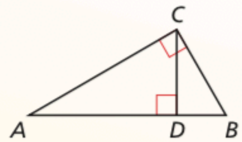
9.3 Similar Right Triangles

Do Now: Complete Mini Lab

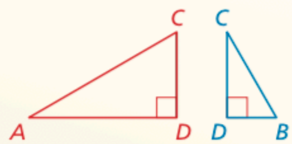
Theorem

Theorem 9.6 Right Triangle Similarity Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

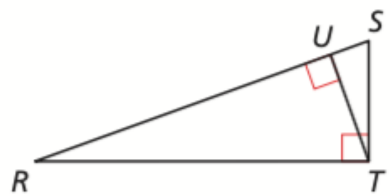


$\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, and $\triangle CBD \sim \triangle ACD$.

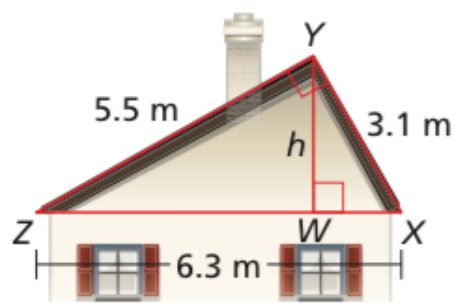


Proof Ex. 45, p. 484

Example 1: Identifying Similar Triangles
Identify the similar triangles in the diagram.



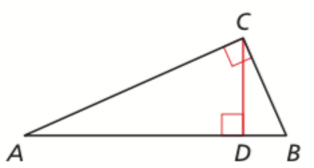
Example 2: Modeling with Mathematics
A roof has a cross section that is a right triangle. The diagram shows the approximate dimensions of this cross section. Find the height h of the roof.



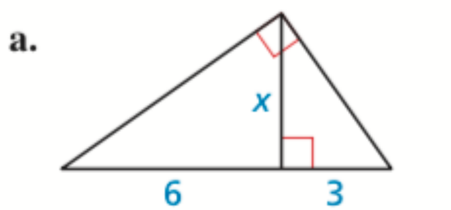
Geometric Mean:
The geometric mean of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$

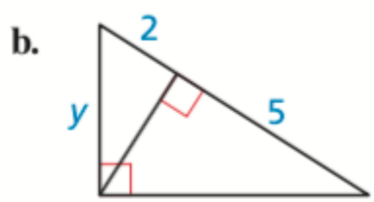
Example 3: Finding a Geometric Mean
Find the geometric mean of 24 and 48.

Identify all the triangles in the diagram. Create an equation to find CD, CB, and AC.

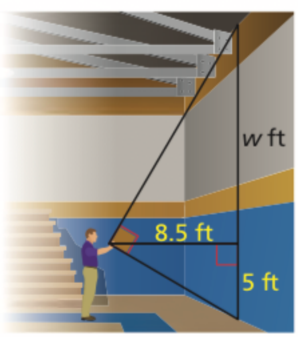


Example 4: Using a Geometric Mean
Find the value of each variable.

