9.1 Properties of Radicals

What are some rules you can recall about radicals?

Simplest form:

- No radicands have a perfect nth power as factors other than 1
- No radicands contain fractions
- No radicands appear in the denominator of a fraction

Example 1: Using the Product Property of Square Roots
Simplify the following

a) $\sqrt{5} \cdot \sqrt{5}$ b. $\sqrt{3} \cdot \sqrt{12}$ c. $\sqrt{108}$ d. $\sqrt{9x^3}$

Example 2: Using the Quotient Property of Square Roots
Simplify the following

a) $\sqrt{\frac{25}{9}}$ b. $\sqrt{\frac{50}{16}}$ c. $\sqrt{\frac{15}{64}}$ d. $\sqrt{\frac{81}{x^2}}$

Example 3: Using Properties of Cube Roots
Simplify the following

a) $\sqrt[3]{64}$ b. $\sqrt[3]{125x^3}$ c. $\sqrt[3]{-27x^7}$ d. $\sqrt[3]{\frac{y}{216}}$

Try on your own:

Simplify the expression.

5. $\sqrt{\frac{23}{9}}$ **6.** $-\sqrt{\frac{17}{100}}$ **7.** $\sqrt{\frac{36}{z^2}}$ **8.** $\sqrt{\frac{4x^2}{64}}$

9. $\sqrt[3]{54}$ **10.** $\sqrt[3]{16x^4}$ **11.** $\sqrt[3]{\frac{a}{-27}}$ **12.** $\sqrt[3]{\frac{25c^7d^3}{64}}$

Rationalizing the Denominator:

a) $\frac{1}{5} + \frac{5}{6} =$

b) $\frac{x}{\sqrt{3}}$

Example 4: Rationalizing the Denominator

a) $\frac{\sqrt{5}}{\sqrt{3n}}$

b) $\frac{2}{\sqrt[3]{9}}$

Example 5: Rationalizing the Denominator Using Conjugates
Simplify the following:

$$\frac{7}{2-\sqrt{3}}$$

Try on your own:

Simplify the expression.

13. $\frac{1}{\sqrt{5}}$ **14.** $\frac{\sqrt{10}}{\sqrt{3}}$ **15.** $\frac{7}{\sqrt{2x}}$ **16.** $\sqrt{\frac{2y^2}{3}}$

17. $\frac{5}{\sqrt[3]{32}}$ **18.** $\frac{8}{1+\sqrt{3}}$ **19.** $\frac{\sqrt{13}}{\sqrt{5}-2}$ **20.** $\frac{12}{\sqrt{2}+\sqrt{7}}$

Example 6: Solving a Real Life Problem
The distance d (in miles) that you can see to the horizon with your eye level h feet above the water is given by $d = \sqrt{\frac{3h}{2}}$. How far can you see when your eye level is 5ft above the water? Find the exact and estimated distance.

Example 7: Modeling with Mathematics
The ratio of the length to the width of a golden rectangle is $(1 + \sqrt{5}) : 2$. The dimensions of the face of the Parthenon in Greece form a golden rectangle. What is the height h of the Parthenon?

Example 8: Adding and Subtracting Radicals
Simplify the following: *(Think like terms)*

A) $5\sqrt{7} + \sqrt{11} - 8\sqrt{7}$ b) $10\sqrt{5} + \sqrt{20}$ c) $6\sqrt[3]{x} + 2\sqrt[3]{x}$

Homework: 5-59odd

9.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The process of eliminating a radical from the denominator of a radical expression is called _______________.

2. **VOCABULARY** What is the conjugate of the binomial $\sqrt{6} + 4$?

3. **WRITING** Are the expressions $\frac{1}{3}\sqrt{2x}$ and $\sqrt{\frac{2x}{9}}$ equivalent? Explain your reasoning.

