7.3 Multiplying and Dividing Rational Expressions

A rational expression is in **SIMPLEST FORM** when its numerator and denominator have no common factors *(other than* ± 1*)*

Spiral Review:
Factor: $x^2 - 2x + 8$

Example 1: Simplifying a rational expression

Simplify: $\frac{x^2-4x-12}{x^2-4}$

Try on your own:
Simplify the rational expression, if possible.

1. $\frac{2(x+1)}{(x+1)(x+3)}$ **2.** $\frac{x+4}{x^2-16}$ **3.** $\frac{4}{x(x+2)}$ **4.** $\frac{x^2-2x-3}{x^2-x-6}$

Example 2: Multiplying Rational Expressions

Find the product $\frac{8x^3y}{2xy^2} \cdot \frac{7x^4y^3}{4y}$

Example 3: Multiplying Rational Expressions

Find the product $\frac{3x-3x^2}{x^2+4x-5} \cdot \frac{x^2+x-20}{3x}$

Example 4: Multiplying a rational expression by a polynomial (Difference of cubes)

Find the product $\frac{x+2}{x^3-27} \cdot (x^2 + 3x + 9)$

Cube of a Binomial

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Try on your own:
Find the product:

5. $\frac{3x^5y^2}{8xy} \cdot \frac{6xy^2}{9x^3y}$ **6.** $\frac{2x^2 - 10x}{x^2 - 25} \cdot \frac{x + 3}{2x^2}$ **7.** $\frac{x + 5}{x^3 - 1} \cdot (x^2 + x + 1)$

Dividing Rational Expressions

Spiral review:
Find the quotient: $\frac{3}{5} \div \frac{7}{3} =$

Example 5: Dividing Rational Expressions

Find the quotient: $\frac{7x}{2x-10} \div \frac{x^2-6x}{x^2-11x+30}$

Example 6: Dividing a rational expression by a polynomial

Find the quotient: $\frac{6x^2+x-15}{4x^2} \div (3x^2 + 5x)$

Try on your own:
Find the quotient:

8. $\frac{4x}{5x - 20} \div \frac{x^2 - 2x}{x^2 - 6x + 8}$

9. $\frac{2x^2 + 3x - 5}{6x} \div (2x^2 + 5x)$

Homework:
3, 6, 10, 11, 14, 17, 26, 29, 31, 32, 37, 44

7.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Describe how to multiply and divide two rational expressions.

2. **WHICH ONE DOESN'T BELONG?** Which rational expression does *not* belong with the other three? Explain your reasoning.

$\frac{x-4}{x^2}$ $\quad \frac{x^2+4x-12}{x^2+6x}$ $\quad \frac{9+x}{3x^2}$ $\quad \frac{x^2-x-12}{x^2-6x}$

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, simplify the expression, if possible. *(See Example 1.)*

3. $\frac{2x^2}{3x^2-4x}$

4. $\frac{7x^3-x^2}{2x^3}$

5. $\frac{x^2-3x-18}{x^2-7x+6}$

6. $\frac{x^2+13x+36}{x^2-7x+10}$

7. $\frac{x^2+11x+18}{x^3+8}$

8. $\frac{x^2-7x+12}{x^3-27}$

9. $\frac{32x^4-50}{4x^3-12x^2-5x+15}$

10. $\frac{3x^3-3x^2+7x-7}{27x^4-147}$

In Exercises 11–20, find the product. *(See Examples 2, 3, and 4.)*

11. $\frac{4xy^3}{x^2y} \cdot \frac{y}{8x}$

12. $\frac{48x^5y^3}{y^4} \cdot \frac{x^2y}{6x^3y^2}$

13. $\frac{x^2(x-4)}{x-3} \cdot \frac{(x-3)(x+6)}{x^3}$

14. $\frac{x^3(x+5)}{x-9} \cdot \frac{(x-9)(x+8)}{3x^3}$

15. $\frac{x^2-3x}{x-2} \cdot \frac{x^2+x-6}{x}$

16. $\frac{x^2-4x}{x-1} \cdot \frac{x^2+3x-4}{2x}$

17. $\frac{x^2+3x-4}{x^2+4x+4} \cdot \frac{2x^2+4x}{x^2-4x+3}$

18. $\frac{x^2-x-6}{4x^3} \cdot \frac{2x^2+2x}{x^2+5x+6}$

19. $\frac{x^2+5x-36}{x^2-49} \cdot (x^2-11x+28)$

20. $\frac{x^2-x-12}{x^2-16} \cdot (x^2+2x-8)$

21. **ERROR ANALYSIS** Describe and correct the error in simplifying the rational expression.

✗ $\frac{x^2+\overset{2}{\cancel{16}}x+\overset{3}{\cancel{48}}}{x^2+\underset{1}{\cancel{8}}x+\underset{1}{\cancel{16}}} = \frac{x^2+2x+3}{x^2+x+1}$

22. **ERROR ANALYSIS** Describe and correct the error in finding the product.

