

## Chapter 7

### Radical Functions

#### 7.1 Inverse Variation

**Do Now:** Consider the following equation:  $y = 2x$ .

- a) As  $x$  increases what happens to  $y$ ?
- b) As  $x$  decreases what happens to  $y$ ?
- c) What can we conclude about the relationship between  $x$  &  $y$ ?

#### Direct Variation and Indirect Variation

Direct variation:  $y = ax$

Indirect variation:  $y = \frac{a}{x}$

#### Example 1: Classifying Equations

Tell whether  $x$  and  $y$  show direct variation, inverse variation, or neither. (Solve each equation for  $y$ )

a)  $xy = 5$

b)  $y = x - 4$

c)  $\frac{y}{2} = x$

#### Key Notes:

A set of data pairs  $(x,y)$  shows direct variation when the ratios  $\frac{x}{y}$  are constant.

A set of data pairs  $(x,y)$  shows indirect variation when the ratios  $xy = a$  are constant.

#### Example 2: Classifying Data

Tell whether  $x$  and  $y$  show direct variation, indirect variation, or neither.

a.

<b>x</b>	2	4	6	8
<b>y</b>	-12	-6	-4	-3

b.

<b>x</b>	1	2	3	4
<b>y</b>	2	4	8	16

**Try on your own:** Tell whether  $x$  and  $y$  show direct variation, indirect variation, or neither.

4.

$x$	-4	-3	-2	-1
$y$	20	15	10	5

5.

$x$	1	2	3	4
$y$	60	30	20	15

**Example 3:** Writing an Inverse Variation Equation

The variables  $x$  and  $y$  vary inversely, and  $y = 4$  when  $x = 3$ . Write an equation that relates  $x$  and  $y$ . Then find  $y$  when  $x = -2$ .

**Example 4:** Modeling with Mathematics

The time  $t$  (in hours) that it takes a group of volunteers to build a playground varies inversely with the number  $n$  of volunteers. It takes a group of 10 volunteers 8 hours to build the playground.

- a) Make a table showing the time that it would take to build the playground when the number of volunteers is 15, 20, 25, and 30.

15	20	25	30

- b) What happens to the time it takes to build the playground as the number of volunteers increases?

## 7.1 Exercises

Dynamic Solutions available at [BigIdeasMath.com](http://BigIdeasMath.com)

### Vocabulary and Core Concept Check

- VOCABULARY** Explain how direct variation equations and inverse variation equations are different.
- DIFFERENT WORDS, SAME QUESTION** Which is different? Find “both” answers.

What is an inverse variation equation relating  $x$  and  $y$  with  $a = 4$ ?

What is an equation for which the ratios  $\frac{y}{x}$  are constant and  $a = 4$ ?

What is an equation for which  $y$  varies inversely with  $x$  and  $a = 4$ ?

What is an equation for which the products  $xy$  are constant and  $a = 4$ ?

### Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, tell whether  $x$  and  $y$  show *direct variation*, *inverse variation*, or *neither*. (See Example 1.)

- $y = \frac{2}{x}$
- $xy = 12$
- $\frac{y}{x} = 8$
- $4x = y$
- $y = x + 4$
- $x + y = 6$
- $8y = x$
- $xy = \frac{1}{5}$

In Exercises 11–14, tell whether  $x$  and  $y$  show *direct variation*, *inverse variation*, or *neither*. (See Example 2.)

11.

$x$	12	18	23	29	34
$y$	132	198	253	319	374

12.

$x$	1.5	2.5	4	7.5	10
$y$	13.5	22.5	36	67.5	90

13.

$x$	4	6	8	8.4	12
$y$	21	14	10.5	10	7

14.

$x$	4	5	6.2	7	11
$y$	16	11	10	9	6

In Exercises 15–22, the variables  $x$  and  $y$  vary inversely. Use the given values to write an equation relating  $x$  and  $y$ . Then find  $y$  when  $x = 3$ . (See Example 3.)

