

## 1.4 Solving Absolute Value Equations

What does an absolute value do? What does it mean?

**Properties of absolute values:**

1)  $|a| \geq 0$

2)  $|-a| = |a|$

3)  $|ab| = |a||b|$

4)  $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, b \neq 0$

**Example 1:** Solving Absolute Value Equations

Solve each equation.

a)  $|x - 4| = 6$

b)  $|3x + 1| = -5$

**Example 2:** Solving an Absolute Value Equations

Solve.

$$|3x + 9| - 10 = -4$$

**Example 3:** Modeling & Writing an Absolute Value Equation.

The cheerleading half time show at all Bergen Tech sporting events has a minimum length of 4 minutes. The maximum length of a routine is 5 minutes. Write an absolute value equation that represents the minimum and maximum lengths.

**Try on your own!**

Solve the equations. Check your solutions

**4.**  $|x - 2| + 5 = 9$

**5.**  $4|2x + 7| = 16$

**6.**  $-2|5x - 1| - 3 = -11$

- 7.** For a poetry contest, the minimum length of a poem is 16 lines. The maximum length is 32 lines. Write an absolute value equation that represents the minimum and maximum lengths.

**Example 4:** Solving Equations with Two Absolute Values

Solve:

**a)**  $|3x - 4| = |x|$

**b)**  $|4x - 10| = 2|3x + 1|$

**Try on your own:** Solve the following.

**8.**  $|x + 8| = |2x + 1|$

**9.**  $3|x - 4| = |2x + 5|$

**Example 5: Identifying Extraneous Solutions**

Solve. Check your solutions.

$$|2x + 12| = 4x$$

**Example 6: Solving Absolute Value Equations**

Solve:

$$|x + 5| = |x + 11|$$

**Try on your own:** Solve.

**10.**  $|x + 6| = 2x$

**11.**  $|3x - 2| = x$

**12.**  $|2 + x| = |x - 8|$

**13.**  $|5x - 2| = |5x + 4|$

Homework:

15-23odd, 26-30, 35, 39, 41, 32, 52, 59\*

# 1.4 Exercises

Dynamic Solutions available at [BigIdeasMath.com](http://BigIdeasMath.com)

## Vocabulary and Core Concept Check

- VOCABULARY** What is an extraneous solution?
- WRITING** Without calculating, how do you know that the equation  $|4x - 7| = -1$  has no solution?

## Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, simplify the expression.

- $|-9|$
- $-|15|$
- $|14| - |-14|$
- $|-3| + |3|$
- $-|-5 \cdot (-7)|$
- $|-0.8 \cdot 10|$
- $\left|\frac{27}{-3}\right|$
- $\left|-\frac{-12}{4}\right|$

In Exercises 11–24, solve the equation. Graph the solution(s), if possible. (See Examples 1 and 2.)

- $|w| = 6$
- $|r| = -2$
- $|y| = -18$
- $|x| = 13$
- $|m + 3| = 7$
- $|q - 8| = 14$
- $|-3d| = 15$
- $\left|\frac{t}{2}\right| = 6$
- $|4b - 5| = 19$
- $|x - 1| + 5 = 2$
- $-4|8 - 5n| = 13$
- $-3\left|1 - \frac{2}{3}v\right| = -9$
- $3 = -2\left|\frac{1}{4}s - 5\right| + 3$
- $9|4p + 2| + 8 = 35$

- WRITING EQUATIONS** The minimum distance from Earth to the Sun is 91.4 million miles. The maximum distance is 94.5 million miles. (See Example 3.)

- Represent these two distances on a number line.
- Write an absolute value equation that represents the minimum and maximum distances.

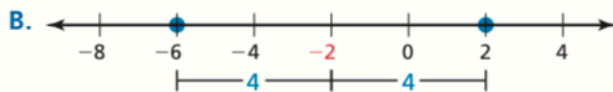
- WRITING EQUATIONS** The shoulder heights of the shortest and tallest miniature poodles are shown.



- Represent these two heights on a number line.
- Write an absolute value equation that represents these heights.

**USING STRUCTURE** In Exercises 27–30, match the absolute value equation with its graph without solving the equation.

- $|x + 2| = 4$
- $|x - 4| = 2$
- $|x - 2| = 4$
- $|x + 4| = 2$



In Exercises 31–34, write an absolute value equation that has the given solutions.

31.  $x = 8$  and  $x = 18$       32.  $x = -6$  and  $x = 10$   
 33.  $x = 2$  and  $x = 9$       34.  $x = -10$  and  $x = -5$

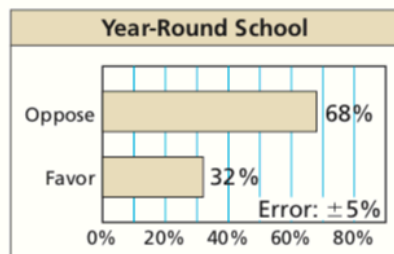
In Exercises 35–44, solve the equation. Check your solutions. (See Examples 4, 5, and 6.)

35.  $|4n - 15| = |n|$       36.  $|2c + 8| = |10c|$   
 37.  $|2b - 9| = |b - 6|$       38.  $|3k - 2| = 2|k + 2|$   
 39.  $4|p - 3| = |2p + 8|$       40.  $2|4w - 1| = 3|4w + 2|$   
 41.  $|3h + 1| = 7h$       42.  $|6a - 5| = 4a$   
 43.  $|f - 6| = |f + 8|$       44.  $|3x - 4| = |3x - 5|$

45. **MODELING WITH MATHEMATICS** Starting from 300 feet away, a car drives toward you. It then passes by you at a speed of 48 feet per second. The distance  $d$  (in feet) of the car from you after  $t$  seconds is given by the equation  $d = |300 - 48t|$ . At what times is the car 60 feet from you?