4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$-\frac{1}{3}\sqrt{6}$	$6\sqrt{3}$	$\frac{1}{6}\sqrt{3}$	$-3\sqrt{3}$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, determine whether the expression is in simplest form. If the expression is not in simplest form, explain why.

5. $\sqrt{19}$
6. $\sqrt{\frac{1}{7}}$
7. $\sqrt{48}$
8. $\sqrt{34}$
9. $\frac{5}{\sqrt{2}}$
10. $\frac{3\sqrt{10}}{4}$
11. $\frac{1}{2 + \sqrt[3]{2}}$
12. $6 - \sqrt[3]{54}$

In Exercises 13–20, simplify the expression. *(See Example 1.)*

13. $\sqrt{20}$
14. $\sqrt{32}$
15. $\sqrt{128}$
16. $-\sqrt{72}$
17. $\sqrt{125b}$
18. $\sqrt{4x^2}$
19. $-\sqrt{81m^3}$
20. $\sqrt{48n^5}$

In Exercises 21–28, simplify the expression. *(See Example 2.)*

21. $\sqrt{\frac{4}{49}}$
22. $-\sqrt{\frac{7}{81}}$
23. $-\sqrt{\frac{23}{64}}$
24. $\sqrt{\frac{65}{121}}$
25. $\sqrt{\frac{a^3}{49}}$
26. $\sqrt{\frac{144}{k^2}}$
27. $\sqrt{\frac{100}{4x^2}}$
28. $\sqrt{\frac{25v^2}{36}}$

In Exercises 29–36, simplify the expression. *(See Example 3.)*

29. $\sqrt[3]{16}$
30. $\sqrt[3]{-108}$
31. $\sqrt[3]{-64x^5}$
32. $-\sqrt[3]{343n^2}$
33. $\sqrt[3]{\frac{6c}{-125}}$
34. $\sqrt[3]{\frac{8h^4}{27}}$
35. $-\sqrt[3]{\frac{81y^2}{1000x^3}}$
36. $\sqrt[3]{\frac{21}{-64a^3b^6}}$

ERROR ANALYSIS **In Exercises 37 and 38, describe and correct the error in simplifying the expression.**

37. ✗

$$\sqrt{72} = \sqrt{4 \cdot 18} = \sqrt{4} \cdot \sqrt{18} = 2\sqrt{18}$$

38. ✗

$$\sqrt[3]{\frac{128y^3}{125}} = \frac{\sqrt[3]{128y^3}}{125} = \frac{\sqrt[3]{64 \cdot 2 \cdot y^3}}{125} = \frac{\sqrt[3]{64} \cdot \sqrt[3]{2} \cdot \sqrt[3]{y^3}}{125} = \frac{4y\sqrt[3]{2}}{125}$$

In Exercises 39–44, write a factor that you can use to rationalize the denominator of the expression.

