7.2 Graphing Rational Functions

What is a rational number?

Rational Functions

A rational function has the from $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $p(x) \neq 0$.

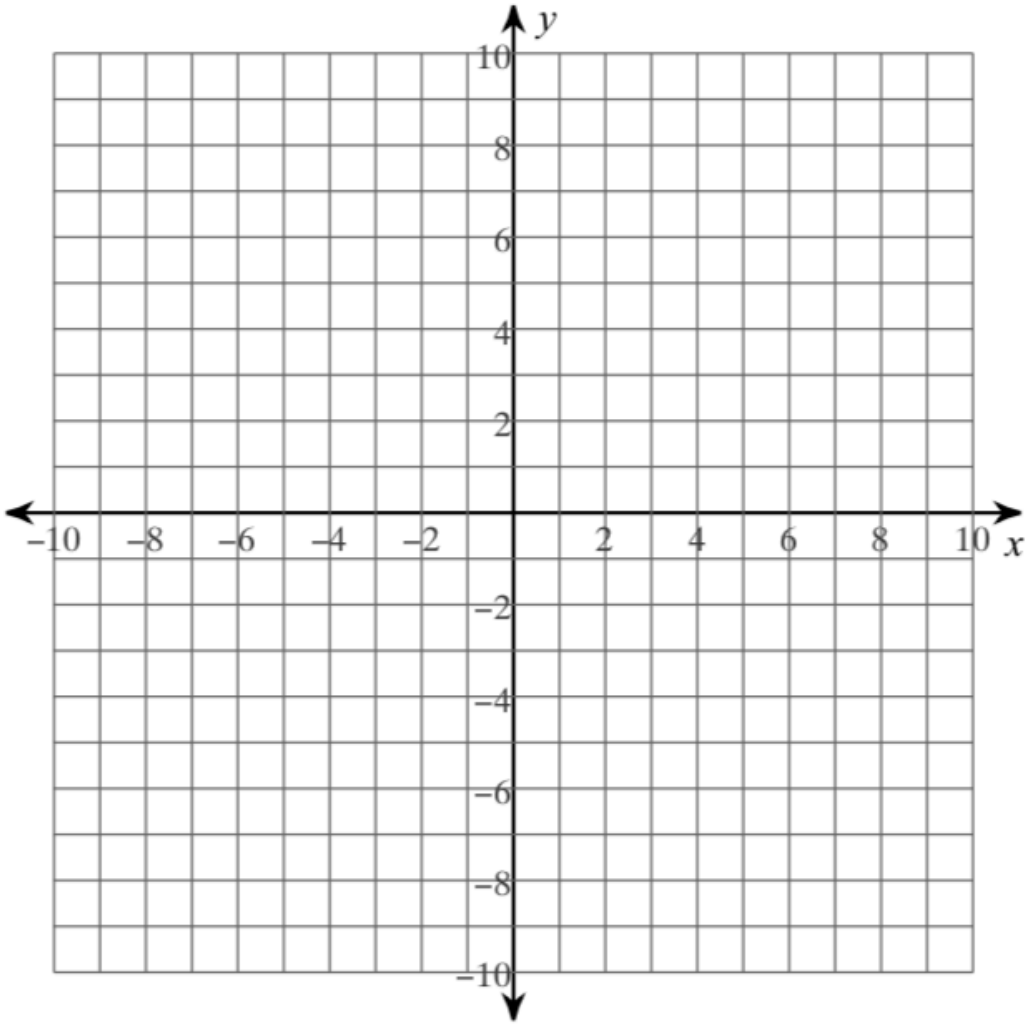
Thinking back:

We just covered inverse variation functions $y = \frac{a}{x}$. This is a rational function.

