

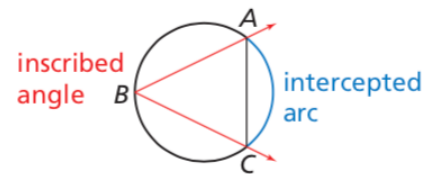
10.4 Inscribed Angles and Polygons

What is an inscribed angle?

Inscribed angle: an angle whose vertex is on a circle and whose sides contain chords of the circle

What is the difference between an inscribed angle and a central angle?

Intercepted Arc: an arc that lies between two lines, rays, or segments



$\angle B$ intercepts \widehat{AC} .
 \widehat{AC} subtends $\angle B$.
 \widehat{AC} subtends $\angle B$.

Theorem: Measure of an Inscribed Angle Theorem

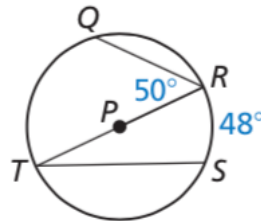
The measure of an inscribed angle is one-half the measure of its intercepts arc.

Example 1: Using Inscribed Angles

Find the indicated measure

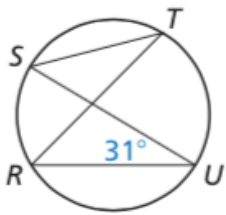
a) $m\angle T =$ _____

b) measure of arc $QR =$ _____



Example 2: Finding the Measure of an Intercepted Arc

Find the measure of arc RS and $m\angle STR$. What do you notice about $\angle STR$ and $\angle RUS$?

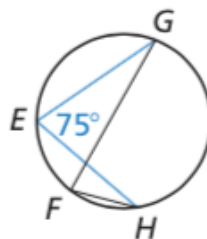


Theorem: Inscribed Angles of a Circle Theorem

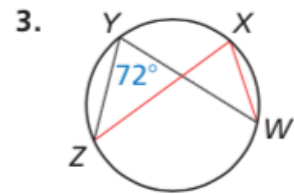
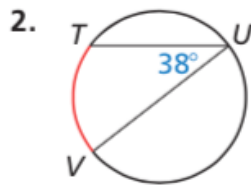
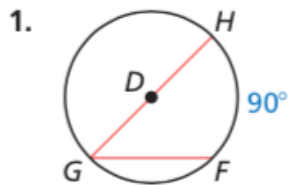
If two inscribed angles of a circle intercept the same arc, then the angles are congruent.

Example 3: Finding the Measure of an Angle

Given $m\angle E = 75^\circ$, find $m\angle F$.



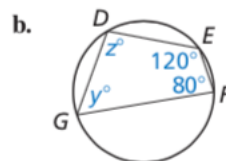
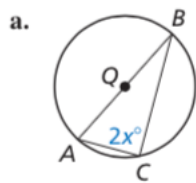
Try on your own: Determine the measure of the red arc or angle.



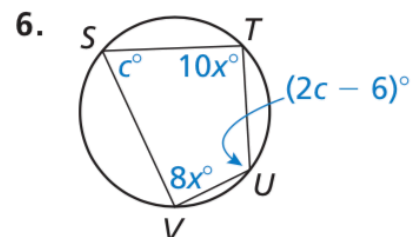
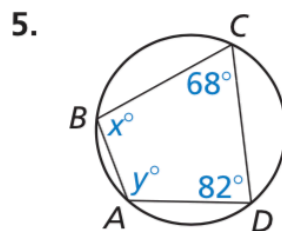
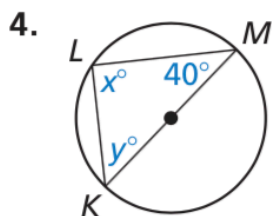
Inscribed Polygons: A polygon is an inscribed polygon when all its vertices lie on a circle.

Example 4: Using Inscribed Polygons

Find the value of each variable:



Try on your own: Find the value of each variable.



Homework:

3-15odd, 19-21, 25-30, 41**

10.4 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

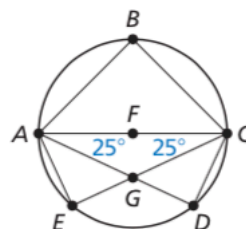
- VOCABULARY** If a circle is circumscribed about a polygon, then the polygon is an _____.
- DIFFERENT WORDS, SAME QUESTION** Which is different?
Find “both” answers.

Find $m\angle ABC$.

Find $m\angle AGC$.

Find $m\angle AEC$.

Find $m\angle ADC$.



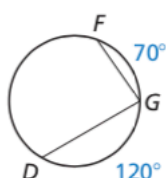
Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, find the indicated measure.
(See Examples 1 and 2.)