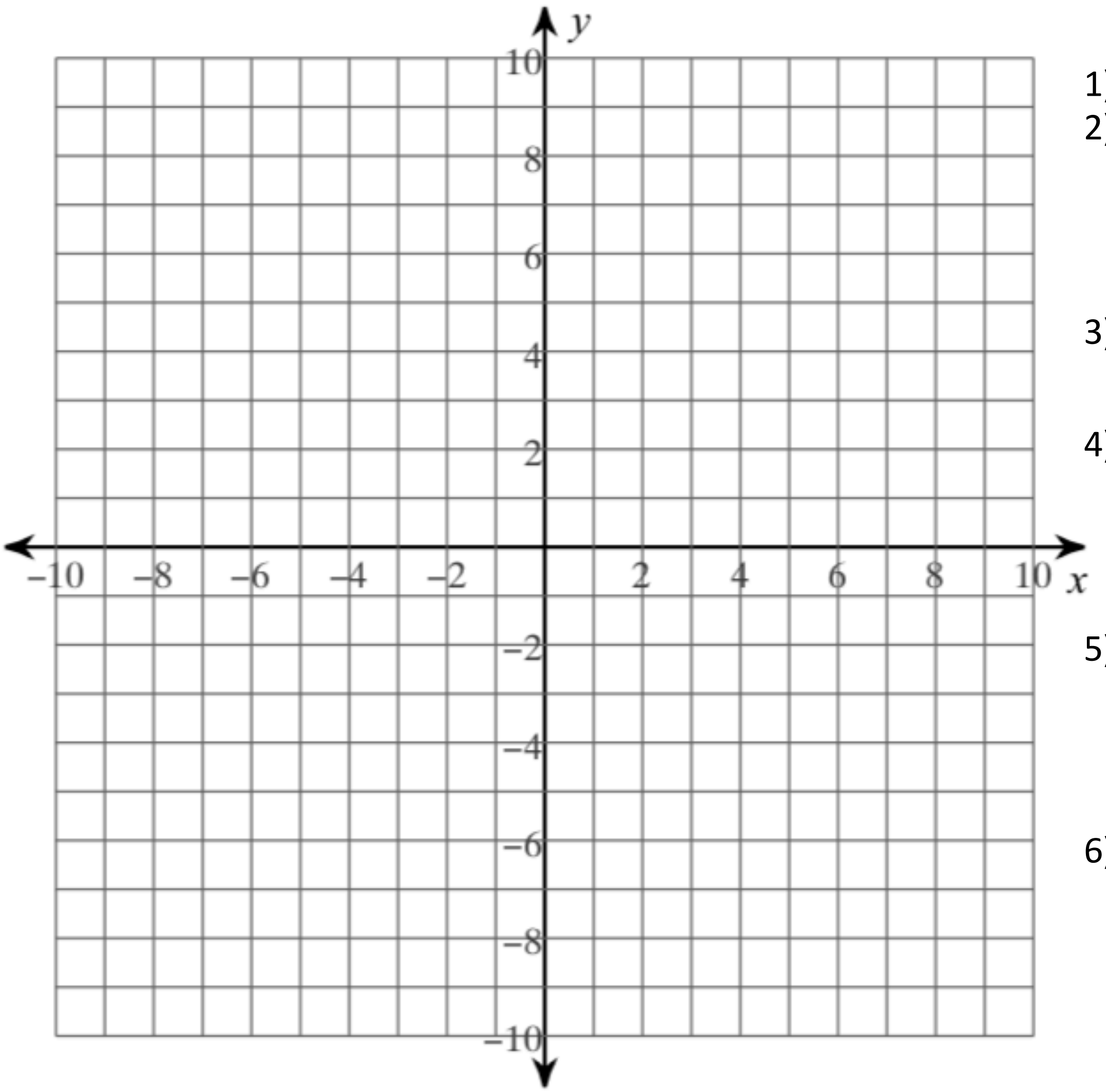




Example 5: Using indirect measuring.
Approximate the height of the gym wall.



Homework:
4, 5, 7, 8, 11-17odd, 20, 21, 22, 24, 25, 31, 33, 34



1) Graph Triangle A(-8,-5) , B(4,1) , C(7,-5)
2) Determine the length of
   I. AB= _____
   II. BC= _____
   III. CA= _____

3) Construct the altitude from point B to line AC
   I. Label the new intersection point D(___ , ___)

4) Determine the side lengths of ΔDBA
   I. DB= _____
   II. BA= _____
   III. AD= _____

5) Determine the side lengths of ΔDBC
   I. DB= _____
   II. BC= _____
   III. CD= _____

6) Compare side lengths of ΔABC, ΔBDA, ΔCDB

9.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

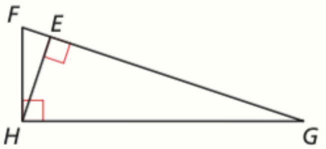
Vocabulary and Core Concept Check

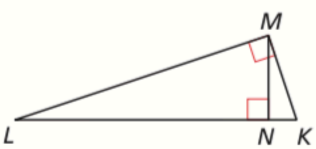
1. **COMPLETE THE SENTENCE** If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and __________________.

