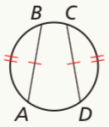
## 10.3 Using Chords

**Recall:** What is a chord?

### 2 Theorems and a converse!

**Theorem 10.6 Congruent Corresponding Chords Theorem**

In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

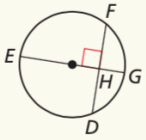



*Proof* Ex. 19, p. 550

$\overset{\frown}{AB} \cong \overset{\frown}{CD}$ if and only if $\overline{AB} \cong \overline{CD}$.

**Theorem 10.7 Perpendicular Chord Bisector Theorem**

If a diameter of a circle is perpendicular to a chord, then the diameter bisects the chord and its arc.

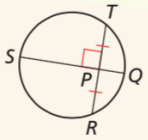



If $\overline{EG}$ is a diameter and $\overline{EG} \perp \overline{DF}$, then $\overline{HD} \cong \overline{HF}$ and $\overset{\frown}{GD} \cong \overset{\frown}{GF}$.

*Proof* Ex. 22, p. 550

**Theorem 10.8 Perpendicular Chord Bisector Converse**

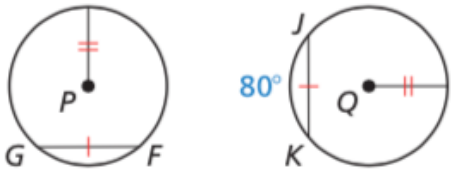
If one chord of a circle is a perpendicular bisector of another chord, then the first chord is a diameter.




If $\overline{QS}$ is a perpendicular bisector of $\overline{TR}$, then $\overline{QS}$ is a diameter of the circle.

*Proof* Ex. 23, p. 550

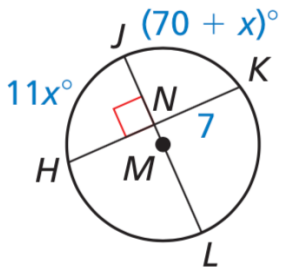
**Example 1:** Using Congruent Chords to Find an Arc Measure
In the diagram $\odot P \cong \odot Q, FG \cong JK,$ and measure of arc $JK = 80^o$. Find measure of arc FG.




**Example 2:** Using a Diameter

a) Find HK

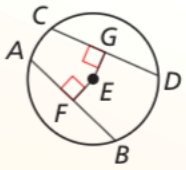
b) Find measure of arc HK



**Example 3:** Real World Applications
Three bushes area arranged in a garden, as shown. Where should you place a sprinkler that is the same distance from each bush?

**Theorem 10.9 Equidistant Chords Theorem**

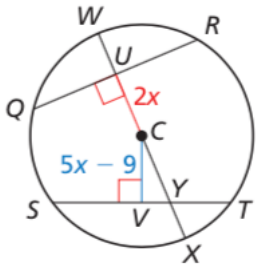
In the same circle, or in congruent circles, two chords are congruent if and only if they are equidistant from the center.




*Proof* Ex. 25, p. 550

$\overline{AB} \cong \overline{CD}$ if and only if $EF = EG$.

**Example 4:** Using Congruent Chords to Find a Circles Radius
In the diagram, $QR = ST = 16, CU = 2x, and\ CV = 5x - 9$. Find the radius of $\odot C$.




Homework:
3, 5, 7, 9, 13, 14, 15, 16

# 10.3 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

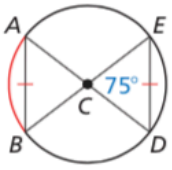
## Vocabulary and Core Concept Check

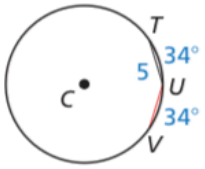
1. **WRITING** Describe what it means to bisect a chord.

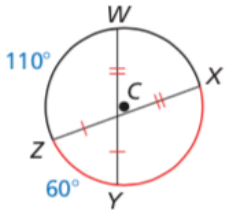
2. **WRITING** Two chords of a circle are perpendicular and congruent. Does one of them have to be a diameter? Explain your reasoning.

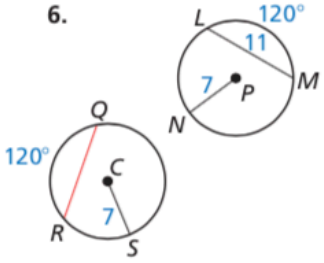
## Monitoring Progress and Modeling with Mathematics

**In Exercises 3–6, find the measure of the red arc or chord in $\odot C$.** *(See Example 1.)*

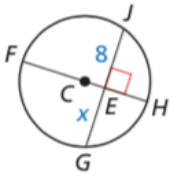
3. 

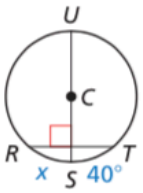

4. 

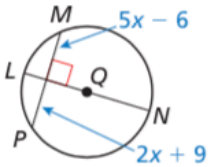

5. 

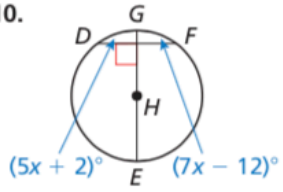

6. 


**In Exercises 7–10, find the value of $x$.** *(See Example 2.)*

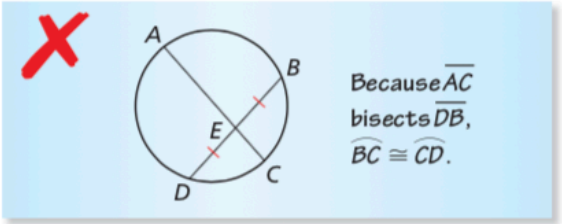
7. 


8. 


9. 

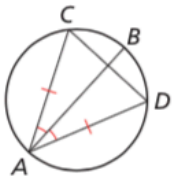

10. 

