

## 9.4 The tangent Ratio

The **tangent** ratio is a trigonometric ratio for acute angles using the leg lengths of a right triangle.

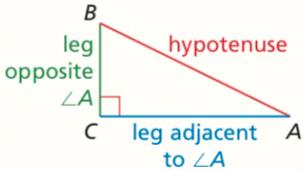
### Core Concept

#### Tangent Ratio

Let  $\triangle ABC$  be a right triangle with acute  $\angle A$ .

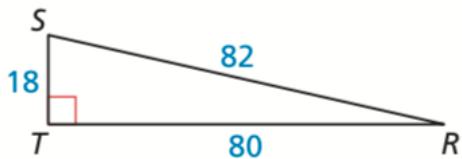
The tangent of  $\angle A$  (written as  $\tan A$ ) is defined as follows.

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A} = \frac{BC}{AC}$$



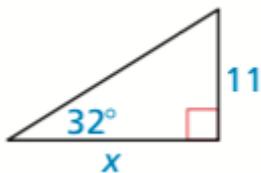
#### Example 1: Finding Tangent Ratios

Find  $\tan S$  and  $\tan R$ . Write each answer as a fraction and a decimal. (4 decimal places)



#### Example 2: Finding a Leg Length

Find the value of  $x$ . Round your answer to the nearest tenth.

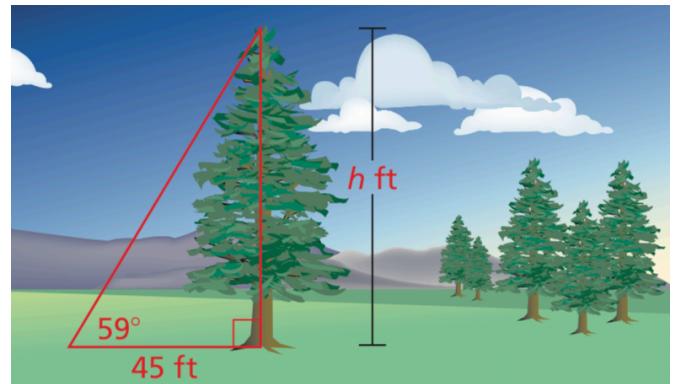


#### Example 3: Using a Special Right Triangle to Find a Tangent

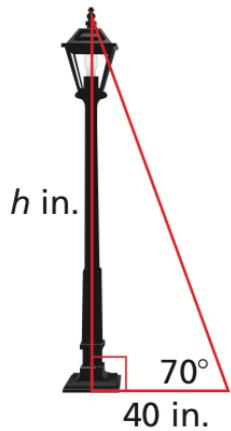
Use a special right triangle to find the tangent of a  $60^\circ$  angle.

**Example 4: Modeling with Mathematics**

You are measuring the height of a tree. You stand 45 feet from the base of the tree. You measure the angle of elevation from the ground to the top of the tree to be  $59^\circ$ . Find the height  $h$  of the tree to the nearest foot.

**Try on your own:**

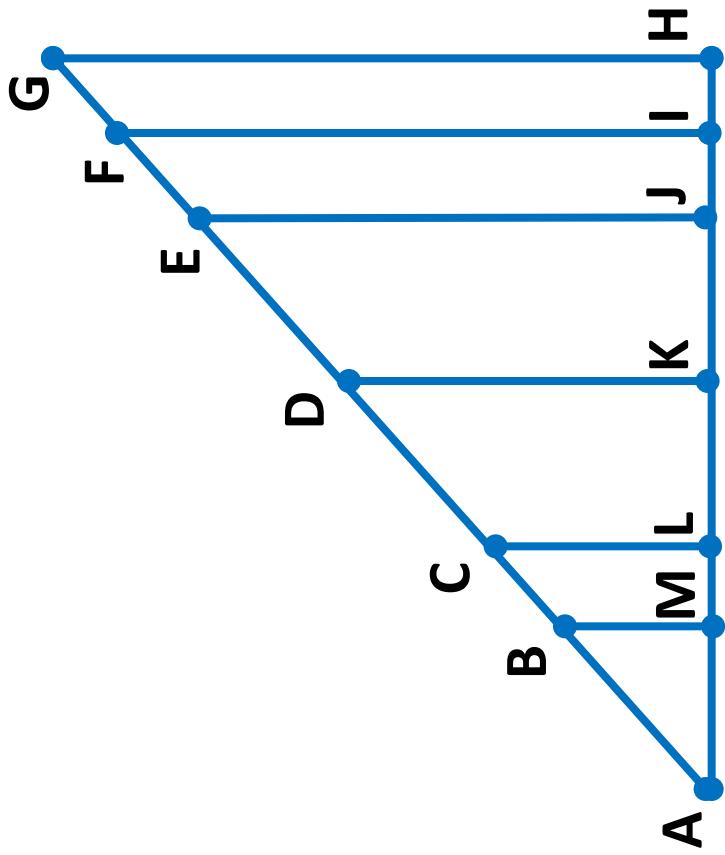
You are measuring the height of a lamp. You stand 40 inches from the base of the lamp. You measure the angle of elevation from the ground to the top of the lamp to be  $70^\circ$ . Find the height  $h$  of the lamp to the nearest inch.



