

1.4 Solving Absolute Value Equations

What does an absolute value do? What does it mean?

Properties of absolute values:

- 1) $|a| \geq 0$
- 2) $|-a| = |a|$
- 3) $|ab| = |a||b|$
- 4) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}, b \neq 0$

Example 1: Solving Absolute Value Equations

Solve each equation.

a) $|x - 4| = 6$ b) $|3x + 1| = -5$

Example 2: Solving an Absolute Value Equations

Solve.

$$|3x + 9| - 10 = -4$$

Example 3: Modeling & Writing an Absolute Value Equation.

The cheerleading half time show at all Bergen Tech sporting events has a minimum length of 4 minutes. The maximum length of a routine is 5 minutes. Write an absolute value equation that represents the minimum and maximum lengths.

Try on your own!

Solve the equations. Check your solutions

$$4. |x - 2| + 5 = 9$$

$$5. 4|2x + 7| = 16$$

$$6. -2|5x - 1| - 3 = -11$$

7. For a poetry contest, the minimum length of a poem is 16 lines. The maximum length is 32 lines. Write an absolute value equation that represents the minimum and maximum lengths.

Example 4: Solving Equations with Two Absolute Values

Solve:

$$a) |3x - 4| = |x|$$

$$b) |4x - 10| = 2|3x + 1|$$

Try on your own: Solve the following.

$$8. |x + 8| = |2x + 1|$$

$$9. 3|x - 4| = |2x + 5|$$

Example 5: Identifying Extraneous Solutions

Solve. Check your solutions.

$$|2x + 12| = 4x$$

Example 6: Solving Absolute Value Equations

Solve:

$$|x + 5| = |x + 11|$$

Try on your own: Solve.

10. $|x + 6| = 2x$

11. $|3x - 2| = x$

12. $|2 + x| = |x - 8|$

13. $|5x - 2| = |5x + 4|$

Homework:

15-23 odd, 26-30, 35, 39, 41, 32, 52, 59*

1.4 Exercises

Dynamic Solutions available at BigIdeasMath.com

Vocabulary and Core Concept Check

- VOCABULARY** What is an extraneous solution?
- WRITING** Without calculating, how do you know that the equation $|4x - 7| = -1$ has no solution?

Monitoring Progress and Modeling with Mathematics

In Exercises 3–10, simplify the expression.

3. $|-9|$
4. $-|15|$
5. $|14| - |-14|$
6. $|-3| + |3|$
7. $-|-5 \cdot (-7)|$
8. $|-0.8 \cdot 10|$
9. $\left| \frac{27}{-3} \right|$
10. $\left| -\frac{-12}{4} \right|$

In Exercises 11–24, solve the equation. Graph the solution(s), if possible. (See Examples 1 and 2.)

11. $|w| = 6$
12. $|r| = -2$
13. $|y| = -18$
14. $|x| = 13$
15. $|m + 3| = 7$
16. $|q - 8| = 14$
17. $|-3d| = 15$
18. $\left| \frac{t}{2} \right| = 6$
19. $|4b - 5| = 19$
20. $|x - 1| + 5 = 2$
21. $-4|8 - 5n| = 13$
22. $-3\left| 1 - \frac{2}{3}v \right| = -9$
23. $3 = -2\left| \frac{1}{4}s - 5 \right| + 3$
24. $9|4p + 2| + 8 = 35$

25. **WRITING EQUATIONS** The minimum distance from Earth to the Sun is 91.4 million miles. The maximum distance is 94.5 million miles. (See Example 3.)

- Represent these two distances on a number line.
- Write an absolute value equation that represents the minimum and maximum distances.

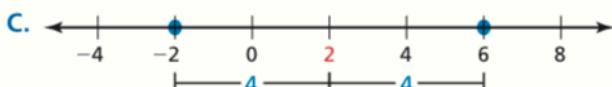
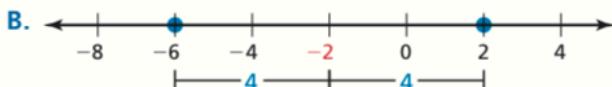
26. **WRITING EQUATIONS** The shoulder heights of the shortest and tallest miniature poodles are shown.



- Represent these two heights on a number line.
- Write an absolute value equation that represents these heights.

USING STRUCTURE In Exercises 27–30, match the absolute value equation with its graph without solving the equation.

27. $|x + 2| = 4$
28. $|x - 4| = 2$
29. $|x - 2| = 4$
30. $|x + 4| = 2$



In Exercises 31–34, write an absolute value equation that has the given solutions.

31. $x = 8$ and $x = 18$

32. $x = -6$ and $x = 10$

33. $x = 2$ and $x = 9$

34. $x = -10$ and $x = -5$

In Exercises 35–44, solve the equation. Check your solutions. (See Examples 4, 5, and 6.)

35. $|4n - 15| = |n|$

36. $|2c + 8| = |10c|$

37. $|2b - 9| = |b - 6|$

38. $|3k - 2| = 2|k + 2|$

39. $4|p - 3| = |2p + 8|$

40. $2|4w - 1| = 3|4w + 2|$

41. $|3h + 1| = 7h$

42. $|6a - 5| = 4a$

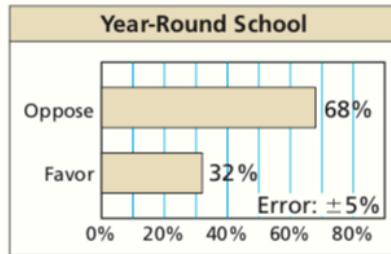
43. $|f - 6| = |f + 8|$

44. $|3x - 4| = |3x - 5|$

45. **MODELING WITH MATHEMATICS** Starting from 300 feet away, a car drives toward you. It then passes by you at a speed of 48 feet per second. The distance d (in feet) of the car from you after t seconds is given by the equation $d = |300 - 48t|$. At what times is the car 60 feet from you?

