRITING** Explain why the acute angles in an isosceles right triangle always measure 45°.

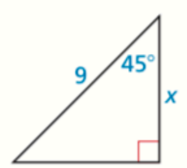
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the value of *x*. Write your answer in simplest form. *(See Example 1.)*

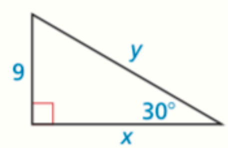
3. 

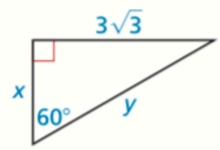
4. 

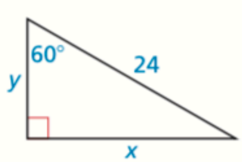
5. 

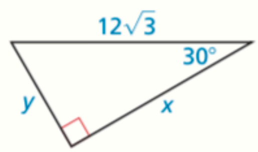
6. 

In Exercises 7–10, find the values of *x* and *y*. Write your answers in simplest form. *(See Example 2.)*

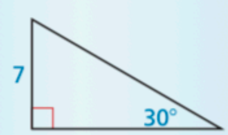
7. 

8. 

9. 

10. 

ERROR ANALYSIS In Exercises 11 and 12, describe and correct the error in finding the length of the hypotenuse.

11. ✗ 

By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 60°. So, the triangle is a 30°-60°-90° triangle.

hypotenuse = shorter leg • $\sqrt{3} = 7\sqrt{3}$

So, the length of the hypotenuse is $7\sqrt{3}$ units.

12. ✗ 

By the Triangle Sum Theorem (Theorem 5.1), the measure of the third angle must be 45°. So, the triangle is a 45°-45°-90° triangle.

hypotenuse = leg • leg • $\sqrt{2} = 5\sqrt{2}$

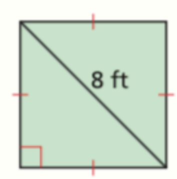
So, the length of the hypotenuse is $5\sqrt{2}$ units.

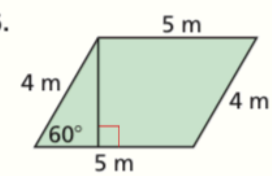
In Exercises 13 and 14, sketch the figure that is described. Find the indicated length. Round decimal answers to the nearest tenth.

13. The side length of an equilateral triangle is 5 centimeters. Find the length of an altitude.

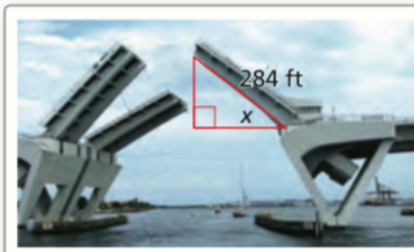
14. The perimeter of a square is 36 inches. Find the length of a diagonal.

In Exercises 15 and 16, find the area of the figure. Round decimal answers to the nearest tenth. *(See Example 3.)*

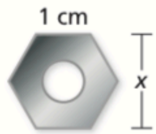
15. 

16. 

17. **PROBLEM SOLVING** Each half of the drawbridge is about 284 feet long. How high does the drawbridge rise when *x* is 30°? 45°? 60°? *(See Example 4.)*



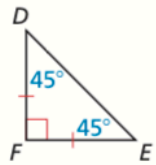
18. MODELING WITH MATHEMATICS A nut is shaped like a regular hexagon with side lengths of 1 centimeter. Find the value of x. (*Hint*: A regular hexagon can be divided into six congruent triangles.)



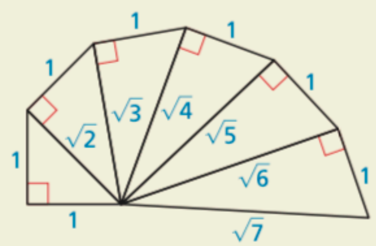
19. PROVING A THEOREM Write a paragraph proof of the 45°-45°-90° Triangle Theorem (Theorem 9.4).

Given $\triangle DEF$ is a 45°-45°-90° triangle.

Prove The hypotenuse is $\sqrt{2}$ times as long as each leg.



20. HOW DO YOU SEE IT? The diagram shows part of the *Wheel of Theodorus*.



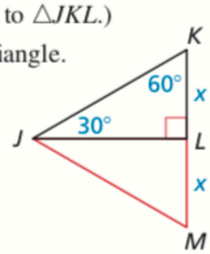
a. Which triangles, if any, are 45°-45°-90° triangles?

b. Which triangles, if any, are 30°-60°-90° triangles?

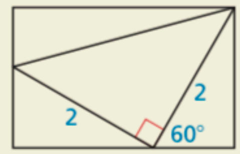
21. PROVING A THEOREM Write a paragraph proof of the 30°-60°-90° Triangle Theorem (Theorem 9.5). (*Hint*: Construct $\triangle JML$ congruent to $\triangle JKL$.)

Given $\triangle JKL$ is a 30°-60°-90° triangle.

Prove The hypotenuse is twice as long as the shorter leg, and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

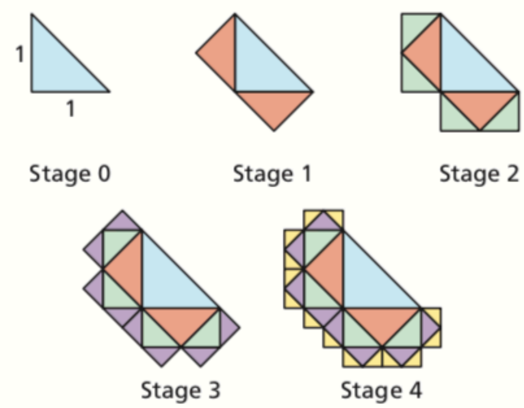


22. THOUGHT PROVOKING The diagram below is called the *Ailles rectangle*. Each triangle in the diagram has rational angle measures and each side length contains at most one square root. Label the sides and angles in the diagram. Describe the triangles.



23. WRITING Describe two ways to show that all isosceles right triangles are similar to each other.

24. MAKING AN ARGUMENT Each triangle in the diagram is a 45°-45°-90° triangle. At Stage 0, the legs of the triangle are each 1 unit long. Your brother claims the lengths of the legs of the triangles added are halved at each stage. So, the length of a leg of a triangle added in Stage 8 will be $\frac{1}{256}$ unit. Is your brother correct? Explain your reasoning.



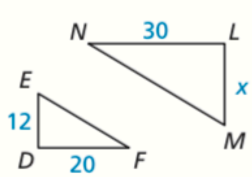
25. USING STRUCTURE $\triangle TUV$ is a 30°-60°-90° triangle, where two vertices are $U(3, -1)$ and $V(-3, -1)$, $\overline{UV}$ is the hypotenuse, and point T is in Quadrant I. Find the coordinates of T.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the value of *x*. *(Section 8.1)*

26. $\triangle DEF \sim \triangle LMN$



27. $\triangle ABC \sim \triangle QRS$