7.4 Adding and Subtracting Rational Expressions

Spiral Review: How do you add fractions?

Example 1: Adding or Subtracting with Like Denominators
Simplify

a) $\frac{7}{4x} + \frac{3}{4x} =$

b) $\frac{2x}{x+6} - \frac{5}{x+6} =$

Example 2: Adding with Unlike Denominators
Find the sum:

$$\frac{7}{9x^2} + \frac{x}{3x^2 + 3x} =$$

Example 3: Adding with Unlike Denominators
Find the sum:

$$\frac{x+2}{2x-2} - \frac{-2x-1}{x^2 - 4x + 3} =$$

Try on your own: Find the sum or difference.

6. $\dfrac{3}{4x} - \dfrac{1}{7}$

7. $\dfrac{1}{3x^2} + \dfrac{x}{9x^2 - 12}$

8. $\dfrac{x}{x^2 - x - 12} + \dfrac{5}{12x - 48}$

Example 4: Rewriting and Graphing a Rational Function

Rewrite the function $g(x) = \dfrac{3x+5}{x+1}$ in the form $g(x) = \dfrac{a}{x-h} + k$. Graph the function.

Example 5: Simplifying Complex Fractions

Simplify

$$\dfrac{\dfrac{5}{x+4}}{\dfrac{1}{x+4} + \dfrac{2}{x}}$$

Homework:
3, 5, 19-25odd, 31, 39, 41, 43, 47, 49

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** A fraction that contains a fraction in its numerator or denominator is called a(n) _____.

2. **WRITING** Explain how adding and subtracting rational expressions is similar to adding and subtracting numerical fractions.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the sum or difference. *(See Example 1.)*

3. $\dfrac{15}{4x} + \dfrac{5}{4x}$

4. $\dfrac{x}{16x^2} - \dfrac{4}{16x^2}$

5. $\dfrac{9}{x+1} - \dfrac{2x}{x+1}$

6. $\dfrac{3x^2}{x-8} + \dfrac{6x}{x-8}$

7. $\dfrac{5x}{x+3} + \dfrac{15}{x+3}$

8. $\dfrac{4x^2}{2x-1} - \dfrac{1}{2x-1}$

In Exercises 9–16, find the least common multiple of the expressions. *(See Example 2.)*

9. $3x,\ 3(x-2)$

10. $2x^2,\ 4x+12$

11. $2x,\ 2x(x-5)$

12. $24x^2,\ 8x^2-16x$

13. $x^2-25,\ x-5$

14. $9x^2-16,\ 3x^2+x-4$

15. $x^2+3x-40,\ x-8$

16. $x^2-2x-63,\ x+7$

ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in finding the sum.

17.
$$\times \quad \frac{2}{5x} + \frac{4}{x^2} = \frac{2+4}{5x+x^2} = \frac{6}{x(5+x)}$$

18.
$$\times \quad \frac{x}{x+2} + \frac{4}{x-5} = \frac{x+4}{(x+2)(x-5)}$$

In Exercises 19–26, find the sum or difference. *(See Examples 3 and 4.)*

19. $\dfrac{12}{5x} - \dfrac{7}{6x}$

20. $\dfrac{8}{3x^2} + \dfrac{5}{4x}$

21. $\dfrac{3}{x+4} - \dfrac{1}{x+6}$

22. $\dfrac{9}{x-3} + \dfrac{2x}{x+1}$

23. $\dfrac{12}{x^2+5x-24} + \dfrac{3}{x-3}$

24. $\dfrac{x^2-5}{x^2+5x-14} - \dfrac{x+3}{x+7}$

25. $\dfrac{x+2}{x-4} + \dfrac{2}{x} + \dfrac{5x}{3x-1}$

26. $\dfrac{x+3}{x^2-25} - \dfrac{x-1}{x-5} + \dfrac{3}{x+3}$

REASONING In Exercises 27 and 28, tell whether the statement is *always*, *sometimes*, or *never* true. Explain.

27. The LCD of two rational expressions is the product of the denominators.

28. The LCD of two rational expressions will have a degree greater than or equal to that of the denominator with the higher degree.

29. **ANALYZING EQUATIONS** How would you begin to rewrite the function $g(x) = \dfrac{4x+1}{x+2}$ to obtain the form $g(x) = \dfrac{a}{x-h} + k$?