46. **MAKING AN ARGUMENT** Your friend says that the absolute value equation  $|3x + 8| - 9 = -5$  has no solution because the constant on the right side of the equation is negative. Is your friend correct? Explain.

47. **MODELING WITH MATHEMATICS** You randomly survey students about year-round school. The results are shown in the graph.



The error given in the graph means that the actual percent could be 5% more or 5% less than the percent reported by the survey.

- a. Write and solve an absolute value equation to find the least and greatest percents of students who could be in favor of year-round school.  
 b. A classmate claims that  $\frac{1}{3}$  of the student body is actually in favor of year-round school. Does this conflict with the survey data? Explain.

48. **MODELING WITH MATHEMATICS** The recommended weight of a soccer ball is 430 grams. The actual weight is allowed to vary by up to 20 grams.



- a. Write and solve an absolute value equation to find the minimum and maximum acceptable soccer ball weights.  
 b. A soccer ball weighs 423 grams. Due to wear and tear, the weight of the ball decreases by 16 grams. Is the weight acceptable? Explain.

**ERROR ANALYSIS** In Exercises 49 and 50, describe and correct the error in solving the equation.

49.  $|2x - 1| = -9$   
 $2x - 1 = -9$  or  $2x - 1 = -(-9)$   
 $2x = -8$        $2x = 10$   
 $x = -4$        $x = 5$   
 The solutions are  $x = -4$  and  $x = 5$ .

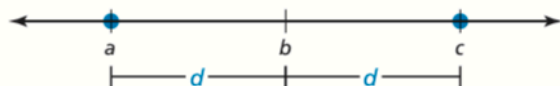
50.  $|5x + 8| = x$   
 $5x + 8 = x$  or  $5x + 8 = -x$   
 $4x + 8 = 0$        $6x + 8 = 0$   
 $4x = -8$        $6x = -8$   
 $x = -2$        $x = -\frac{4}{3}$   
 The solutions are  $x = -2$  and  $x = -\frac{4}{3}$ .

51. **ANALYZING EQUATIONS** Without solving completely, place each equation into one of the three categories.

No solution	One solution	Two solutions
$ x - 2  + 6 = 0$	$ x + 3  - 1 = 0$	
$ x + 8  + 2 = 7$	$ x - 1  + 4 = 4$	
$ x - 6  - 5 = -9$	$ x + 5  - 8 = -8$	

52. **USING STRUCTURE** Fill in the equation

$|x - \boxed{\phantom{a}}| = \boxed{\phantom{a}}$  with  $a, b, c$ , or  $d$  so that the equation is graphed correctly.



**ABSTRACT REASONING** In Exercises 53–56, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.

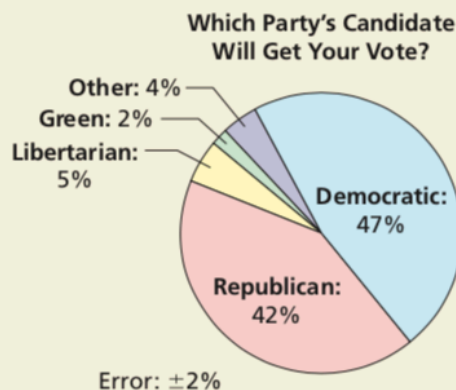
53. If  $x^2 = a^2$ , then  $|x|$  is \_\_\_\_\_ equal to  $|a|$ .
54. If  $a$  and  $b$  are real numbers, then  $|a - b|$  is \_\_\_\_\_ equal to  $|b - a|$ .
55. For any real number  $p$ , the equation  $|x - 4| = p$  will \_\_\_\_\_ have two solutions.
56. For any real number  $p$ , the equation  $|x - p| = 4$  will \_\_\_\_\_ have two solutions.
57. **WRITING** Explain why absolute value equations can have no solution, one solution, or two solutions. Give an example of each case.

58. **THOUGHT PROVOKING** Describe a real-life situation that can be modeled by an absolute value equation with the solutions  $x = 62$  and  $x = 72$ .

59. **CRITICAL THINKING** Solve the equation shown. Explain how you found your solution(s).

$$8|x + 2| - 6 = 5|x + 2| + 3$$

60. **HOW DO YOU SEE IT?** The circle graph shows the results of a survey of registered voters the day of an election.



The error given in the graph means that the actual percent could be 2% more or 2% less than the percent reported by the survey.

- What are the minimum and maximum percents of voters who could vote Republican? Green?
  - How can you use absolute value equations to represent your answers in part (a)?
  - One candidate receives 44% of the vote. Which party does the candidate belong to? Explain.
61. **ABSTRACT REASONING** How many solutions does the equation  $a|x + b| + c = d$  have when  $a > 0$  and  $c = d$ ? when  $a < 0$  and  $c > d$ ? Explain your reasoning.

## Maintaining Mathematical Proficiency Reviewing what you learned in previous grades and lessons

Identify the property of equality that makes Equation 1 and Equation 2 equivalent. (*Section 1.1*)

62.

Equation 1  $3x + 8 = x - 1$

Equation 2  $3x + 9 = x$

63.

Equation 1  $4y = 28$

Equation 2  $y = 7$

Use a geometric formula to solve the problem. (*Skills Review Handbook*)

64. A square has an area of 81 square meters. Find the side length.
65. A circle has an area of  $36\pi$  square inches. Find the radius.
66. A triangle has a height of 8 feet and an area of 48 square feet. Find the base.
67. A rectangle has a width of 4 centimeters and a perimeter of 26 centimeters. Find the length.