1.4 Solving Linear Systems

Do Now: Solve the system of equations

$2x - 3y = -2$

$4x + y = 24$

You have solved systems of linear equations with substitution, elimination, and graphing. Let us take a look in a new dimension! (X , Y , Z)

Example 1: Solving a Three-Variable System

Solve the system:

$$4x + 2y + 3z = 12$$
$$2x - 3y + 5z = -7$$
$$6x - y + 4z = -3$$

Example 2: Solving a Three-Variable System

Solve the system:

$$x + y + z = 2$$
$$5x + 5y + 5z = 3$$
$$4x + y - 3z = -6$$

Example 3: Solving a Three-Variable System
Solve the system

$$x - y + z = -3$$
$$x - y - z = -3$$
$$5x - 5y + z = -15$$

Try on your own:

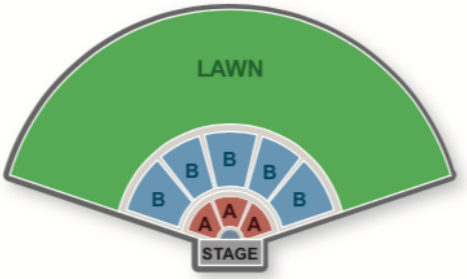
1. $x - 2y + z = -11$
$3x + 2y - z = 7$
$-x + 2y + 4z = -9$

2. $x + y - z = -1$
$4x + 4y - 4z = -2$
$3x + 2y + z = 0$

3. $x + y + z = 8$
$x - y + z = 8$
$2x + y + 2z = 16$

Example 4: Modeling with Mathematics

PNC Bank Art Center is an amphitheater that changes $75 for each seat in Section A, $55 for each seat in Section B, and $30 for each lawn seat. There are three times as many seats in Section B as in Section A. The revenue from selling all 23,000 seats is $870,000. How many seats are in each section of the amphitheater?



Homework:

3, 5, 7, 12, 14, 18, 34, 41, 43*

1.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **VOCABULARY** The solution of a system of three linear equations is expressed as a(n)_____.
2. **WRITING** Explain how you know when a linear system in three variables has infinitely many solutions.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, solve the system using the elimination method. *(See Example 1.)*

3. $x + y - 2z = 5$
   $-x + 2y + z = 2$
   $2x + 3y - z = 9$

4. $x + 4y - 6z = -1$
   $2x - y + 2z = -7$
   $-x + 2y - 4z = 5$

5. $2x + y - z = 9$
   $-x + 6y + 2z = -17$
   $5x + 7y + z = 4$

6. $3x + 2y - z = 8$
   $-3x + 4y + 5z = -14$
   $x - 3y + 4z = -14$

7. $2x + 2y + 5z = -1$
   $2x - y + z = 2$
   $2x + 4y - 3z = 14$

8. $3x + 2y - 3z = -2$
   $7x - 2y + 5z = -14$
   $2x + 4y + z = 6$

ERROR ANALYSIS **In Exercises 9 and 10, describe and correct the error in the first step of solving the system of linear equations.**

$$4x - y + 2z = -18$$
$$-x + 2y + z = 11$$
$$3x + 3y - 4z = 44$$

9. ✗
$$\begin{array}{r} 4x - y + 2z = -18 \\ -4x + 2y + z = 11 \\ \hline y + 3z = -7 \end{array}$$

10. ✗
$$\begin{array}{r} 12x - 3y + 6z = -18 \\ 3x + 3y - 4z = 44 \\ \hline 15x + 2z = 26 \end{array}$$

In Exercises 11–16, solve the system using the elimination method. *(See Examples 2 and 3.)*

