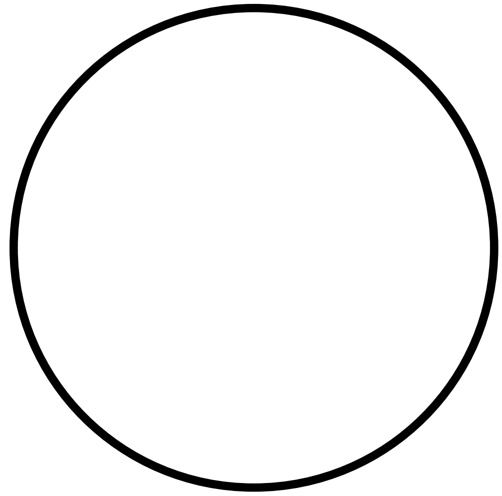
Chapter 10
Circles

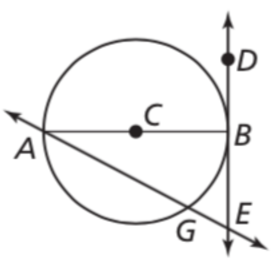
10.1 Lines and Segments That Intersect Circles

Do Now: Draw the following parts of the circle

- **Center Point** P
- **Radius** PA
- **Diameter** BC
- **Chord** DE
- **Secant** FG
- **Tangent** JK



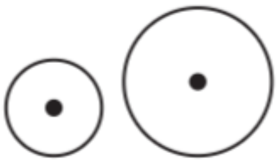
Example 1: Identifying Special Segments and Lines
Identify all the special segments and lines in the diagram below.

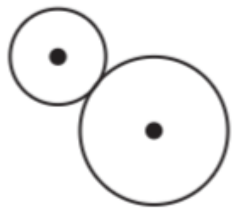


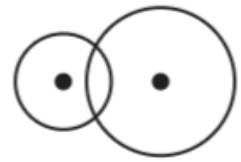
Drawing and Identifying Common Tangents:
Sketch a circle with:

2 intersections 1 intersection 0 intersections

Example 2: Drawing and Identifying Common Tangents
Tell how many common tangents the circles have and draw them.

a. 

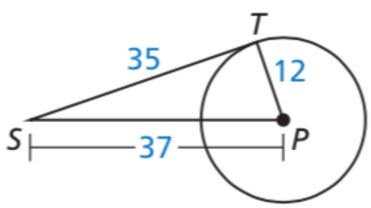
b. 

c. 

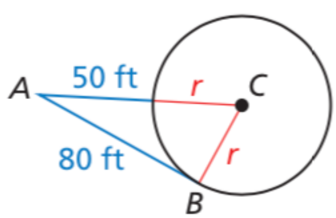
Properties of Tangents:

1) In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle.
2) Tangent segments that form a common external point are congruent

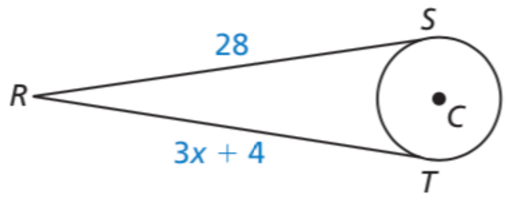
Example 3: Verifying a Tangent to a Circle
Is segment ST tangent to $\odot P$?



Example 4: Finding the Radius of a Circle
In the diagram, point B is a point of tangency. Find the radius of $\odot C$

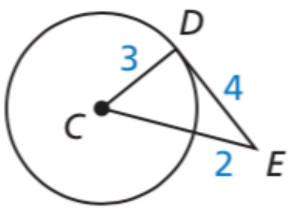


Example 5: Using Properties of Tangents
Segment RS is tangent to $\odot C$ at S, and segment RT is tangent to $\odot C$ at T. Find the value of x.

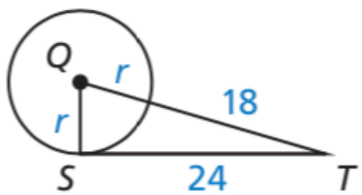


Try on your own:

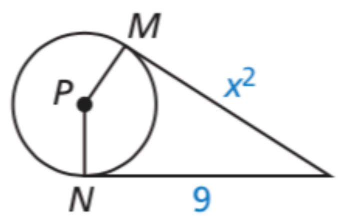
6. Is $\overline{DE}$ tangent to $\odot C$?