   Ⓐ $g(x) = \dfrac{4(x+2)-7}{x+2}$

   Ⓑ $g(x) = \dfrac{4(x+2)+1}{x+2}$

   Ⓒ $g(x) = \dfrac{(x+2)+(3x-1)}{x+2}$

   Ⓓ $g(x) = \dfrac{4x+2-1}{x+2}$

30. **ANALYZING EQUATIONS** How would you begin to rewrite the function $g(x) = \dfrac{x}{x-5}$ to obtain the form $g(x) = \dfrac{a}{x-h} + k$?

   Ⓐ $g(x) = \dfrac{x(x+5)(x-5)}{x-5}$

   Ⓑ $g(x) = \dfrac{x-5+5}{x-5}$

   Ⓒ $g(x) = \dfrac{x}{x-5+5}$

   Ⓓ $g(x) = \dfrac{x}{x} - \dfrac{x}{5}$

In Exercises 31–38, rewrite the function g in the form $g(x) = \dfrac{a}{x - h} + k$. Graph the function. Describe the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$. (See Example 5.)

31. $g(x) = \dfrac{5x - 7}{x - 1}$

32. $g(x) = \dfrac{6x + 4}{x + 5}$

33. $g(x) = \dfrac{12x}{x - 5}$

34. $g(x) = \dfrac{8x}{x + 13}$

35. $g(x) = \dfrac{2x + 3}{x}$

36. $g(x) = \dfrac{4x - 6}{x}$

37. $g(x) = \dfrac{3x + 11}{x - 3}$

38. $g(x) = \dfrac{7x - 9}{x + 10}$

In Exercises 39–44, simplify the complex fraction. (See Example 6.)

39. $\dfrac{\dfrac{x}{3} - 6}{10 + \dfrac{4}{x}}$

40. $\dfrac{15 - \dfrac{2}{x}}{\dfrac{x}{5} + 4}$

41. $\dfrac{\dfrac{1}{2x - 5} - \dfrac{7}{8x - 20}}{\dfrac{x}{2x - 5}}$

42. $\dfrac{\dfrac{16}{x - 2}}{\dfrac{4}{x + 1} + \dfrac{6}{x}}$

43. $\dfrac{\dfrac{1}{3x^2 - 3}}{\dfrac{5}{x + 1} - \dfrac{x + 4}{x^2 - 3x - 4}}$

44. $\dfrac{\dfrac{3}{x - 2} - \dfrac{6}{x^2 - 4}}{\dfrac{3}{x + 2} + \dfrac{1}{x - 2}}$

45. **PROBLEM SOLVING** The total time T (in hours) needed to fly from New York to Los Angeles and back can be modeled by the equation below, where d is the distance (in miles) each way, a is the average airplane speed (in miles per hour), and j is the average speed (in miles per hour) of the jet stream. Simplify the equation. Then find the total time it takes to fly 2468 miles when $a = 510$ miles per hour and $j = 115$ miles per hour.

$$T = \dfrac{d}{a - j} + \dfrac{d}{a + j}$$
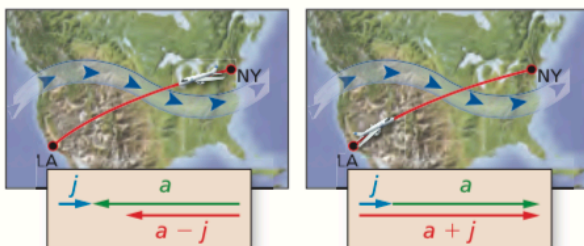
46. **REWRITING A FORMULA** The total resistance R_t of two resistors in a parallel circuit with resistances R_1 and R_2 (in ohms) is given by the equation shown. Simplify the complex fraction. Then find the total resistance when $R_1 = 2000$ ohms and $R_2 = 5600$ ohms.

$$R_t = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$$
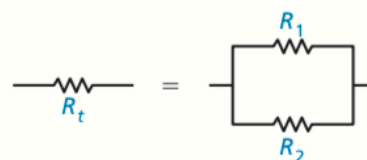
47. **PROBLEM SOLVING** You plan a trip that involves a 40-mile bus ride and a train ride. The entire trip is 140 miles. The time (in hours) the bus travels is $y_1 = \dfrac{40}{x}$, where x is the average speed (in miles per hour) of the bus. The time (in hours) the train travels is $y_2 = \dfrac{100}{x + 30}$. Write and simplify a model that shows the total time y of the trip.

48. **PROBLEM SOLVING** You participate in a sprint triathlon that involves swimming, bicycling, and running. The table shows the distances (in miles) and your average speed for each portion of the race.