2. **WRITING** In your own words, explain *geometric mean*.

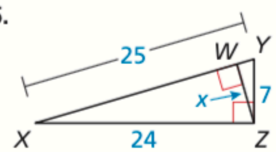
Monitoring Progress and Modeling with Mathematics

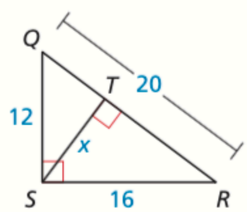
In Exercises 3 and 4, identify the similar triangles. *(See Example 1.)*

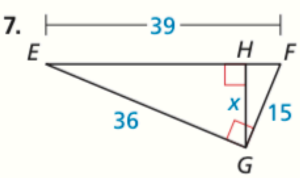
3. 

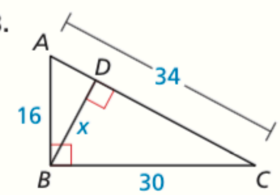
4. 

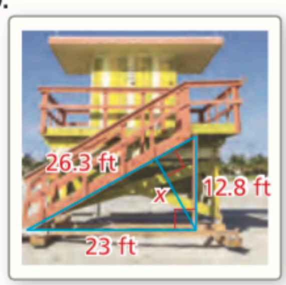
In Exercises 5–10, find the value of *x*. *(See Example 2.)*

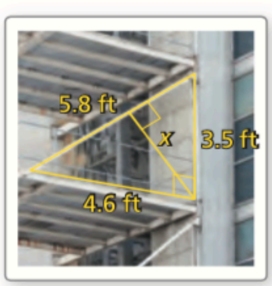
5. 

6. 

7. 

8. 

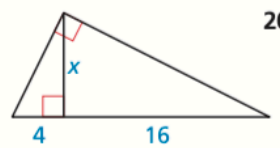
9. 

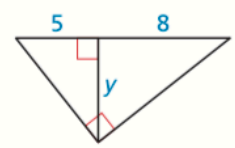
10. 

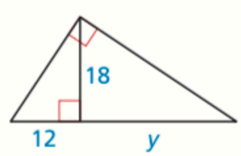
In Exercises 11–18, find the geometric mean of the two numbers. *(See Example 3.)*

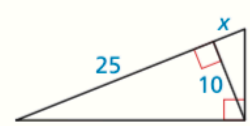
11. 8 and 32
12. 9 and 16
13. 14 and 20
14. 25 and 35
15. 16 and 25
16. 8 and 28
17. 17 and 36
18. 24 and 45

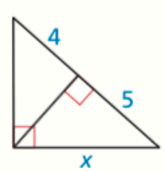
In Exercises 19–26, find the value of the variable. *(See Example 4.)*

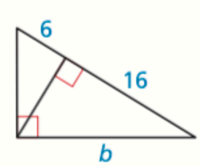
19. 

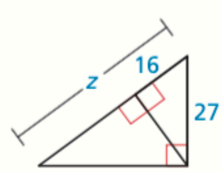
20. 

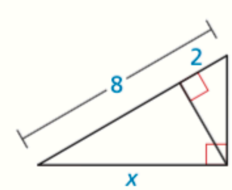
21. 

22. 

23. 

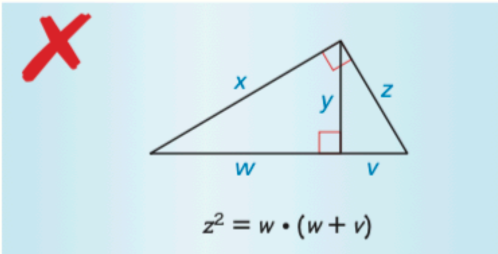
24. 