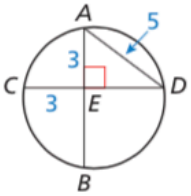

11. **ERROR ANALYSIS** Describe and correct the error in reasoning.



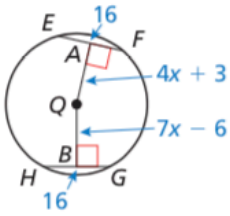

12. **PROBLEM SOLVING** In the cross section of the submarine shown, the control panels are parallel and the same length. Describe a method you can use to find the center of the cross section. Justify your method. *(See Example 3.)*

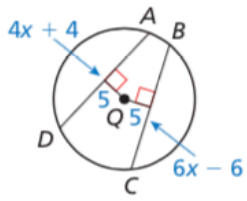
**In Exercises 13 and 14, determine whether $\overline{AB}$ is a diameter of the circle. Explain your reasoning.**

13. 

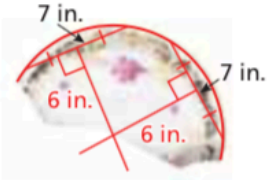

14. 


**In Exercises 15 and 16, find the radius of $\odot Q$.** *(See Example 4.)*

15. 

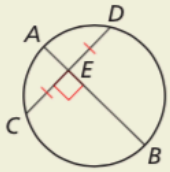

16. 


17. **PROBLEM SOLVING** An archaeologist finds part of a circular plate. What was the diameter of the plate to the nearest tenth of an inch? Justify your answer.

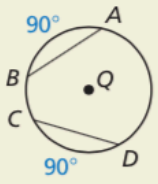


**18. HOW DO YOU SEE IT?** What can you conclude from each diagram? Name a theorem that justifies your answer.

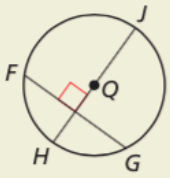
**a.**



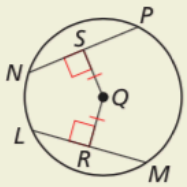

**b.**



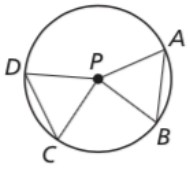

**c.**




**d.**



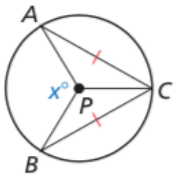

**19. PROVING A THEOREM** Use the diagram to prove each part of the biconditional in the Congruent Corresponding Chords Theorem (Theorem 10.6).



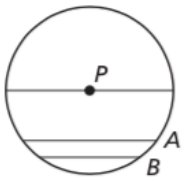

**a. Given** $\overline{AB}$ and $\overline{CD}$ are congruent chords.
**Prove** $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

**b. Given** $\overset{\frown}{AB} \cong \overset{\frown}{CD}$
**Prove** $\overline{AB} \cong \overline{CD}$

**20. MATHEMATICAL CONNECTIONS** In $\odot P$, all the arcs shown have integer measures. Show that $x$ must be even.



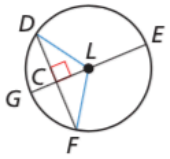

**21. REASONING** In $\odot P$, the lengths of the parallel chords are 20, 16, and 12. Find $m\overset{\frown}{AB}$. Explain your reasoning.




**22. PROVING A THEOREM** Use congruent triangles to prove the Perpendicular Chord Bisector Theorem (Theorem 10.7).

**Given** $\overline{EG}$ is a diameter of $\odot L$.
$\overline{EG} \perp \overline{DF}$

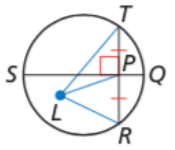
**Prove** $\overline{DC} \cong \overline{FC}$, $\overset{\frown}{DG} \cong \overset{\frown}{FG}$




**23. PROVING A THEOREM** Write a proof of the Perpendicular Chord Bisector Converse (Theorem 10.8).

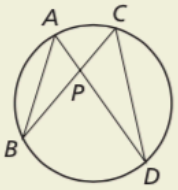
**Given** $\overline{QS}$ is a perpendicular bisector of $\overline{RT}$.

**Prove** $\overline{QS}$ is a diameter of the circle $L$.



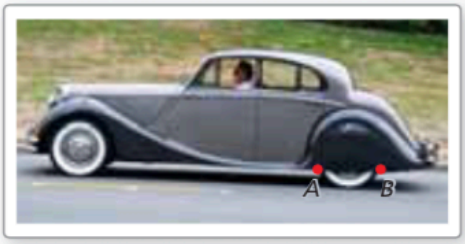

(*Hint*: Plot the center $L$ and draw $\triangle LPT$ and $\triangle LPR$.)

**24. THOUGHT PROVOKING** Consider two chords that intersect at point $P$. Do you think that $\frac{AP}{BP} = \frac{CP}{DP}$? Justify your answer.




**25. PROVING A THEOREM** Use the diagram with the Equidistant Chords Theorem (Theorem 10.9) on page 548 to prove both parts of the biconditional of this theorem.

**26. MAKING AN ARGUMENT** A car is designed so that the rear wheel is only partially visible below the body of the car. The bottom edge of the panel is parallel to the ground. Your friend claims that the point where the tire touches the ground bisects $\overset{\frown}{AB}$. Is your friend correct? Explain your reasoning.




## Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

**Find the missing interior angle measure.** *(Section 7.1)*

**27.** Quadrilateral $JKLM$ has angle measures $m\angle J = 32°$, $m\angle K = 25°$, and $m\angle L = 44°$. Find $m\angle M$.

**28.** Pentagon $PQRST$ has angle measures $m\angle P = 85°$, $m\angle Q = 134°$, $m\angle R = 97°$, and $m\angle S = 102°$. Find $m\angle T$.