# TANGENT

Using a ruler (in millimeters) determine the following  
Ratios of the sides.

Sides	$\frac{MB}{AM}$	$\frac{LC}{AI}$	$\frac{KD}{AK}$	$\frac{JE}{AJ}$	$\frac{IF}{AI}$	$\frac{HG}{AH}$
Ratio						



## Table of $\tan(\text{angle})$

Angle	$\tan(a)$
0.0	0.00
1.0	.0175
2.0	.0349
3.0	.0524
4.0	.0699
5.0	.0875
6.0	.1051
7.0	.1228
8.0	.1405
9.0	.1584
10.0	.1763
11.0	.1944
12.0	.2126
13.0	.2309
14.0	.2493
15.0	.2679
16.0	.2867
17.0	.3057
18.0	.3249
19.0	.3443
20.0	.3640
21.0	.3839
22.0	.4040
23.0	.4245
24.0	.4452

Angle	$\tan(a)$
25.0	.4663
26.0	.4877
27.0	.5095
28.0	.5317
29.0	.5543
30.0	.5773
31.0	.6009
32.0	.6249
33.0	.6494
34.0	.6745
35.0	.7002
36.0	.7265
37.0	.7535
38.0	.7813
39.0	.8098
40.0	.8391
41.0	.8693
42.0	.9004
43.0	.9325
44.0	.9657
45.0	1.000

Angle	$\tan(a)$
46.0	1.0355
47.0	1.0724
48.0	1.1106
49.0	1.1504
50.0	1.1918
51.0	1.2349
52.0	1.2799
53.0	1.3270
54.0	1.3764
55.0	1.4281
56.0	1.4826
57.0	1.5399
58.0	1.6003
59.0	1.6643
60.0	1.7321
61.0	1.8040
62.0	1.8907
63.0	1.9626
64.0	2.0503
65.0	2.1445
66.0	2.2460
67.0	2.3559
68.0	2.4751
69.0	2.6051
70.0	2.7475

Angle	$\tan(a)$
71.0	2.9042
72.0	3.0777
73.0	3.2709
74.0	3.4874
75.0	3.7321
76.0	4.0108
77.0	4.3315
78.0	4.7046
79.0	5.1446
80.0	5.6713
81.0	6.3138
82.0	7.1154
83.0	8.1443
84.0	9.5144
85.0	11.430
86.0	14.301
87.0	19.081
88.0	28.636
89.0	57.290
90.0	infinite

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## 9.4 Exercises

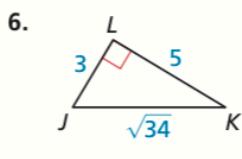
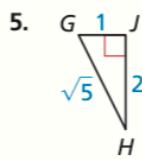
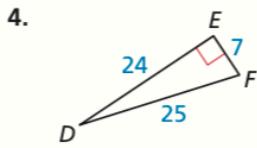
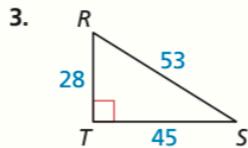
Dynamic Solutions available at [BigIdeasMath.com](http://BigIdeasMath.com)

### Vocabulary and Core Concept Check

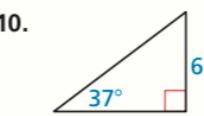
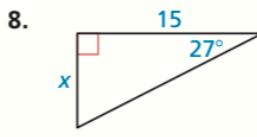
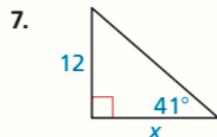
1. **COMPLETE THE SENTENCE** The tangent ratio compares the length of \_\_\_\_\_ to the length of \_\_\_\_\_.
2. **WRITING** Explain how you know the tangent ratio is constant for a given angle measure.

### Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, find the tangents of the acute angles in the right triangle. Write each answer as a fraction and as a decimal rounded to four decimal places.  
(See Example 1.)

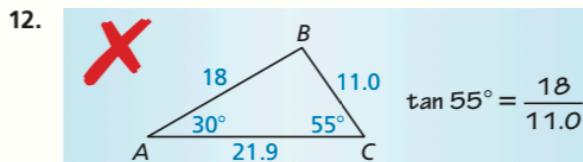
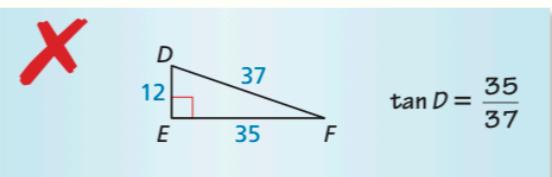


In Exercises 7–10, find the value of  $x$ . Round your answer to the nearest tenth. (See Example 2.)



**ERROR ANALYSIS** In Exercises 11 and 12, describe the error in the statement of the tangent ratio. Correct the error if possible. Otherwise, write not possible.

11.



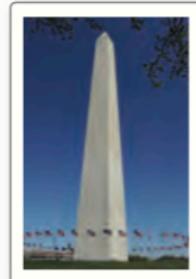
In Exercises 13 and 14, use a special right triangle to find the tangent of the given angle measure.  
(See Example 3.)