Critical Thinking: Complete the table and then graph the function $f(x) = \frac{1}{x}$

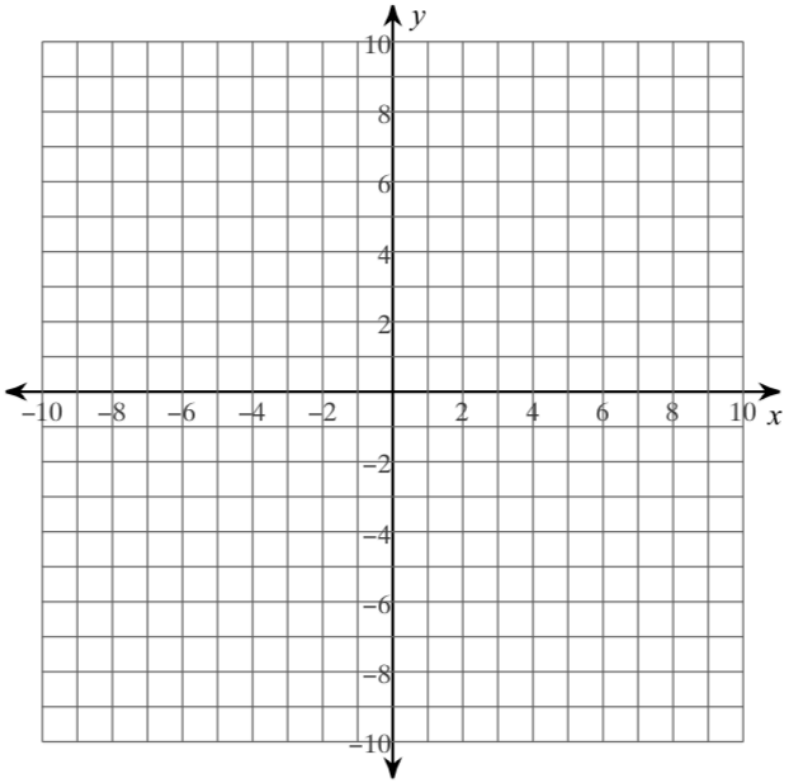
Don't cheat and use a graphing calculator!

x	-10	-4	-2	-1	1	2	4	10
$f(x)$								



Example 1: Graphing a Rational Function of the form $y = \frac{a}{x}$

Graph $g(x) = \frac{4}{x}$. Compare the graph with the graph of $f(x) = \frac{1}{x}$



Try on your own: Use a graphing calculator to compare the graph with the graph of $f(x) = \frac{1}{x}$

a) $s(x) = \frac{10}{x}$ b) $t(x) = \frac{0.1}{x}$ c) $u(x) = \frac{-6}{x}$

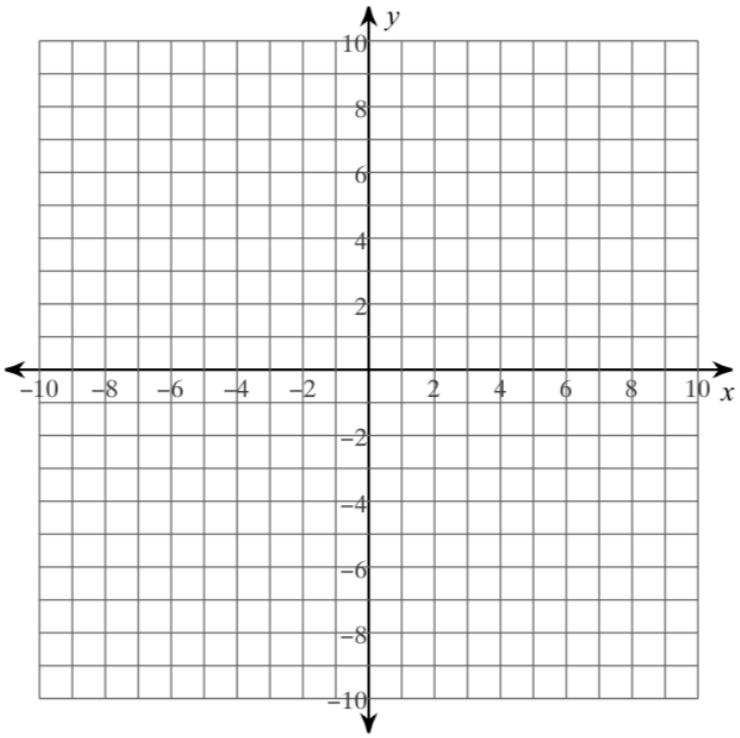
Translations of Simple Rational Functions: $y = \frac{a}{x-h} + k$

h meaning:

k meaning:

Example 2: Graphing a Translation of a Rational Function

Graph $g(x) = \frac{-4}{x+2} - 1$. State the domain and range.