✗ $\frac{x^2-25}{3-x} \cdot \frac{x-3}{x+5} = \frac{(x+5)(x-5)}{3-x} \cdot \frac{x-3}{x+5}$

$= \frac{\cancel{(x+5)}(x-5)\cancel{(x-3)}}{\cancel{(3-x)}\cancel{(x+5)}}$

$= x-5, x \neq 3, x \neq -5$

23. **USING STRUCTURE** Which rational expression is in simplified form?

Ⓐ $\frac{x^2-x-6}{x^2+3x+2}$ $\quad$ Ⓑ $\frac{x^2+6x+8}{x^2+2x-3}$

Ⓒ $\frac{x^2-6x+9}{x^2-2x-3}$ $\quad$ Ⓓ $\frac{x^2+3x-4}{x^2+x-2}$

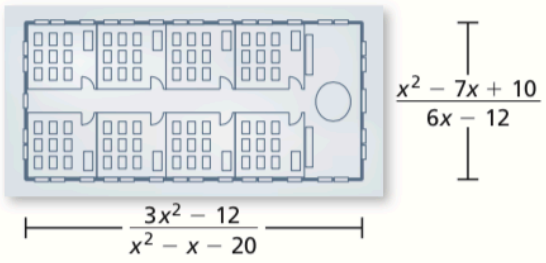
24. **COMPARING METHODS** Find the product below by multiplying the numerators and denominators, then simplifying. Then find the product by simplifying each expression, then multiplying. Which method do you prefer? Explain.

$\frac{4x^2y}{2x^3} \cdot \frac{12y^4}{24x^2}$

25. WRITING Compare the function

$f(x) = \dfrac{(3x - 7)(x + 6)}{(3x - 7)}$ to the function $g(x) = x + 6$.

26. MODELING WITH MATHEMATICS Write a model in terms of x for the total area of the base of the building.



In Exercises 27–34, find the quotient. *(See Examples 5 and 6.)*

27. $\dfrac{32x^3y}{y^8} \div \dfrac{y^7}{8x^4}$

28. $\dfrac{2xyz}{x^3z^3} \div \dfrac{6y^4}{2x^2z^2}$

29. $\dfrac{x^2 - x - 6}{2x^4 - 6x^3} \div \dfrac{x + 2}{4x^3}$

30. $\dfrac{2x^2 - 12x}{x^2 - 7x + 6} \div \dfrac{2x}{3x - 3}$

31. $\dfrac{x^2 - x - 6}{x + 4} \div (x^2 - 6x + 9)$

32. $\dfrac{x^2 - 5x - 36}{x + 2} \div (x^2 - 18x + 81)$

33. $\dfrac{x^2 + 9x + 18}{x^2 + 6x + 8} \div \dfrac{x^2 - 3x - 18}{x^2 + 2x - 8}$

34. $\dfrac{x^2 - 3x - 40}{x^2 + 8x - 20} \div \dfrac{x^2 + 13x + 40}{x^2 + 12x + 20}$

In Exercises 35 and 36, use the following information.

Manufacturers often package products in a way that uses the least amount of material. One measure of the efficiency of a package is the ratio of its surface area S to its volume V. The smaller the ratio, the more efficient the packaging.

35. You are examining three cylindrical containers.

a. Write an expression for the efficiency ratio $\dfrac{S}{V}$ of a cylinder.

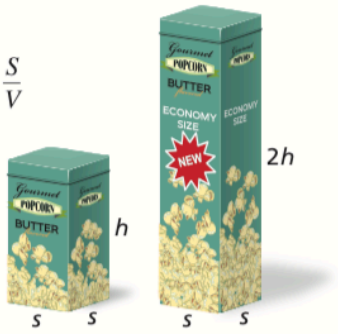
b. Find the efficiency ratio for each cylindrical can listed in the table. Rank the three cans according to efficiency.

	Soup	Coffee	Paint
Height, *h*	10.2 cm	15.9 cm	19.4 cm
Radius, *r*	3.4 cm	7.8 cm	8.4 cm

36. PROBLEM SOLVING A popcorn company is designing a new tin with the same square base and twice the height of the old tin.

a. Write an expression for the efficiency ratio $\dfrac{S}{V}$ of each tin.

b. Did the company make a good decision by creating the new tin? Explain.