39. $\dfrac{4}{\sqrt{6}}$ **40.** $\dfrac{1}{\sqrt{13z}}$

41. $\dfrac{2}{\sqrt[3]{x^2}}$ **42.** $\dfrac{3m}{\sqrt[3]{4}}$

43. $\dfrac{\sqrt{2}}{\sqrt{5}-8}$ **44.** $\dfrac{5}{\sqrt{3}+\sqrt{7}}$

In Exercises 45–54, simplify the expression. *(See Example 4.)*

45. $\dfrac{2}{\sqrt{2}}$ **46.** $\dfrac{4}{\sqrt{3}}$

47. $\dfrac{\sqrt{5}}{\sqrt{48}}$ **48.** $\sqrt{\dfrac{4}{52}}$

49. $\dfrac{3}{\sqrt{a}}$ **50.** $\dfrac{1}{\sqrt{2x}}$

51. $\sqrt{\dfrac{3d^2}{5}}$ **52.** $\dfrac{\sqrt{8}}{\sqrt{3n^3}}$

53. $\dfrac{4}{\sqrt[3]{25}}$ **54.** $\sqrt[3]{\dfrac{1}{108y^2}}$

In Exercises 55–60, simplify the expression. *(See Example 5.)*

55. $\dfrac{1}{\sqrt{7}+1}$ **56.** $\dfrac{2}{5-\sqrt{3}}$

57. $\dfrac{\sqrt{10}}{7-\sqrt{2}}$ **58.** $\dfrac{\sqrt{5}}{6+\sqrt{5}}$

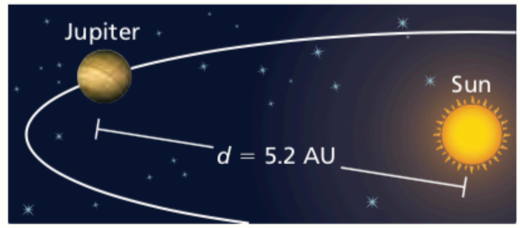
59. $\dfrac{3}{\sqrt{5}-\sqrt{2}}$ **60.** $\dfrac{\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

61. MODELING WITH MATHEMATICS The time t (in seconds) it takes an object to hit the ground is given by $t = \sqrt{\dfrac{h}{16}}$, where h is the height (in feet) from which the object was dropped. *(See Example 6.)*

a. How long does it take an earring to hit the ground when it falls from the roof of the building?

b. How much sooner does the earring hit the ground when it is dropped from two stories (22 feet) below the roof?

62. MODELING WITH MATHEMATICS The orbital period of a planet is the time it takes the planet to travel around the Sun. You can find the orbital period P (in Earth years) using the formula $P = \sqrt{d^3}$, where d is the average distance (in astronomical units, abbreviated AU) of the planet from the Sun.



a. Simplify the formula.

b. What is Jupiter's orbital period?

63. MODELING WITH MATHEMATICS The electric current I (in amperes) an appliance uses is given by the formula $I = \sqrt{\dfrac{P}{R}}$, where P is the power (in watts) and R is the resistance (in ohms). Find the current an appliance uses when the power is 147 watts and the resistance is 5 ohms.

64. MODELING WITH MATHEMATICS You can find the average annual interest rate r (in decimal form) of a savings account using the formula $r = \sqrt{\dfrac{V_2}{V_0}} - 1$, where V_0 is the initial investment and V_2 is the balance of the account after 2 years. Use the formula to compare the savings accounts. In which account would you invest money? Explain.

Account	Initial investment	Balance after 2 years
1	\$275	\$293
2	\$361	\$382
3	\$199	\$214
4	\$254	\$272
5	\$386	\$406

In Exercises 65–68, evaluate the function for the given value of x. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

65. $h(x) = \sqrt{5x};\ x = 10$

66. $g(x) = \sqrt{3x};\ x = 60$

67. $r(x) = \sqrt{\dfrac{3x}{3x^2 + 6}};\ x = 4$

68. $p(x) = \sqrt{\dfrac{x - 1}{5x}};\ x = 8$

In Exercises 69–72, evaluate the expression when $a = -2$, $b = 8$, and $c = \frac{1}{2}$. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

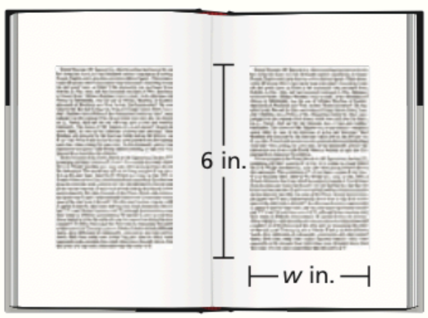
69. $\sqrt{a^2 + bc}$

70. $-\sqrt{4c - 6ab}$

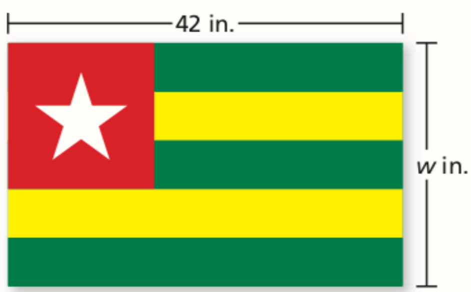
71. $-\sqrt{2a^2 + b^2}$

72. $\sqrt{b^2 - 4ac}$

73. **MODELING WITH MATHEMATICS** The text in the book shown forms a golden rectangle. What is the width w of the text? *(See Example 7.)*



74. **MODELING WITH MATHEMATICS** The flag of Togo is approximately the shape of a golden rectangle. What is the width w of the flag?