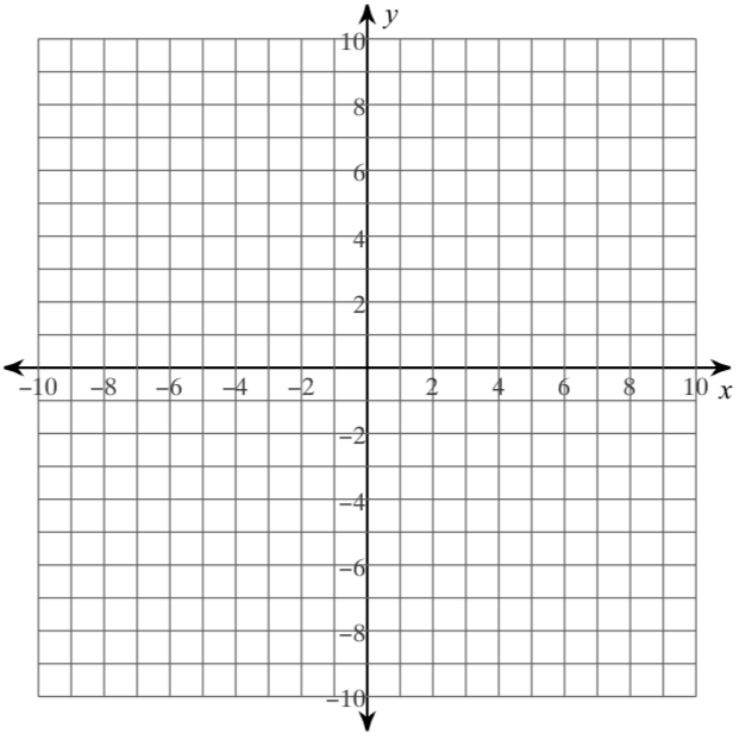
Graphing Other Rational Functions:

All Rational Functions of the form $g(x) = \frac{ax+b}{cx+d}$ also have hyperbolic graphs.

Vertical asymptote $x = -\frac{d}{c}$ Horizontal asymptote $y = \frac{a}{c}$

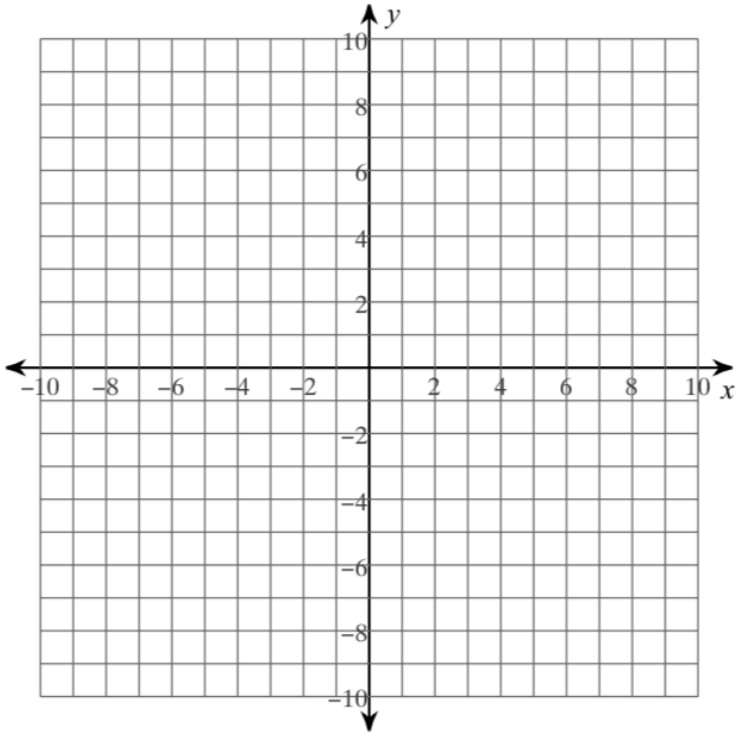
Example 3: Graphing a Rational Function of the form $y = \frac{ax+b}{cx+d}$

Graph $f(x) = \frac{2x+1}{x-3}$. State the domain and range.



