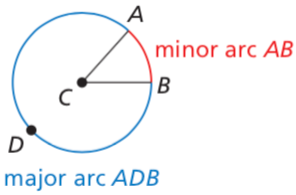
10.2 Finding Arc Measures

Definitions:

1) **Central Angle:** an angle whose vertex is the center of the circle.
2) **Minor arc:** an arc formed by central angle less than 180^{o}
3) **Major arc:** an arc formed by central angle more than 180^{o}
4) **Semicircle:** an arc formed by a central angle exactly 180^{o}

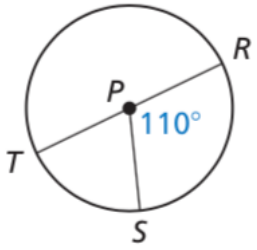


Example 1: Finding Measures of Arcs
Find the measure of each arc of $\odot P$, where segment RT is a diameter.

a) Arc RS

b) Arc RTS

c) Arc RST



Spiral Review: If points A, B and C are collinear and in that order. If AB=7 and BC=9 then AC= _____

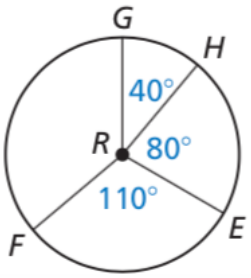
Angle Addition Postulate:
If two arcs of the same circle are adjacent and intersecting at exactly one point you can add the measures of the two adjacent arcs.

Example 2: Using the Arc Addition Postulate
Find the measure of each arc.

a) Arc GE

b) Arc GEF

c) Arc GF



Example 3: Finding Measures of Arcs

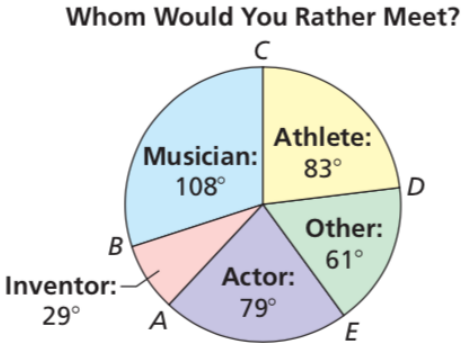
A recent survey asked teenagers whether they would rather meet a famous musician, athlete, actor, inventor, or other person. The circle graph shows the results. Find the indicated angle measures.

A) Measure of arc AC

B) Measure of arc ACD

C) Measure of arc ADC

D) Measure of arc EBD



Proving circles are congruent:

We have learned that we could use SSS, SAS, ASA, AAS, & HL can be used to prove that triangles are congruent. How do you think we can prove that circles are congruent?

Congruent Circles Theorem

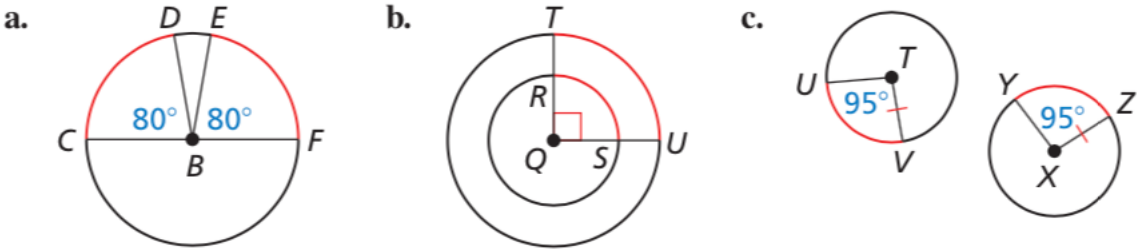
Two circles are congruent if and only if _____

Congruent Central Angles Theorem

Two minor arcs are congruent if and only if their corresponding central angles are congruent

Example 4: Identifying Congruent Arcs

Tell whether the red arcs are congruent. Explain why or why not.