In Exercises 75–82, simplify the expression. *(See Example 8.)*

75. $\sqrt{3} - 2\sqrt{2} + 6\sqrt{2}$

76. $\sqrt{5} - 5\sqrt{13} - 8\sqrt{5}$

77. $2\sqrt{6} - 5\sqrt{54}$

78. $9\sqrt{32} + \sqrt{2}$

79. $\sqrt{12} + 6\sqrt{3} + 2\sqrt{6}$

80. $3\sqrt{7} - 5\sqrt{14} + 2\sqrt{28}$

81. $\sqrt[3]{-81} + 4\sqrt[3]{3}$

82. $6\sqrt[3]{128t} - 2\sqrt[3]{2t}$

In Exercises 83–90, simplify the expression. *(See Example 9.)*

83. $\sqrt{2}(\sqrt{45} + \sqrt{5})$

84. $\sqrt{3}(\sqrt{72} - 3\sqrt{2})$

85. $\sqrt{5}(2\sqrt{6x} - \sqrt{96x})$

86. $\sqrt{7y}(\sqrt{27y} + 5\sqrt{12y})$

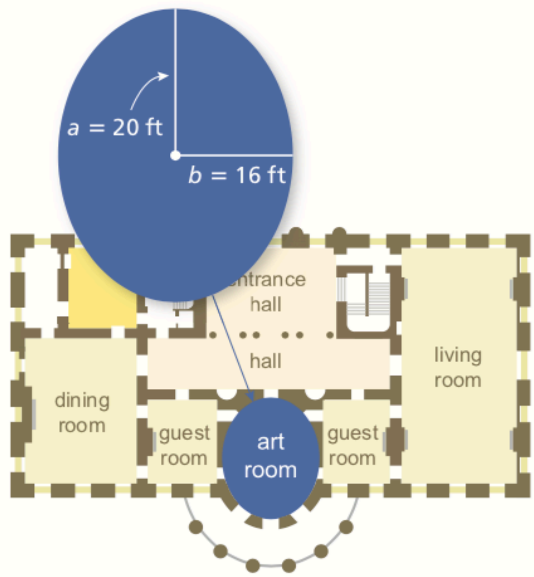
87. $(4\sqrt{2} - \sqrt{98})^2$

88. $(\sqrt{3} + \sqrt{48})(\sqrt{20} - \sqrt{5})$

89. $\sqrt[3]{3}(\sqrt[3]{4} + \sqrt[3]{32})$

90. $\sqrt[3]{2}(\sqrt[3]{135} - 4\sqrt[3]{5})$

91. **MODELING WITH MATHEMATICS** The circumference C of the art room in a mansion is approximated by the formula $C \approx 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$. Approximate the circumference of the room.



92. **CRITICAL THINKING** Determine whether each expression represents a *rational* or an *irrational* number. Justify your answer.

a. $4 + \sqrt{6}$

b. $\dfrac{\sqrt{48}}{\sqrt{3}}$

c. $\dfrac{8}{\sqrt{12}}$

d. $\sqrt{3} + \sqrt{7}$

e. $\dfrac{a}{\sqrt{10} - \sqrt{2}}$, where a is a positive integer

f. $\dfrac{2 + \sqrt{5}}{2b + \sqrt{5b^2}}$, where b is a positive integer

In Exercises 93–98, simplify the expression.