Example 4: Rewriting and Graphing a Rational Function

Rewrite $g(x) = \frac{3x+5}{x+1}$ in the form of $g(x) = \frac{a}{x-h} + k$. Graph the function.



Example 5: Modeling with Mathematics

A 3-D printer builds up layers of materials to make three-dimensional models. Each deposited layer bonds to the layer below it. A company decides to make small display models of engine components using a 3-D printer. The printer costs $1000. The material for each model costs $50.

- Estimate how many models must be printed for the average cost per model to fall to $90.
- What happens to the average cost as more models are printed?

Homework:
3-17odd, 21-24, 25-39odd, 41, 42, 51

7.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The function $y = \frac{7}{x + 4} + 3$ has a(n) _____ of all real numbers except 3 and a(n) _____ of all real numbers except -4.

2. **WRITING** Is $f(x) = \frac{-3x + 5}{2^x + 1}$ a rational function? Explain your reasoning.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, graph the function. Compare the graph with the graph of $f(x) = \frac{1}{x}$. *(See Example 1.)*

3. $g(x) = \frac{3}{x}$

4. $g(x) = \frac{10}{x}$

5. $g(x) = \frac{-5}{x}$

6. $g(x) = \frac{-9}{x}$

7. $g(x) = \frac{15}{x}$

8. $g(x) = \frac{-12}{x}$

9. $g(x) = \frac{-0.5}{x}$

10. $g(x) = \frac{0.1}{x}$

In Exercises 11–18, graph the function. State the domain and range. *(See Example 2.)*

11. $g(x) = \frac{4}{x} + 3$

12. $y = \frac{2}{x} - 3$

13. $h(x) = \frac{6}{x - 1}$

14. $y = \frac{1}{x + 2}$

15. $h(x) = \frac{-3}{x + 2}$

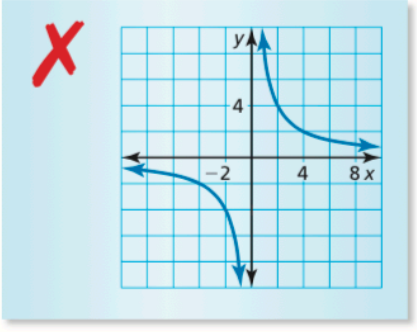
16. $f(x) = \frac{-2}{x - 7}$

17. $g(x) = \frac{-3}{x - 4} - 1$

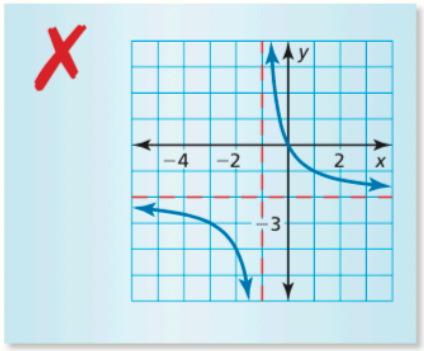
18. $y = \frac{10}{x + 7} - 5$

ERROR ANALYSIS **In Exercises 19 and 20, describe and correct the error in graphing the rational function.**

19. $y = \frac{-8}{x}$



20. $y = \frac{2}{x - 1} - 2$



ANALYZING RELATIONSHIPS **In Exercises 21–24, match the function with its graph. Explain your reasoning.**

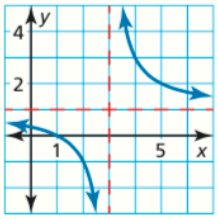
21. $g(x) = \frac{2}{x - 3} + 1$

22. $h(x) = \frac{2}{x + 3} + 1$

23. $f(x) = \frac{2}{x - 3} - 1$

24. $y = \frac{2}{x + 3} - 1$

A.