Important note! All circles are _____

Homework:

3-17, 21, 22-24, 31, 34*, 36

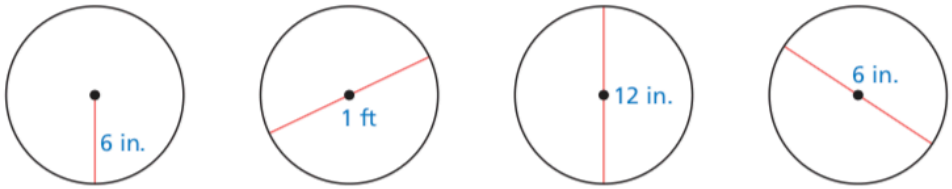
10.2 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

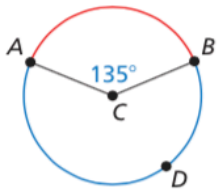
1. **VOCABULARY** Copy and complete: If $\angle ACB$ and $\angle DCE$ are congruent central angles of $\odot C$, then $\overset{\frown}{AB}$ and $\overset{\frown}{DE}$ are _____.

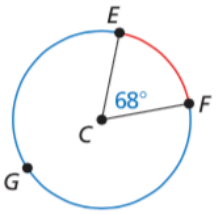
2. **WHICH ONE DOESN'T BELONG?** Which circle does *not* belong with the other three? Explain your reasoning.

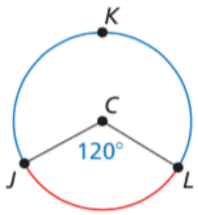


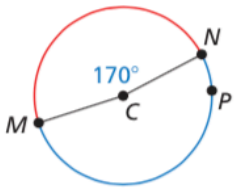
Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, name the red minor arc and find its measure. Then name the blue major arc and find its measure.

3. 

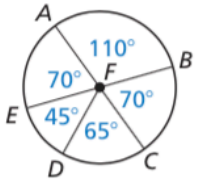
4. 

5. 

6. 

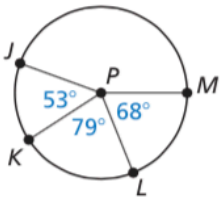
In Exercises 7–14, identify the given arc as a *major arc*, *minor arc*, or *semicircle*. Then find the measure of the arc. *(See Example 1.)*

7. $\overset{\frown}{BC}$
8. $\overset{\frown}{DC}$
9. $\overset{\frown}{ED}$
10. $\overset{\frown}{AE}$
11. $\overset{\frown}{EAB}$
12. $\overset{\frown}{ABC}$
13. $\overset{\frown}{BAC}$
14. $\overset{\frown}{EBD}$

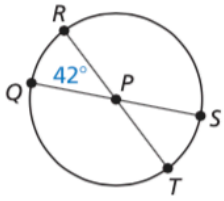


In Exercises 15 and 16, find the measure of each arc. *(See Example 2.)*

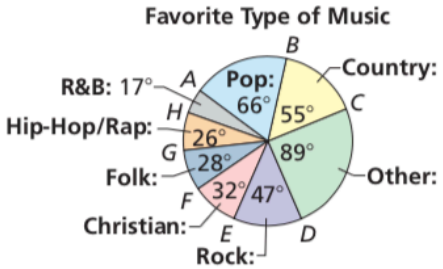
15. a. $\overset{\frown}{JL}$
    b. $\overset{\frown}{KM}$
    c. $\overset{\frown}{JLM}$
    d. $\overset{\frown}{JM}$



16. a. $\overset{\frown}{RS}$
    b. $\overset{\frown}{QRS}$
    c. $\overset{\frown}{QST}$
    d. $\overset{\frown}{QT}$