46. **MAKING AN ARGUMENT** Your friend says that the absolute value equation $|3x + 8| - 9 = -5$ has no solution because the constant on the right side of the equation is negative. Is your friend correct? Explain.

47. **MODELING WITH MATHEMATICS** You randomly survey students about year-round school. The results are shown in the graph.



The error given in the graph means that the actual percent could be 5% more or 5% less than the percent reported by the survey.

- Write and solve an absolute value equation to find the least and greatest percents of students who could be in favor of year-round school.
- A classmate claims that $\frac{1}{3}$ of the student body is actually in favor of year-round school. Does this conflict with the survey data? Explain.

48. **MODELING WITH MATHEMATICS** The recommended weight of a soccer ball is 430 grams. The actual weight is allowed to vary by up to 20 grams.

- Write and solve an absolute value equation to find the minimum and maximum acceptable soccer ball weights.
- A soccer ball weighs 423 grams. Due to wear and tear, the weight of the ball decreases by 16 grams. Is the weight acceptable? Explain.



ERROR ANALYSIS In Exercises 49 and 50, describe and correct the error in solving the equation.

49.



$$|2x - 1| = -9$$

$$2x - 1 = -9 \quad \text{or} \quad 2x - 1 = -(-9)$$

$$2x = -8 \quad \quad \quad 2x = 10$$

$$x = -4 \quad \quad \quad x = 5$$

The solutions are $x = -4$ and $x = 5$.

50.



$$|5x + 8| = x$$

$$5x + 8 = x \quad \text{or} \quad 5x + 8 = -x$$

$$4x + 8 = 0 \quad \quad \quad 6x + 8 = 0$$

$$4x = -8 \quad \quad \quad 6x = -8$$

$$x = -2 \quad \quad \quad x = -\frac{4}{3}$$

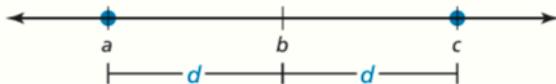
The solutions are $x = -2$ and $x = -\frac{4}{3}$.

51. **ANALYZING EQUATIONS** Without solving completely, place each equation into one of the three categories.

No solution	One solution	Two solutions
$ x - 2 + 6 = 0$	$ x + 3 - 1 = 0$	
$ x + 8 + 2 = 7$	$ x - 1 + 4 = 4$	
$ x - 6 - 5 = -9$	$ x + 5 - 8 = -8$	

52. **USING STRUCTURE** Fill in the equation

$|x - \boxed{\quad}| = \boxed{\quad}$ with a, b, c , or d so that the equation is graphed correctly.



ABSTRACT REASONING In Exercises 53–56, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.

53. If $x^2 = a^2$, then $|x|$ is _____ equal to $|a|$.

54. If a and b are real numbers, then $|a - b|$ is _____ equal to $|b - a|$.

55. For any real number p , the equation $|x - 4| = p$ will _____ have two solutions.

56. For any real number p , the equation $|x - p| = 4$ will _____ have two solutions.

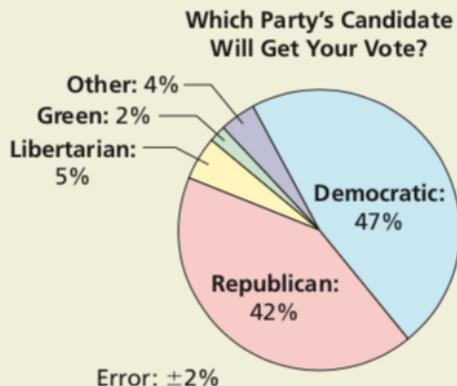
57. **WRITING** Explain why absolute value equations can have no solution, one solution, or two solutions. Give an example of each case.

58. **THOUGHT PROVOKING** Describe a real-life situation that can be modeled by an absolute value equation with the solutions $x = 62$ and $x = 72$.

59. **CRITICAL THINKING** Solve the equation shown. Explain how you found your solution(s).

$$8|x + 2| - 6 = 5|x + 2| + 3$$

60. **HOW DO YOU SEE IT?** The circle graph shows the results of a survey of registered voters the day of an election.



The error given in the graph means that the actual percent could be 2% more or 2% less than the percent reported by the survey.

- What are the minimum and maximum percents of voters who could vote Republican? Green?
- How can you use absolute value equations to represent your answers in part (a)?
- One candidate receives 44% of the vote. Which party does the candidate belong to? Explain.

61. **ABSTRACT REASONING** How many solutions does the equation $a|x + b| + c = d$ have when $a > 0$ and $c = d$? when $a < 0$ and $c > d$? Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Identify the property of equality that makes Equation 1 and Equation 2 equivalent. (*Section 1.1*)

62.

Equation 1 $3x + 8 = x - 1$

Equation 2 $3x + 9 = x$

63.

Equation 1 $4y = 28$

Equation 2 $y = 7$

Use a geometric formula to solve the problem. (*Skills Review Handbook*)

64. A square has an area of 81 square meters. Find the side length.

65. A circle has an area of 36π square inches. Find the radius.

66. A triangle has a height of 8 feet and an area of 48 square feet. Find the base.

67. A rectangle has a width of 4 centimeters and a perimeter of 26 centimeters. Find the length.