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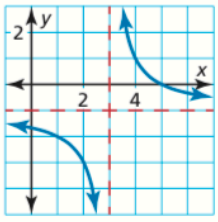

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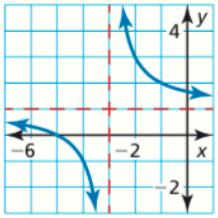

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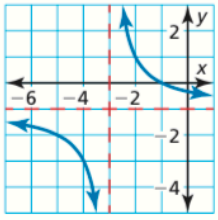


In Exercises 25–32, graph the function. State the domain and range. *(See Example 3.)*

25. $f(x) = \dfrac{x + 4}{x - 3}$

26. $y = \dfrac{x - 1}{x + 5}$

27. $y = \dfrac{x + 6}{4x - 8}$

28. $h(x) = \dfrac{8x + 3}{2x - 6}$

29. $f(x) = \dfrac{-5x + 2}{4x + 5}$

30. $g(x) = \dfrac{6x - 1}{3x - 1}$

31. $h(x) = \dfrac{-5x}{-2x - 3}$

32. $y = \dfrac{-2x + 3}{-x + 10}$

In Exercises 33–40, rewrite the function in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$. *(See Example 4.)*

33. $g(x) = \dfrac{5x + 6}{x + 1}$

34. $g(x) = \dfrac{7x + 4}{x - 3}$

35. $g(x) = \dfrac{2x - 4}{x - 5}$

36. $g(x) = \dfrac{4x - 11}{x - 2}$

37. $g(x) = \dfrac{x + 18}{x - 6}$

38. $g(x) = \dfrac{x + 2}{x - 8}$

39. $g(x) = \dfrac{7x - 20}{x + 13}$

40. $g(x) = \dfrac{9x - 3}{x + 7}$

41. PROBLEM SOLVING Your school purchases a math software program. The program has an initial cost of \$500 plus \$20 for each student that uses the program. *(See Example 5.)*

a. Estimate how many students must use the program for the average cost per student to fall to \$30.

b. What happens to the average cost as more students use the program?

42. PROBLEM SOLVING To join a rock climbing gym, you must pay an initial fee of \$100 and a monthly fee of \$59.

a. Estimate how many months you must purchase a membership for the average cost per month to fall to \$69.

b. What happens to the average cost as the number of months that you are a member increases?

43. USING STRUCTURE What is the vertical asymptote of the graph of the function $y = \dfrac{2}{x + 4} + 7$?

Ⓐ $x = -7$ Ⓑ $x = -4$

Ⓒ $x = 4$ Ⓓ $x = 7$

44. REASONING What are the x-intercept(s) of the graph of the function $y = \dfrac{x - 5}{x^2 - 1}$?

Ⓐ $1, -1$ Ⓑ 5

Ⓒ 1 Ⓓ -5

45. USING TOOLS The time t (in seconds) it takes for sound to travel 1 kilometer can be modeled by

$$t = \frac{1000}{0.6T + 331}$$

where T is the air temperature (in degrees Celsius).

a. You are 1 kilometer from a lightning strike. You hear the thunder 2.9 seconds later. Use a graph to find the approximate air temperature.

b. Find the average rate of change in the time it takes sound to travel 1 kilometer as the air temperature increases from 0°C to 10°C.

46. MODELING WITH MATHEMATICS A business is studying the cost to remove a pollutant from the ground at its site. The function $y = \dfrac{15x}{1.1 - x}$ models the estimated cost y (in thousands of dollars) to remove x percent (expressed as a decimal) of the pollutant.

a. Graph the function. Describe a reasonable domain and range.

b. How much does it cost to remove 20% of the pollutant? 40% of the pollutant? 80% of the pollutant? Does doubling the percentage of the pollutant removed double the cost? Explain.

USING TOOLS In Exercises 47–50, use a graphing calculator to graph the function. Then determine whether the function is *even*, *odd*, or *neither*.