11. $3x - y + 2z = 4$
    $6x - 2y + 4z = -8$
    $2x - y + 3z = 10$

12. $5x + y - z = 6$
    $x + y + z = 2$
    $12x + 4y = 10$

13. $x + 3y - z = 2$
    $x + y - z = 0$
    $3x + 2y - 3z = -1$

14. $x + 2y - z = 3$
    $-2x - y + z = -1$
    $6x - 3y - z = -7$

15. $x + 2y + 3z = 4$
    $-3x + 2y - z = 12$
    $-2x - 2y - 4z = -14$

16. $-2x - 3y + z = -6$
    $x + y - z = 5$
    $7x + 8y - 6z = 31$

17. **MODELING WITH MATHEMATICS** Three orders are placed at a pizza shop. Two small pizzas, a liter of soda, and a salad cost \$14; one small pizza, a liter of soda, and three salads cost \$15; and three small pizzas, a liter of soda, and two salads cost \$22. How much does each item cost?

18. **MODELING WITH MATHEMATICS** Sam's Furniture Store places the following advertisement in the local newspaper. Write a system of equations for the three combinations of furniture. What is the price of each piece of furniture? Explain.

In Exercises 19–28, solve the system of linear equations using the substitution method. *(See Example 4.)*

19. $-2x + y + 6z = 1$
$3x + 2y + 5z = 16$
$7x + 3y - 4z = 11$

20. $x - 6y - 2z = -8$
$-x + 5y + 3z = 2$
$3x - 2y - 4z = 18$

21. $x + y + z = 4$
$5x + 5y + 5z = 12$
$x - 4y + z = 9$

22. $x + 2y = -1$
$-x + 3y + 2z = -4$
$-x + y - 4z = 10$

23. $2x - 3y + z = 10$
$y + 2z = 13$
$z = 5$

24. $x = 4$
$x + y = -6$
$4x - 3y + 2z = 26$

25. $x + y - z = 4$
$3x + 2y + 4z = 17$
$-x + 5y + z = 8$

26. $2x - y - z = 15$
$4x + 5y + 2z = 10$
$-x - 4y + 3z = -20$

27. $4x + y + 5z = 5$
$8x + 2y + 10z = 10$
$x - y - 2z = -2$

28. $x + 2y - z = 3$
$2x + 4y - 2z = 6$
$-x - 2y + z = -6$

29. PROBLEM SOLVING The number of left-handed people in the world is one-tenth the number of right-handed people. The percent of right-handed people is nine times the percent of left-handed people and ambidextrous people combined. What percent of people are ambidextrous?

30. MODELING WITH MATHEMATICS Use a system of linear equations to model the data in the following newspaper article. Solve the system to find how many athletes finished in each place.

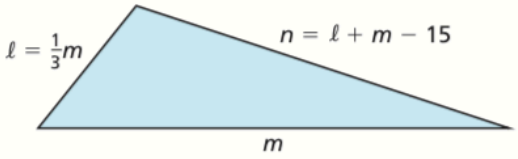
> Lawrence High prevailed in Saturday's track meet with the help of 20 individual-event placers earning a combined 68 points. A first-place finish earns 5 points, a second-place finish earns 3 points, and a third-place finish earns 1 point. Lawrence had a strong second-place showing, with as many second place finishers as first- and third-place finishers combined.

31. WRITING Explain when it might be more convenient to use the elimination method than the substitution method to solve a linear system. Give an example to support your claim.

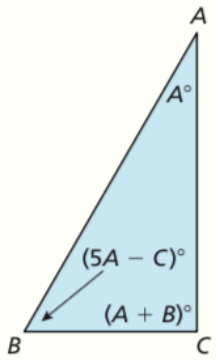
32. REPEATED REASONING Using what you know about solving linear systems in two and three variables, plan a strategy for how you would solve a system that has *four* linear equations in *four* variables.

MATHEMATICAL CONNECTIONS In Exercises 33 and 34, write and use a linear system to answer the question.