93. $\sqrt[5]{\dfrac{13}{5x^5}}$

94. $\sqrt[4]{\dfrac{10}{81}}$

95. $\sqrt[4]{256y}$

96. $\sqrt[5]{160x^6}$

97. $6\sqrt[4]{9} - \sqrt[5]{9} + 3\sqrt[4]{9}$

98. $\sqrt[5]{2}(\sqrt[4]{7} + \sqrt[5]{16})$

REASONING **In Exercises 99 and 100, use the table shown.**

	2	$\frac{1}{4}$	0	$\sqrt{3}$	$-\sqrt{3}$	π
2						
$\frac{1}{4}$						
0						
$\sqrt{3}$						
$-\sqrt{3}$						
π						

99. Copy and complete the table by (a) finding each sum $\left(2 + 2, 2 + \frac{1}{4}, \text{etc.}\right)$ and (b) finding each product $\left(2 \cdot 2, 2 \cdot \frac{1}{4}, \text{etc.}\right)$.

100. Use your answers in Exercise 99 to determine whether each statement is *always*, *sometimes*, or *never* true. Justify your answer.

a. The sum of a rational number and a rational number is rational.

b. The sum of a rational number and an irrational number is irrational.

c. The sum of an irrational number and an irrational number is irrational.

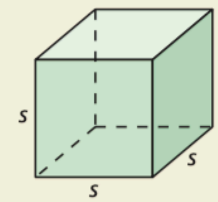
d. The product of a rational number and a rational number is rational.

e. The product of a nonzero rational number and an irrational number is irrational.

f. The product of an irrational number and an irrational number is irrational.

101. **REASONING** Let m be a positive integer. For what values of m will the simplified form of the expression $\sqrt{2^m}$ contain a radical? For what values will it *not* contain a radical? Explain.

102. **HOW DO YOU SEE IT?** The edge length s of a cube is an irrational number, the surface area is an irrational number, and the volume is a rational number. Give a possible value of s.



103. **REASONING** Let a and b be positive numbers. Explain why $\sqrt{ab}$ lies between a and b on a number line. (*Hint:* Let $a < b$ and multiply each side of $a < b$ by a. Then let $a < b$ and multiply each side by b.)

104. **MAKING AN ARGUMENT** Your friend says that you can rationalize the denominator of the expression $\frac{2}{4 + \sqrt[3]{5}}$ by multiplying the numerator and denominator by $4 - \sqrt[3]{5}$. Is your friend correct? Explain.

105. **PROBLEM SOLVING** The ratio of consecutive terms $\frac{a_n}{a_{n-1}}$ in the Fibonacci sequence gets closer and closer to the golden ratio $\frac{1 + \sqrt{5}}{2}$ as n increases. Find the term that precedes 610 in the sequence.

106. **THOUGHT PROVOKING** Use the golden ratio $\frac{1 + \sqrt{5}}{2}$ and the golden ratio conjugate $\frac{1 - \sqrt{5}}{2}$ for each of the following.

a. Show that the golden ratio and golden ratio conjugate are both solutions of $x^2 - x - 1 = 0$.

b. Construct a geometric diagram that has the golden ratio as the length of a part of the diagram.

107. **CRITICAL THINKING** Use the special product pattern $(a + b)(a^2 - ab + b^2) = a^3 + b^3$ to simplify the expression $\frac{2}{\sqrt[3]{x} + 1}$. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph the linear equation. Identify the *x*-intercept. *(Section 3.5)*

108. $y = x - 4$ **109.** $y = -2x + 6$ **110.** $y = -\frac{1}{3}x - 1$ **111.** $y = \frac{3}{2}x + 6$

Solve the equation. Check your solution. *(Section 6.5)*

112. $32 = 2^x$ **113.** $27^x = 3^{x-6}$ **114.** $\left(\frac{1}{6}\right)^{2x} = 216^{1-x}$ **115.** $625^x = \left(\frac{1}{25}\right)^{x+2}$