7. $\overline{ST}$ is tangent to $\odot Q$. Find the radius of $\odot Q$.



8. Points M and N are points of tangency. Find the value(s) of x.



Homework:
5-11, 14, 15, 18, 19, 21, 23, 26, 29, 32, 37, 38*, 40*, 45*

10.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** How are chords and secants alike? How are they different?

2. **WRITING** Explain how you can determine from the context whether the words *radius* and *diameter* are referring to segments or lengths.

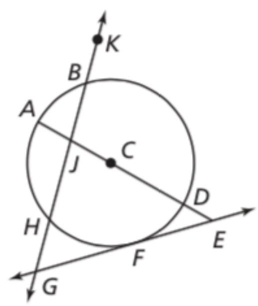
3. **COMPLETE THE SENTENCE** Coplanar circles that have a common center are called ________.

4. **WHICH ONE DOESN'T BELONG?** Which segment does *not* belong with the other three? Explain your reasoning.

chord	radius	tangent	diameter

Monitoring Progress and Modeling with Mathematics

In Exercises 5–10, use the diagram. *(See Example 1.)*



5. Name the circle.

6. Name two radii.

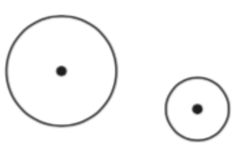
7. Name two chords.

8. Name a diameter.

9. Name a secant.

10. Name a tangent and a point of tangency.

In Exercises 11–14, copy the diagram. Tell how many common tangents the circles have and draw them. *(See Example 2.)*

11. 

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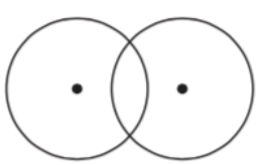
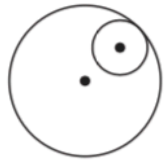
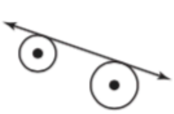
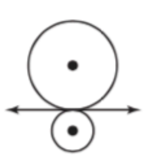
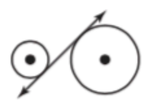
13. 

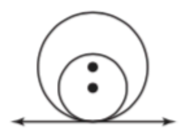
14. 

In Exercises 15–18, tell whether the common tangent is *internal* or *external*.

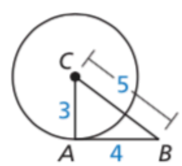
15. 

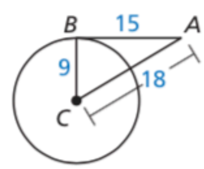
16. 

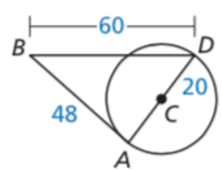
17. 

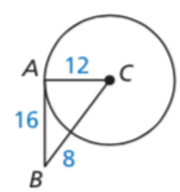
18. 

In Exercises 19–22, tell whether $\overline{AB}$ is tangent to $\odot C$. Explain your reasoning. *(See Example 3.)*

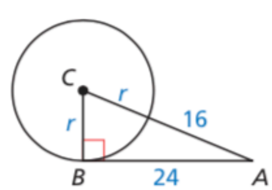
19. 

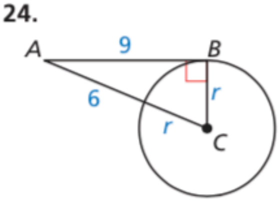
20. 

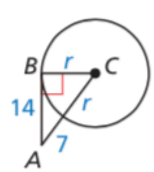
21. 

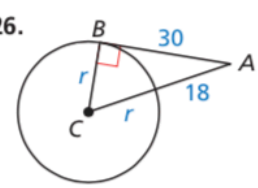
22. 

In Exercises 23–26, point *B* is a point of tangency. Find the radius *r* of $\odot C$. *(See Example 4.)*

23. 

24. 