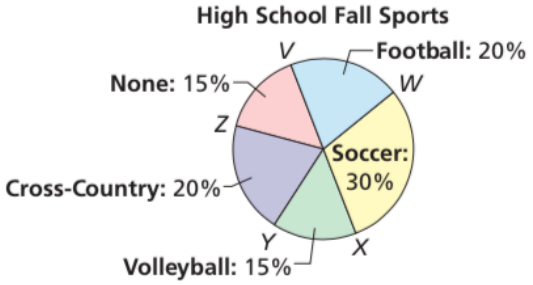


17. **MODELING WITH MATHEMATICS** A recent survey asked high school students their favorite type of music. The results are shown in the circle graph. Find each indicated arc measure. *(See Example 3.)*



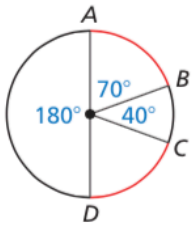
a. $m\overset{\frown}{AE}$
b. $m\overset{\frown}{ACE}$
c. $m\overset{\frown}{GDC}$
d. $m\overset{\frown}{BHC}$
e. $m\overset{\frown}{FD}$
f. $m\overset{\frown}{FBD}$

18. ABSTRACT REASONING The circle graph shows the percentages of students enrolled in fall sports at a high school. Is it possible to find the measure of each minor arc? If so, find the measure of the arc for each category shown. If not, explain why it is not possible.



In Exercises 19–22, tell whether the red arcs are congruent. Explain why or why not. *(See Example 4.)*

19.



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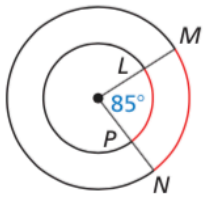

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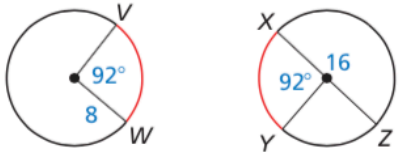
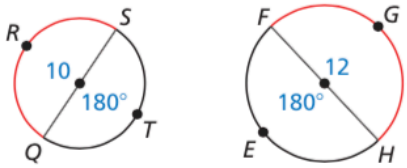
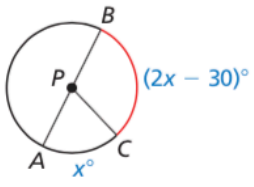


22.

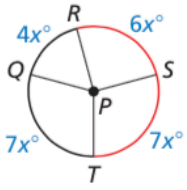


MATHEMATICAL CONNECTIONS In Exercises 23 and 24, find the value of x. Then find the measure of the red arc.

23.

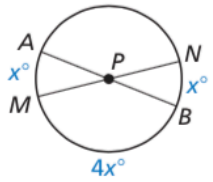


24.

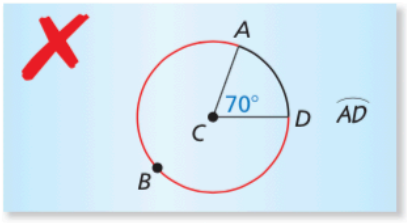


25. MAKING AN ARGUMENT Your friend claims that any two arcs with the same measure are similar. Your cousin claims that any two arcs with the same measure are congruent. Who is correct? Explain.

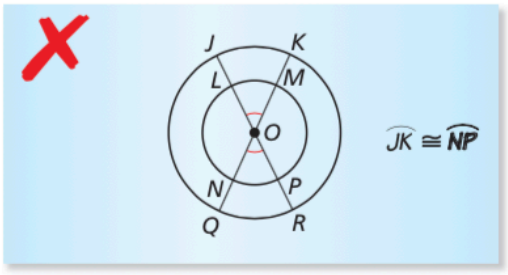
26. MAKING AN ARGUMENT Your friend claims that there is not enough information given to find the value of x. Is your friend correct? Explain your reasoning.



27. ERROR ANALYSIS Describe and correct the error in naming the red arc.



28. ERROR ANALYSIS Describe and correct the error in naming congruent arcs.



29. ATTENDING TO PRECISION Two diameters of $\odot P$ are $\overline{AB}$ and $\overline{CD}$. Find $m\overset{\frown}{ACD}$ and $m\overset{\frown}{AC}$ when $m\overset{\frown}{AD} = 20°$.