	Distance (miles)	Speed (miles per hour)
Swimming	0.5	r
Bicycling	22	$15r$
Running	6	$r + 5$

a. Write a model in simplified form for the total time (in hours) it takes to complete the race.

b. How long does it take to complete the race if you can swim at an average speed of 2 miles per hour? Justify your answer.

49. **MAKING AN ARGUMENT** Your friend claims that the least common multiple of two numbers is always greater than each of the numbers. Is your friend correct? Justify your answer.

50. HOW DO YOU SEE IT?
Use the graph of the function $f(x) = \dfrac{a}{x - h} + k$ to determine the values of h and k.
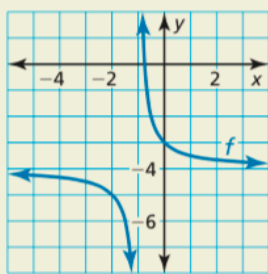
51. REWRITING A FORMULA You borrow P dollars to buy a car and agree to repay the loan over t years at a monthly interest rate of i (expressed as a decimal). Your monthly payment M is given by either formula below.

$$M = \frac{Pi}{1 - \left(\dfrac{1}{1+i}\right)^{12t}} \quad \text{or} \quad M = \frac{Pi(1 + i)^{12t}}{(1 + i)^{12t} - 1}$$

a. Show that the formulas are equivalent by simplifying the first formula.

b. Find your monthly payment when you borrow $15,500 at a monthly interest rate of 0.5% and repay the loan over 4 years.

52. THOUGHT PROVOKING Is it possible to write two rational functions whose sum is a quadratic function? Justify your answer.

53. USING TOOLS Use technology to rewrite the function $g(x) = \dfrac{(97.6)(0.024) + x(0.003)}{12.2 + x}$ in the form $g(x) = \dfrac{a}{x - h} + k$. Describe the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$.

**54. MATHEMATICAL CONNECTIONS** Find an expression for the surface area of the box.
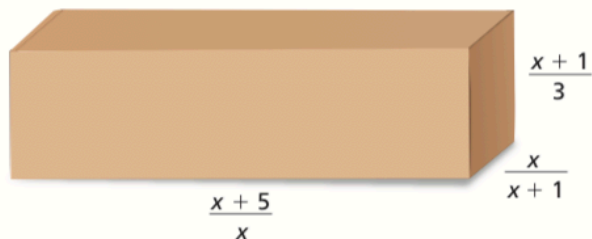
55. PROBLEM SOLVING You are hired to wash the new cars at a car dealership with two other employees. You take an average of 40 minutes to wash a car ($R_1 = 1/40$ car per minute). The second employee washes a car in x minutes. The third employee washes a car in $x + 10$ minutes.

a. Write expressions for the rates that each employee can wash a car.

b. Write a single expression R for the combined rate of cars washed per minute by the group.

c. Evaluate your expression in part (b) when the second employee washes a car in 35 minutes. How many cars per hour does this represent? Explain your reasoning.

56. MODELING WITH MATHEMATICS The amount A (in milligrams) of aspirin in a person's bloodstream can be modeled by

$$A = \frac{391t^2 + 0.112}{0.218t^4 + 0.991t^2 + 1}$$

where t is the time (in hours) after one dose is taken.
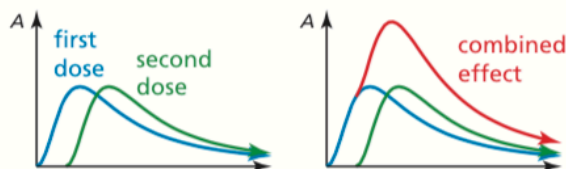
a. A second dose is taken 1 hour after the first dose. Write an equation to model the amount of the second dose in the bloodstream.

b. Write a model for the *total* amount of aspirin in the bloodstream after the second dose is taken.

57. FINDING A PATTERN Find the next two expressions in the pattern shown. Then simplify all five expressions. What value do the expressions approach?

$$1 + \cfrac{1}{2 + \cfrac{1}{2}}, \; 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}, \; 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}}, \ldots$$

Maintaining Mathematical Proficiency
Reviewing what you learned in previous grades and lessons

Solve the system by graphing. *(Section 3.5)*

58. $y = x^2 + 6$
$y = 3x + 4$

59. $2x^2 - 3x - y = 0$
$\frac{5}{2}x - y = \frac{9}{4}$

60. $3 = y - x^2 - x$
$y = -x^2 - 3x - 5$

61. $y = (x + 2)^2 - 3$
$y = x^2 + 4x + 5$