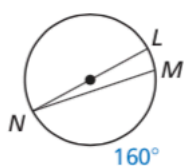
3. $m\angle A$



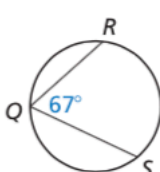
4. $m\angle G$



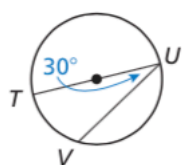
5. $m\angle N$



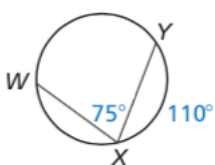
6. $m\widehat{RS}$



7. $m\widehat{VU}$

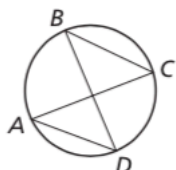


8. $m\widehat{WX}$

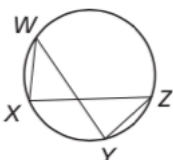


In Exercises 9 and 10, name two pairs of congruent angles.

9.

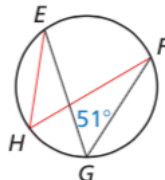


10.

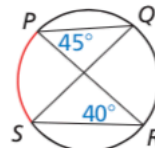


In Exercises 11 and 12, find the measure of the red arc or angle. (See Example 3.)

11.

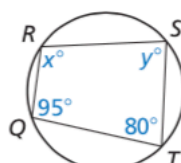


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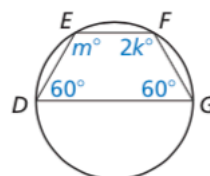


In Exercises 13–16, find the value of each variable.
(See Example 4.)

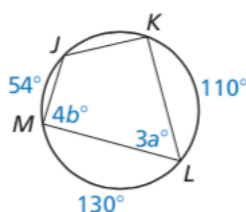
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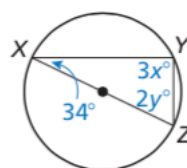
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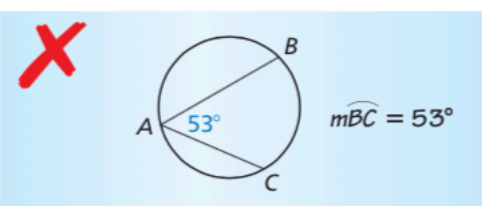
15.



16.



17. **ERROR ANALYSIS** Describe and correct the error in finding $m\widehat{BC}$.

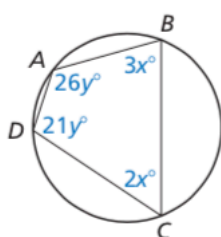


18. **MODELING WITH MATHEMATICS** A carpenter's square is an L-shaped tool used to draw right angles. You need to cut a circular piece of wood into two semicircles. How can you use the carpenter's square to draw a diameter on the circular piece of wood? (See Example 5.)

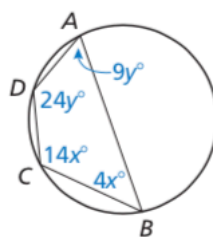


MATHEMATICAL CONNECTIONS In Exercises 19–21, find the values of x and y . Then find the measures of the interior angles of the polygon.

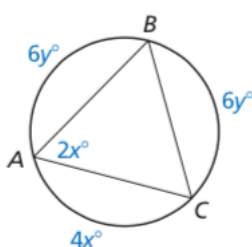
19.



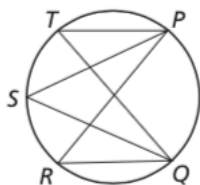
20.



21.



22. **MAKING AN ARGUMENT** Your friend claims that $\angle PTQ \cong \angle PSQ \cong \angle PRQ$. Is your friend correct? Explain your reasoning.



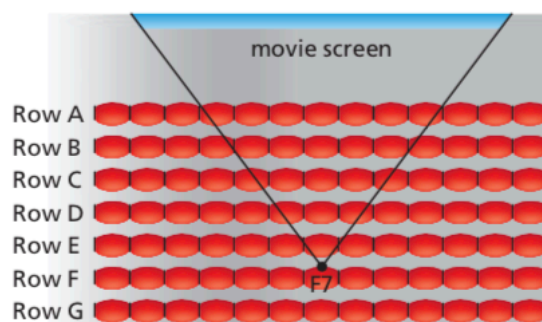
23. **CONSTRUCTION** Construct an equilateral triangle inscribed in a circle.
24. **CONSTRUCTION** The side length of an inscribed regular hexagon is equal to the radius of the circumscribed circle. Use this fact to construct a regular hexagon inscribed in a circle.

REASONING In Exercises 25–30, determine whether a quadrilateral of the given type can always be inscribed inside a circle. Explain your reasoning.

25. square
26. rectangle
27. parallelogram
28. kite
29. rhombus
30. isosceles trapezoid

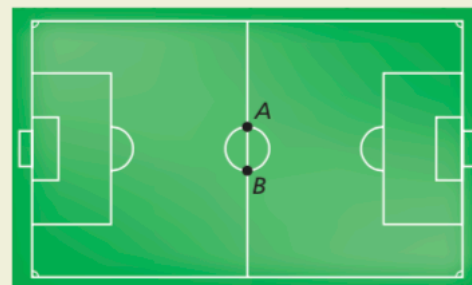
31. **MODELING WITH MATHEMATICS** Three moons, A, B, and C, are in the same circular orbit 100,000 kilometers above the surface of a planet. The planet is 20,000 kilometers in diameter and $m\angle ABC = 90^\circ$. Draw a diagram of the situation. How far is moon A from moon C?