13.  $45^\circ$

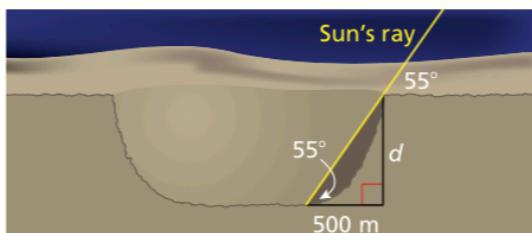
14.  $30^\circ$

15. **MODELING WITH MATHEMATICS**

A surveyor is standing 118 feet from the base of the Washington Monument. The surveyor measures the angle of elevation from the ground to the top of the monument to be  $78^\circ$ . Find the height  $h$  of the Washington Monument to the nearest foot.  
(See Example 4.)



16. **MODELING WITH MATHEMATICS** Scientists can measure the depths of craters on the moon by looking at photos of shadows. The length of the shadow cast by the edge of a crater is 500 meters. The angle of elevation of the rays of the Sun is  $55^\circ$ . Estimate the depth  $d$  of the crater.



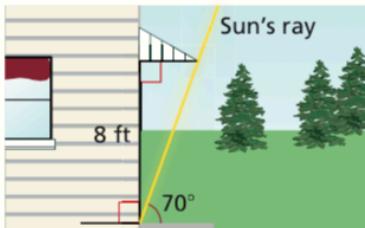
17. **USING STRUCTURE** Find the tangent of the smaller acute angle in a right triangle with side lengths 5, 12, and 13.

18. **USING STRUCTURE** Find the tangent of the larger acute angle in a right triangle with side lengths 3, 4, and 5.

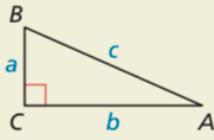
19. **REASONING** How does the tangent of an acute angle in a right triangle change as the angle measure increases? Justify your answer.

20. **CRITICAL THINKING** For what angle measure(s) is the tangent of an acute angle in a right triangle equal to 1? greater than 1? less than 1? Justify your answer.

21. **MAKING AN ARGUMENT** Your family room has a sliding-glass door. You want to buy an awning for the door that will be just long enough to keep the Sun out when it is at its highest point in the sky. The angle of elevation of the rays of the Sun at this point is  $70^\circ$ , and the height of the door is 8 feet. Your sister claims you can determine how far the overhang should extend by multiplying 8 by  $\tan 70^\circ$ . Is your sister correct? Explain.

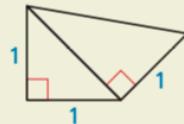


22. **HOW DO YOU SEE IT?** Write expressions for the tangent of each acute angle in the right triangle. Explain how the tangent of one acute angle is related to the tangent of the other acute angle. What kind of angle pair is  $\angle A$  and  $\angle B$ ?

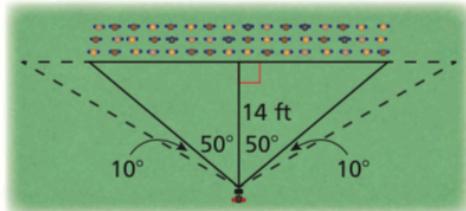


23. **REASONING** Explain why it is not possible to find the tangent of a right angle or an obtuse angle.

24. **THOUGHT PROVOKING** To create the diagram below, you begin with an isosceles right triangle with legs 1 unit long. Then the hypotenuse of the first triangle becomes the leg of a second triangle, whose remaining leg is 1 unit long. Continue the diagram until you have constructed an angle whose tangent is  $\frac{1}{\sqrt{6}}$ . Approximate the measure of this angle.

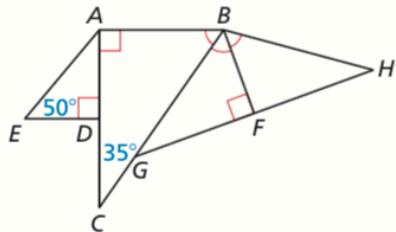


25. **PROBLEM SOLVING** Your class is having a class picture taken on the lawn. The photographer is positioned 14 feet away from the center of the class. The photographer turns  $50^\circ$  to look at either end of the class.



a. What is the distance between the ends of the class?  
 b. The photographer turns another  $10^\circ$  either way to see the end of the camera range. If each student needs 2 feet of space, about how many more students can fit at the end of each row? Explain.

26. **PROBLEM SOLVING** Find the perimeter of the figure, where  $AC = 26$ ,  $AD = BF$ , and  $D$  is the midpoint of  $\overline{AC}$ .

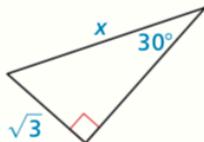


## Maintaining Mathematical Proficiency

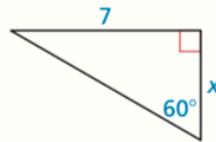
Reviewing what you learned in previous grades and lessons

Find the value of  $x$ . (Section 9.2)

27.



28.



29.