30. REASONING In $\odot R$, $m\overset{\frown}{AB} = 60°$, $m\overset{\frown}{BC} = 25°$, $m\overset{\frown}{CD} = 70°$, and $m\overset{\frown}{DE} = 20°$. Find two possible measures of $\overset{\frown}{AE}$.

31. MODELING WITH MATHEMATICS On a regulation dartboard, the outermost circle is divided into twenty congruent sections. What is the measure of each arc in this circle?

32. MODELING WITH MATHEMATICS You can use the time zone wheel to find the time in different locations across the world. For example, to find the time in Tokyo when it is 4 P.M. in San Francisco, rotate the small wheel until 4 P.M. and San Francisco line up, as shown. Then look at Tokyo to see that it is 9 A.M. there.

a. What is the arc measure between each time zone on the wheel?

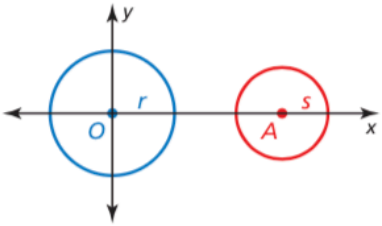
b. What is the measure of the minor arc from the Tokyo zone to the Anchorage zone?

c. If two locations differ by 180° on the wheel, then it is 3 P.M. at one location when it is _____ at the other location.

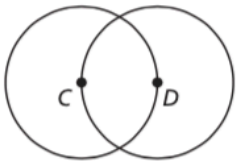
33. PROVING A THEOREM Write a coordinate proof of the Similar Circles Theorem (Theorem 10.5).

Given $\odot O$ with center $O(0, 0)$ and radius r, $\odot A$ with center $A(a, 0)$ and radius s

Prove $\odot O \sim \odot A$



34. ABSTRACT REASONING Is there enough information to tell whether $\odot C \cong \odot D$? Explain your reasoning.



35. PROVING A THEOREM Use the diagram on page 540 to prove each part of the biconditional in the Congruent Circles Theorem (Theorem 10.3).

a. Given $\overline{AC} \cong \overline{BD}$
Prove $\odot A \cong \odot B$

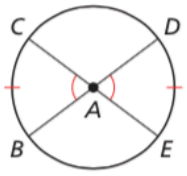
b. Given $\odot A \cong \odot B$
Prove $\overline{AC} \cong \overline{BD}$

36. HOW DO YOU SEE IT? Are the circles on the target *similar* or *congruent*? Explain your reasoning.

37. PROVING A THEOREM Use the diagram to prove each part of the biconditional in the Congruent Central Angles Theorem (Theorem 10.4).

a. Given $\angle BAC \cong \angle DAE$
Prove $\overset{\frown}{BC} \cong \overset{\frown}{DE}$

b. Given $\overset{\frown}{BC} \cong \overset{\frown}{DE}$
Prove $\angle BAC \cong \angle DAE$



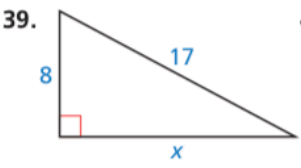
38. THOUGHT PROVOKING Write a formula for the length of a circular arc. Justify your answer.

Maintaining Mathematical Proficiency

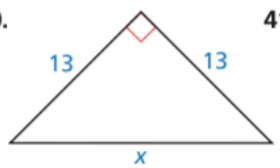
Reviewing what you learned in previous grades and lessons

Find the value of x. Tell whether the side lengths form a Pythagorean triple. *(Section 9.1)*

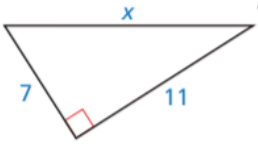
39.



40.



41.



42.