25. 

26. 

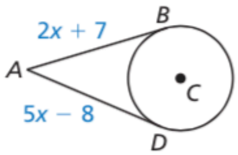
CONSTRUCTION **In Exercises 27 and 28, construct ⊙*C* with the given radius and point *A* outside of ⊙*C*. Then construct a line tangent to ⊙*C* that passes through *A*.**

27. $r = 2$ in.

28. $r = 4.5$ cm

In Exercises 29–32, points *B* and *D* are points of tangency. Find the value(s) of *x*. *(See Example 5.)*

29.



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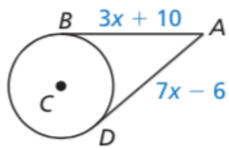

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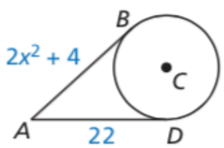

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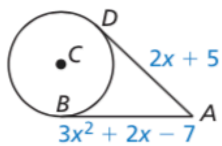
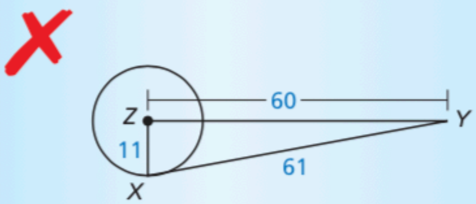
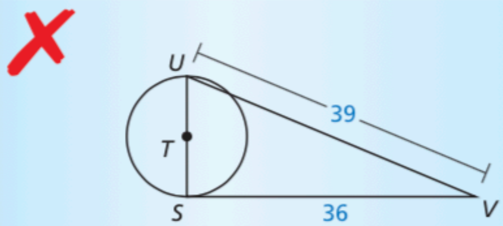


33. **ERROR ANALYSIS** Describe and correct the error in determining whether $\overline{XY}$ is tangent to ⊙*Z*.



Because $11^2 + 60^2 = 61^2$, △*XYZ* is a right triangle. So, $\overline{XY}$ is tangent to ⊙*Z*.

34. **ERROR ANALYSIS** Describe and correct the error in finding the radius of ⊙*T*.



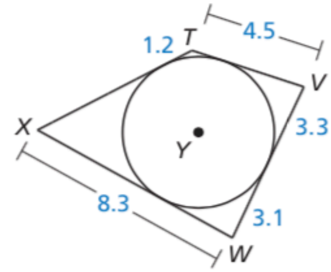
$39^2 - 36^2 = 15^2$

So, the radius is 15.

35. **ABSTRACT REASONING** For a point outside of a circle, how many lines exist tangent to the circle that pass through the point? How many such lines exist for a point on the circle? inside the circle? Explain your reasoning.

36. **CRITICAL THINKING** When will two lines tangent to the same circle not intersect? Justify your answer.

37. **USING STRUCTURE** Each side of quadrilateral *TVWX* is tangent to ⊙*Y*. Find the perimeter of the quadrilateral.

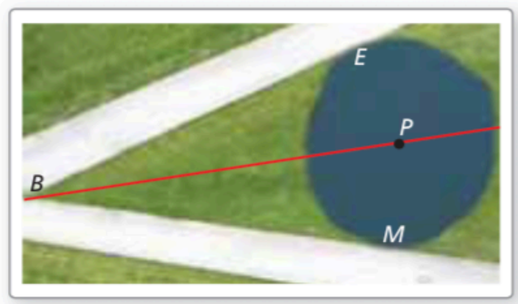


38. **LOGIC** In ⊙*C*, radii $\overline{CA}$ and $\overline{CB}$ are perpendicular. $\overleftrightarrow{BD}$ and $\overleftrightarrow{AD}$ are tangent to ⊙*C*.

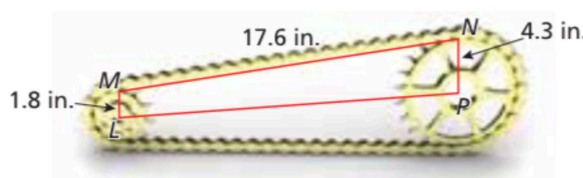
a. Sketch ⊙*C*, $\overline{CA}$, $\overline{CB}$, $\overleftrightarrow{BD}$, and $\overleftrightarrow{AD}$.

b. What type of quadrilateral is *CADB*? Explain your reasoning.