25. 

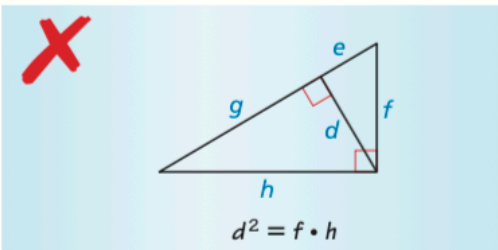
26. 

ERROR ANALYSIS In Exercises 27 and 28, describe and correct the error in writing an equation for the given diagram.

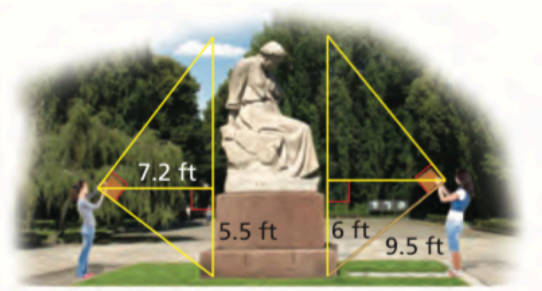
27.



28.



MODELING WITH MATHEMATICS In Exercises 29 and 30, use the diagram. *(See Example 5.)*



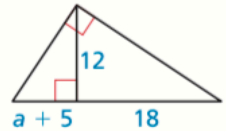
Ex. 29 Ex. 30

29. You want to determine the height of a monument at a local park. You use a cardboard square to line up the top and bottom of the monument, as shown at the above left. Your friend measures the vertical distance from the ground to your eye and the horizontal distance from you to the monument. Approximate the height of the monument.

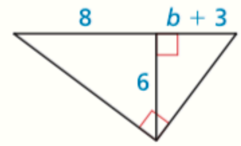
30. Your classmate is standing on the other side of the monument. She has a piece of rope staked at the base of the monument. She extends the rope to the cardboard square she is holding lined up to the top and bottom of the monument. Use the information in the diagram above to approximate the height of the monument. Do you get the same answer as in Exercise 29? Explain your reasoning.

MATHEMATICAL CONNECTIONS In Exercises 31–34, find the value(s) of the variable(s).

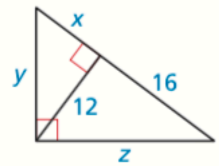
31.



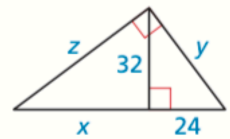
32.



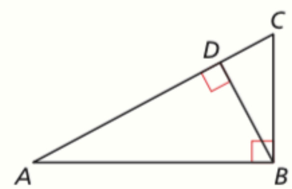
33.



34.



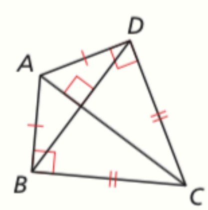
35. REASONING Use the diagram. Decide which proportions are true. Select all that apply.



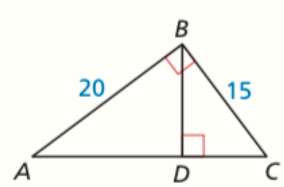
Ⓐ $\frac{DB}{DC} = \frac{DA}{DB}$ Ⓑ $\frac{BA}{CB} = \frac{CB}{BD}$

Ⓒ $\frac{CA}{BA} = \frac{BA}{CA}$ Ⓓ $\frac{DB}{BC} = \frac{DA}{BA}$

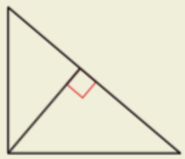
36. ANALYZING RELATIONSHIPS You are designing a diamond-shaped kite. You know that $AD = 44.8$ centimeters, $DC = 72$ centimeters, and $AC = 84.8$ centimeters. You want to use a straight crossbar $\overline{BD}$. About how long should it be? Explain your reasoning.