37. MODELING WITH MATHEMATICS The total amount I (in millions of dollars) of healthcare expenditures and the residential population P (in millions) in the United States can be modeled by

$$I = \frac{171{,}000t + 1{,}361{,}000}{1 + 0.018t} \quad \text{and}$$

$$P = 2.96t + 278.649$$

where t is the number of years since 2000. Find a model M for the annual healthcare expenditures per resident. Estimate the annual healthcare expenditures per resident in 2010. *(See Example 7.)*

38. MODELING WITH MATHEMATICS The total amount I (in millions of dollars) of school expenditures from prekindergarten to a college level and the enrollment P (in millions) in prekindergarten through college in the United States can be modeled by

$$I = \frac{17{,}913t + 709{,}569}{1 - 0.028t} \text{ and } P = 0.5906t + 70.219$$

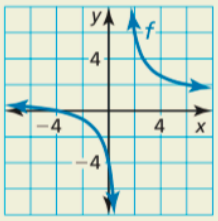
where t is the number of years since 2001. Find a model M for the annual education expenditures per student. Estimate the annual education expenditures per student in 2009.

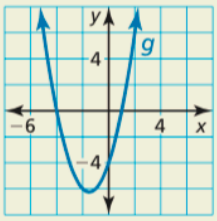
39. USING EQUATIONS Refer to the population model P in Exercise 37.

a. Interpret the meaning of the coefficient of t.

b. Interpret the meaning of the constant term.

40. HOW DO YOU SEE IT? Use the graphs of f and g to determine the excluded values of the functions $h(x) = (fg)(x)$ and $k(x) = \left(\frac{f}{g}\right)(x)$. Explain your reasoning.





41. DRAWING CONCLUSIONS Complete the table for the function $y = \frac{x + 4}{x^2 - 16}$. Then use the *trace* feature of a graphing calculator to explain the behavior of the function at $x = -4$.

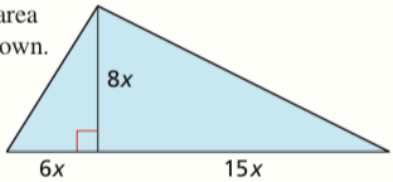
x	y
−3.5	
−3.8	
−3.9	
−4.1	
−4.2	

42. MAKING AN ARGUMENT You and your friend are asked to state the domain of the expression below.

$$\frac{x^2 + 6x - 27}{x^2 + 4x - 45}$$

Your friend claims the domain is all real numbers except 5. You claim the domain is all real numbers except −9 and 5. Who is correct? Explain.

43. MATHEMATICAL CONNECTIONS Find the ratio of the perimeter to the area of the triangle shown.



44. CRITICAL THINKING Find the expression that makes the following statement true. Assume $x \neq -2$ and $x \neq 5$.

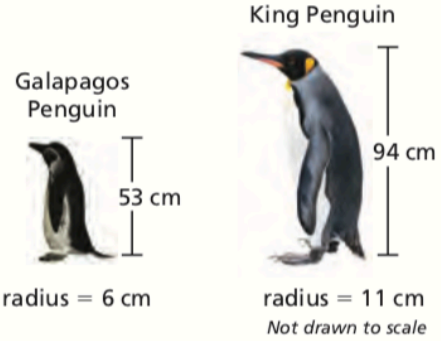
$$\frac{x - 5}{x^2 + 2x - 35} \div \frac{\square}{x^2 - 3x - 10} = \frac{x + 2}{x + 7}$$

USING STRUCTURE In Exercises 45 and 46, perform the indicated operations.

45. $\frac{2x^2 + x - 15}{2x^2 - 11x - 21} \cdot (6x + 9) \div \frac{2x - 5}{3x - 21}$

46. $(x^3 + 8) \cdot \frac{x - 2}{x^2 - 2x + 4} \div \frac{x^2 - 4}{x - 6}$

47. REASONING Animals that live in temperatures several degrees colder than their bodies must avoid losing heat to survive. Animals can better conserve body heat as their surface area to volume ratios decrease. Find the surface area to volume ratio of each penguin shown by using cylinders to approximate their shapes. Which penguin is better equipped to live in a colder environment? Explain your reasoning.



48. THOUGHT PROVOKING Is it possible to write two radical functions whose product when graphed is a parabola and whose quotient when graphed is a hyperbola? Justify your answer.

49. REASONING Find two rational functions f and g that have the stated product and quotient.

$$(fg)(x) = x^2, \left(\frac{f}{g}\right)(x) = \frac{(x - 1)^2}{(x + 2)^2}$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. *(Skills Review Handbook)*

50. $\frac{1}{2}x + 4 = \frac{3}{2}x + 5$ **51.** $\frac{1}{3}x - 2 = \frac{3}{4}x$ **52.** $\frac{1}{4}x - \frac{3}{5} = \frac{9}{2}x - \frac{4}{5}$ **53.** $\frac{1}{2}x + \frac{1}{3} = \frac{3}{4}x - \frac{1}{5}$

Write the prime factorization of the number. If the number is prime, then write *prime*. *(Skills Review Handbook)*

54. 42 **55.** 91 **56.** 72 **57.** 79