39. **MAKING AN ARGUMENT** Two bike paths are tangent to an approximately circular pond. Your class is building a nature trail that begins at the intersection *B* of the bike paths and runs between the bike paths and over a bridge through the center *P* of the pond. Your classmate uses the Converse of the Angle Bisector Theorem (Theorem 6.4) to conclude that the trail must bisect the angle formed by the bike paths. Is your classmate correct? Explain your reasoning.



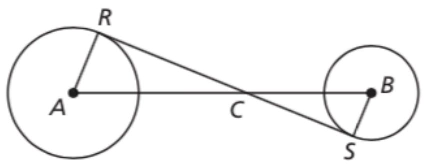
40. **MODELING WITH MATHEMATICS** A bicycle chain is pulled tightly so that $\overline{MN}$ is a common tangent of the gears. Find the distance between the centers of the gears.



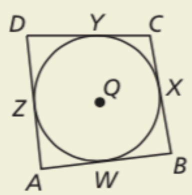
41. **WRITING** Explain why the diameter of a circle is the longest chord of the circle.

42. HOW DO YOU SEE IT? In the figure, $\overrightarrow{PA}$ is tangent to the dime, $\overrightarrow{PC}$ is tangent to the quarter, and $\overrightarrow{PB}$ is a common internal tangent. How do you know that $\overline{PA} \cong \overline{PB} \cong \overline{PC}$?

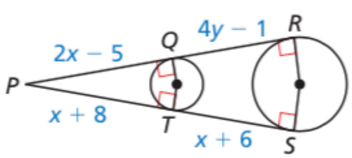
43. PROOF In the diagram, $\overline{RS}$ is a common internal tangent to ⊙A and ⊙B. Prove that $\frac{AC}{BC} = \frac{RC}{SC}$.



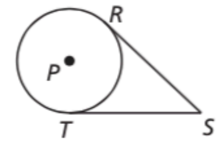
44. THOUGHT PROVOKING A polygon is *circumscribed* about a circle when every side of the polygon is tangent to the circle. In the diagram, quadrilateral $ABCD$ is circumscribed about ⊙Q. Is it always true that $AB + CD = AD + BC$? Justify your answer.



45. MATHEMATICAL CONNECTIONS Find the values of x and y. Justify your answer.



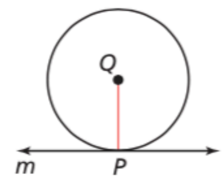
46. PROVING A THEOREM Prove the External Tangent Congruence Theorem (Theorem 10.2).



Given $\overline{SR}$ and $\overline{ST}$ are tangent to ⊙P.

Prove $\overline{SR} \cong \overline{ST}$

47. PROVING A THEOREM Use the diagram to prove each part of the biconditional in the Tangent Line to Circle Theorem (Theorem 10.1).



a. Prove indirectly that if a line is tangent to a circle, then it is perpendicular to a radius. (*Hint*: If you assume line m is not perpendicular to $\overline{QP}$, then the perpendicular segment from point Q to line m must intersect line m at some other point R.)

Given Line m is tangent to ⊙Q at point P.

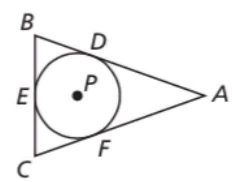
Prove $m \perp \overline{QP}$

b. Prove indirectly that if a line is perpendicular to a radius at its endpoint, then the line is tangent to the circle.

Given $m \perp \overline{QP}$

Prove Line m is tangent to ⊙Q.

48. REASONING In the diagram, $AB = AC = 12$, $BC = 8$, and all three segments are tangent to ⊙P. What is the radius of ⊙P? Justify your answer.

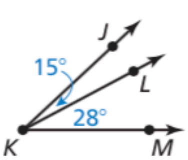


Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Find the indicated measure. *(Section 1.2 and Section 1.5)*

49. $m\angle JKM$



50. AB