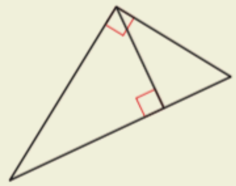


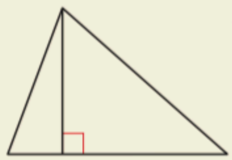
37. ANALYZING RELATIONSHIPS Use the Geometric Mean Theorems (Theorems 9.7 and 9.8) to find AC and BD.

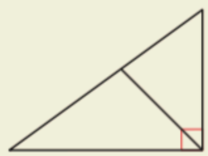


38. HOW DO YOU SEE IT? In which of the following triangles does the Geometric Mean (Altitude) Theorem (Theorem 9.7) apply?

Ⓐ 

Ⓑ 

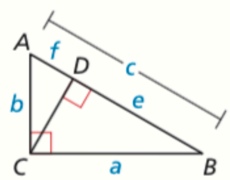
Ⓒ 

Ⓓ 

39. PROVING A THEOREM Use the diagram of $\triangle ABC$. Copy and complete the proof of the Pythagorean Theorem (Theorem 9.1).

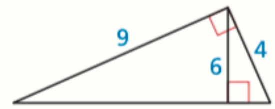
Given In $\triangle ABC$, $\angle BCA$ is a right angle.

Prove $c^2 = a^2 + b^2$



STATEMENTS	REASONS
1. In $\triangle ABC$, $\angle BCA$ is a right angle.	1. ________
2. Draw a perpendicular segment (altitude) from C to $\overline{AB}$.	2. Perpendicular Postulate (Postulate 3.2)
3. $ce = a^2$ and $cf = b^2$	3. ________
4. $ce + b^2 =$ ____ $+ b^2$	4. Addition Property of Equality
5. $ce + cf = a^2 + b^2$	5. ________
6. $c(e + f) = a^2 + b^2$	6. ________
7. $e + f =$ ____	7. Segment Addition Postulate (Postulate 1.2)
8. $c \cdot c = a^2 + b^2$	8. ________
9. $c^2 = a^2 + b^2$	9. Simplify.

40. MAKING AN ARGUMENT Your friend claims the geometric mean of 4 and 9 is 6, and then labels the triangle, as shown. Is your friend correct? Explain your reasoning.



In Exercises 41 and 42, use the given statements to prove the theorem.

Given $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

41. PROVING A THEOREM Prove the Geometric Mean (Altitude) Theorem (Theorem 9.7) by showing that $CD^2 = AD \cdot BD$.

42. PROVING A THEOREM Prove the Geometric Mean (Leg) Theorem (Theorem 9.8) by showing that $CB^2 = DB \cdot AB$ and $AC^2 = AD \cdot AB$.

43. CRITICAL THINKING Draw a right isosceles triangle and label the two leg lengths x. Then draw the altitude to the hypotenuse and label its length y. Now, use the Right Triangle Similarity Theorem (Theorem 9.6) to draw the three similar triangles from the image and label any side length that is equal to either x or y. What can you conclude about the relationship between the two smaller triangles? Explain your reasoning.

44. THOUGHT PROVOKING The arithmetic mean and geometric mean of two nonnegative numbers x and y are shown.

$$\text{arithmetic mean} = \frac{x + y}{2}$$

$$\text{geometric mean} = \sqrt{xy}$$

Write an inequality that relates these two means. Justify your answer.

45. PROVING A THEOREM Prove the Right Triangle Similarity Theorem (Theorem 9.6) by proving three similarity statements.

Given $\triangle ABC$ is a right triangle.
Altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

Prove $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ACD$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation for x. *(Skills Review Handbook)*

46. $13 = \frac{x}{5}$ **47.** $29 = \frac{x}{4}$ **48.** $9 = \frac{78}{x}$ **49.** $30 = \frac{115}{x}$