33. The triangle has a perimeter of 65 feet. What are the lengths of sides ℓ, m, and n?



34. What are the measures of angles A, B, and C?



35. OPEN-ENDED Consider the system of linear equations below. Choose nonzero values for a, b, and c so the system satisfies the given condition. Explain your reasoning.

$$x + y + z = 2$$
$$ax + by + cz = 10$$
$$x - 2y + z = 4$$

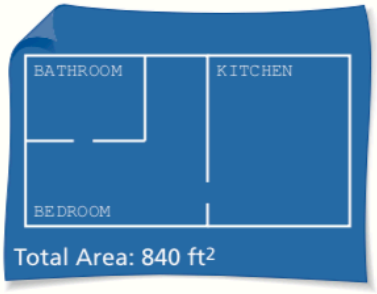
a. The system has no solution.

b. The system has exactly one solution.

c. The system has infinitely many solutions.

36. MAKING AN ARGUMENT A linear system in three variables has no solution. Your friend concludes that it is not possible for two of the three equations to have any points in common. Is your friend correct? Explain your reasoning.

37. PROBLEM SOLVING A contractor is hired to build an apartment complex. Each 840-square-foot unit has a bedroom, kitchen, and bathroom. The bedroom will be the same size as the kitchen. The owner orders 980 square feet of tile to completely cover the floors of two kitchens and two bathrooms. Determine how many square feet of carpet is needed for each bedroom.



38. THOUGHT PROVOKING Does the system of linear equations have more than one solution? Justify your answer.

$$4x + y + z = 0$$
$$2x + \tfrac{1}{2}y - 3z = 0$$
$$-x - \tfrac{1}{4}y - z = 0$$

39. PROBLEM SOLVING A florist must make 5 identical bridesmaid bouquets for a wedding. The budget is \$160, and each bouquet must have 12 flowers. Roses cost \$2.50 each, lilies cost \$4 each, and irises cost \$2 each. The florist wants twice as many roses as the other two types of flowers combined.

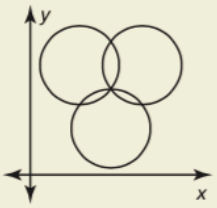
a. Write a system of equations to represent this situation, assuming the florist plans to use the maximum budget.

b. Solve the system to find how many of each type of flower should be in each bouquet.

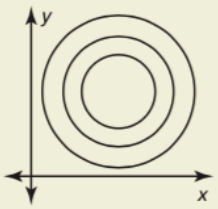
c. Suppose there is no limitation on the total cost of the bouquets. Does the problem still have exactly one solution? If so, find the solution. If not, give three possible solutions.

40. HOW DO YOU SEE IT? Determine whether the system of equations that represents the circles has *no solution*, *one solution*, or *infinitely many solutions*. Explain your reasoning.

a.



b.



41. CRITICAL THINKING Find the values of a, b, and c so that the linear system shown has $(-1, 2, -3)$ as its only solution. Explain your reasoning.

$$x + 2y - 3z = a$$
$$-x - y + z = b$$
$$2x + 3y - 2z = c$$

42. ANALYZING RELATIONSHIPS Determine which arrangement(s) of the integers -5, 2, and 3 produce a solution of the linear system that consist of only integers. Justify your answer.

$$x - 3y + 6z = 21$$
$$\_\_x + \_\_y + \_\_z = -30$$
$$2x - 5y + 2z = -6$$

43. ABSTRACT REASONING Write a linear system to represent the first three pictures below. Use the system to determine how many tangerines are required to balance the apple in the fourth picture. *Note:* The first picture shows that one tangerine and one apple balance one grapefruit.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify. *(Skills Review Handbook)*

44. $(x - 2)^2$ **45.** $(3m + 1)^2$ **46.** $(2z - 5)^2$ **47.** $(4 - y)^2$

Write a function g described by the given transformation of $f(x) = |x| - 5$. *(Section 1.2)*

48. translation 2 units to the left **49.** reflection in the x-axis

50. translation 4 units up **51.** vertical stretch by a factor of 3