- $x = 5, y = -4$
- $x = 1, y = 9$
- $x = -3, y = 8$
- $x = 7, y = 2$
- $x = \frac{3}{4}, y = 28$
- $x = -4, y = -\frac{5}{4}$
- $x = -12, y = -\frac{1}{6}$
- $x = \frac{5}{3}, y = -7$

**ERROR ANALYSIS** In Exercises 23 and 24, the variables  $x$  and  $y$  vary inversely. Describe and correct the error in writing an equation relating  $x$  and  $y$ .

23.  $x = 8, y = 5$

**X**

$$y = ax$$

$$5 = a(8)$$

$$\frac{5}{8} = a$$

$$\text{So, } y = \frac{5}{8}x.$$

24.  $x = 5, y = 2$

**X**

$$xy = a$$

$$5 \cdot 2 = a$$

$$10 = a$$

$$\text{So, } y = 10x.$$

25. **MODELING WITH MATHEMATICS** The number  $y$  of songs that can be stored on an MP3 player varies inversely with the average size  $x$  of a song. A certain MP3 player can store 2500 songs when the average size of a song is 4 megabytes (MB). (See Example 4.)

- Make a table showing the numbers of songs that will fit on the MP3 player when the average size of a song is 2 MB, 2.5 MB, 3 MB, and 5 MB.
- What happens to the number of songs as the average song size increases?

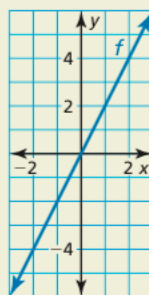
26. **MODELING WITH MATHEMATICS** When you stand on snow, the average pressure  $P$  (in pounds per square inch) that you exert on the snow varies inversely with the total area  $A$  (in square inches) of the soles of your footwear. Suppose the pressure is 0.43 pound per square inch when you wear the snowshoes shown. Write an equation that gives  $P$  as a function of  $A$ . Then find the pressure when you wear the boots shown.



27. **PROBLEM SOLVING** Computer chips are etched onto silicon wafers. The table compares the area  $A$  (in square millimeters) of a computer chip with the number  $c$  of chips that can be obtained from a silicon wafer. Write a model that gives  $c$  as a function of  $A$ . Then predict the number of chips per wafer when the area of a chip is 81 square millimeters.

Area ( $\text{mm}^2$ ), $A$	58	62	66	70
Number of chips, $c$	448	424	392	376

28. **HOW DO YOU SEE IT?** Does the graph of  $f$  represent inverse variation or direct variation? Explain your reasoning.



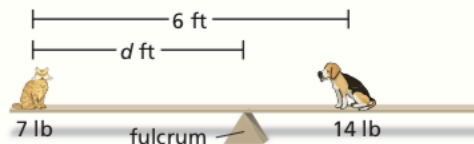
29. **MAKING AN ARGUMENT** You have enough money to buy 5 hats for \$10 each or 10 hats for \$5 each. Your friend says this situation represents inverse variation. Is your friend correct? Explain your reasoning.

30. **THOUGHT PROVOKING** The weight  $w$  (in pounds) of an object varies inversely with the square of the distance  $d$  (in miles) of the object from the center of Earth. At sea level (3978 miles from the center of the Earth), an astronaut weighs 210 pounds. How much does the astronaut weigh 200 miles above sea level?

31. **OPEN-ENDED** Describe a real-life situation that can be modeled by an inverse variation equation.

32. **CRITICAL THINKING** Suppose  $x$  varies inversely with  $y$  and  $y$  varies inversely with  $z$ . How does  $x$  vary with  $z$ ? Justify your answer.

33. **USING STRUCTURE** To balance the board in the diagram, the distance (in feet) of each animal from the center of the board must vary inversely with its weight (in pounds). What is the distance of each animal from the fulcrum? Justify your answer.



## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Divide.** (Section 4.3)

34.  $(x^2 + 2x - 99) \div (x + 11)$

35.  $(3x^4 - 13x^2 - x^3 + 6x - 30) \div (3x^2 - x + 5)$

**Graph the function. Then state the domain and range.** (Section 6.4)

36.  $f(x) = 5^x + 4$

37.  $g(x) = e^{x-1}$

38.  $y = \ln 3x - 6$

39.  $h(x) = 2 \ln(x + 9)$