47. $h(x) = \dfrac{6}{x^2 + 1}$

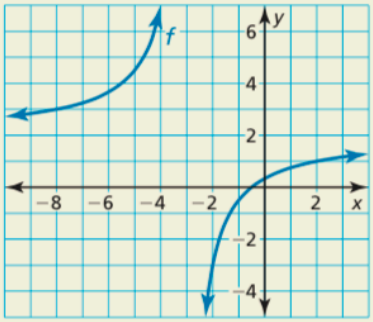
48. $f(x) = \dfrac{2x^2}{x^2 - 9}$

49. $y = \dfrac{x^3}{3x^2 + x^4}$

50. $f(x) = \dfrac{4x^2}{2x^3 - x}$

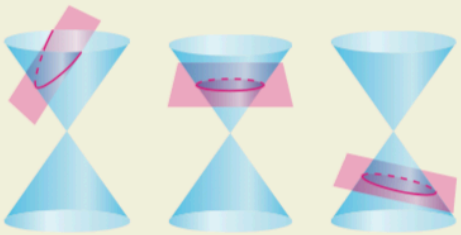
51. MAKING AN ARGUMENT Your friend claims it is possible for a rational function to have two vertical asymptotes. Is your friend correct? Justify your answer.

52. HOW DO YOU SEE IT? Use the graph of f to determine the equations of the asymptotes. Explain.



53. DRAWING CONCLUSIONS In what line(s) is the graph of $y = \frac{1}{x}$ symmetric? What does this symmetry tell you about the inverse of the function $f(x) = \frac{1}{x}$?

54. THOUGHT PROVOKING There are four basic types of conic sections: parabola, circle, ellipse, and hyperbola. Each of these can be represented by the intersection of a double-napped cone and a plane. The intersections for a parabola, circle, and ellipse are shown below. Sketch the intersection for a hyperbola.



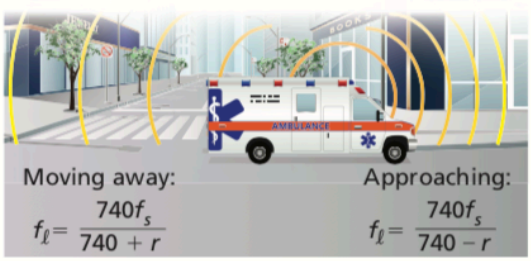
55. REASONING The graph of the rational function f is a hyperbola. The asymptotes of the graph of f intersect at (3, 2). The point (2, 1) is on the graph. Find another point on the graph. Explain your reasoning.

56. ABSTRACT REASONING Describe the intervals where the graph of $y = \frac{a}{x}$ is increasing or decreasing when (a) $a > 0$ and (b) $a < 0$. Explain your reasoning.

57. PROBLEM SOLVING An Internet service provider charges a \$50 installation fee and a monthly fee of \$43. The table shows the average monthly costs y of a competing provider for x months of service. Under what conditions would a person choose one provider over the other? Explain your reasoning.

Months, x	Average monthly cost (dollars), y
6	\$49.83
12	\$46.92
18	\$45.94
24	\$45.45

58. MODELING WITH MATHEMATICS The Doppler effect occurs when the source of a sound is moving relative to a listener, so that the frequency f_ℓ (in hertz) heard by the listener is different from the frequency f_s (in hertz) at the source. In both equations below, r is the speed (in miles per hour) of the sound source.



a. An ambulance siren has a frequency of 2000 hertz. Write two equations modeling the frequencies heard when the ambulance is approaching and when the ambulance is moving away.

b. Graph the equations in part (a) using the domain $0 \le r \le 60$.

c. For any speed r, how does the frequency heard for an approaching sound source compare with the frequency heard when the source moves away?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the polynomial. *(Skills Review Handbook)*

59. $4x^2 - 4x - 80$ **60.** $3x^2 - 3x - 6$ **61.** $2x^2 - 2x - 12$ **62.** $10x^2 + 31x - 14$

Simplify the expression. *(Section 5.2)*

63. $3^2 \cdot 3^4$ **64.** $2^{1/2} \cdot 2^{3/5}$ **65.** $\frac{6^{5/6}}{6^{1/6}}$ **66.** $\frac{6^8}{6^{10}}$