32. **MODELING WITH MATHEMATICS** At the movie theater, you want to choose a seat that has the best viewing angle, so that you can be close to the screen and still see the whole screen without moving your eyes. You previously decided that seat F7 has the best viewing angle, but this time someone else is already sitting there. Where else can you sit so that your seat has the same viewing angle as seat F7? Explain.



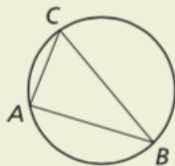
33. **WRITING** A right triangle is inscribed in a circle, and the radius of the circle is given. Explain how to find the length of the hypotenuse.

34. **HOW DO YOU SEE IT?** Let point Y represent your location on the soccer field below. What type of angle is $\angle AYB$ if you stand anywhere on the circle except at point A or point B?



35. **WRITING** Explain why the diagonals of a rectangle inscribed in a circle are diameters of the circle.

36. **THOUGHT PROVOKING** The figure shows a circle that is circumscribed about $\triangle ABC$. Is it possible to circumscribe a circle about any triangle? Justify your answer.



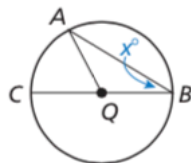
37. **PROVING A THEOREM** If an angle is inscribed in $\odot Q$, the center Q can be on a side of the inscribed angle, inside the inscribed angle, or outside the inscribed angle. Prove each case of the Measure of an Inscribed Angle Theorem (Theorem 10.10).

a. Case 1

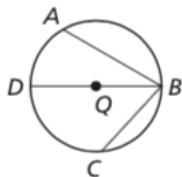
Given $\angle ABC$ is inscribed in $\odot Q$.
Let $m\angle B = x^\circ$.
Center Q lies on \overline{AC} .

Prove $m\angle ABC = \frac{1}{2} m\widehat{AC}$

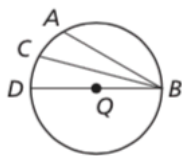
(Hint: Show that $\triangle AQB$ is isosceles. Then write $m\widehat{AC}$ in terms of x .)



- b. Case 2** Use the diagram and auxiliary line to write **Given** and **Prove** statements for Case 2. Then write a proof.



- c. Case 3** Use the diagram and auxiliary line to write **Given** and **Prove** statements for Case 3. Then write a proof.



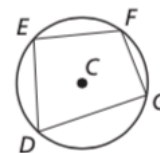
38. **PROVING A THEOREM** Write a paragraph proof of the Inscribed Angles of a Circle Theorem (Theorem 10.11). First, draw a diagram and write **Given** and **Prove** statements.

39. **PROVING A THEOREM** The Inscribed Right Triangle Theorem (Theorem 10.12) is written as a conditional statement and its converse. Write a plan for proof for each statement.

40. **PROVING A THEOREM** Copy and complete the paragraph proof for one part of the Inscribed Quadrilateral Theorem (Theorem 10.13).

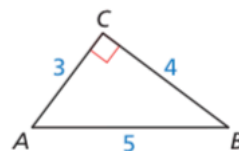
Given $\odot C$ with inscribed quadrilateral $DEFG$

Prove $m\angle D + m\angle F = 180^\circ$,
 $m\angle E + m\angle G = 180^\circ$

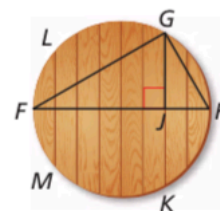


By the Arc Addition Postulate (Postulate 10.1), $m\widehat{EFG} + \underline{\hspace{1cm}} = 360^\circ$ and $m\widehat{FGD} + m\widehat{DEF} = 360^\circ$. Using the Theorem, $m\widehat{EDG} = 2m\angle F$, $m\widehat{EFG} = 2m\angle D$, $m\widehat{DEF} = 2m\angle G$, and $m\widehat{FGD} = 2m\angle E$. By the Substitution Property of Equality, $2m\angle D + \underline{\hspace{1cm}} = 360^\circ$, so . Similarly, .

41. **CRITICAL THINKING** In the diagram, $\angle C$ is a right angle. If you draw the smallest possible circle through C tangent to \overline{AB} , the circle will intersect \overline{AC} at J and \overline{BC} at K . Find the exact length of \overline{JK} .



42. **CRITICAL THINKING** You are making a circular cutting board. To begin, you glue eight 1-inch boards together, as shown. Then you draw and cut a circle with an 8-inch diameter from the boards.



- a. \overline{FH} is a diameter of the circular cutting board. Write a proportion relating GJ and JH . State a theorem to justify your answer.
- b. Find FJ , JH , and GJ . What is the length of the cutting board seam labeled \overline{GK} ?

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the equation. Check your solution. (Skills Review Handbook)

43. $3x = 145$

44. $\frac{1}{2}x = 63$

45. $240 = 2x$

46. $75 = \frac{1}{2}(x - 30)$