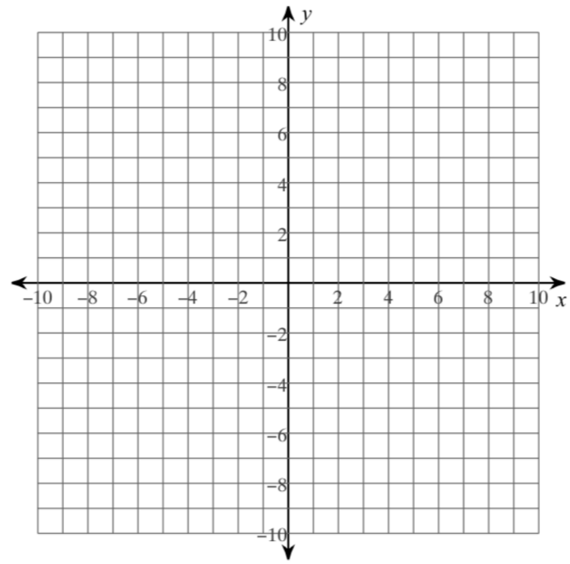
Chapter 2
2.1 Transformations of Quadratic Functions

The U-shape graph produced by a quadratic function is called a ________________

Horizontal and Vertical Translations
A horizontal translation is INSIDE the function and a vertical translation is OUTSIDE the function.

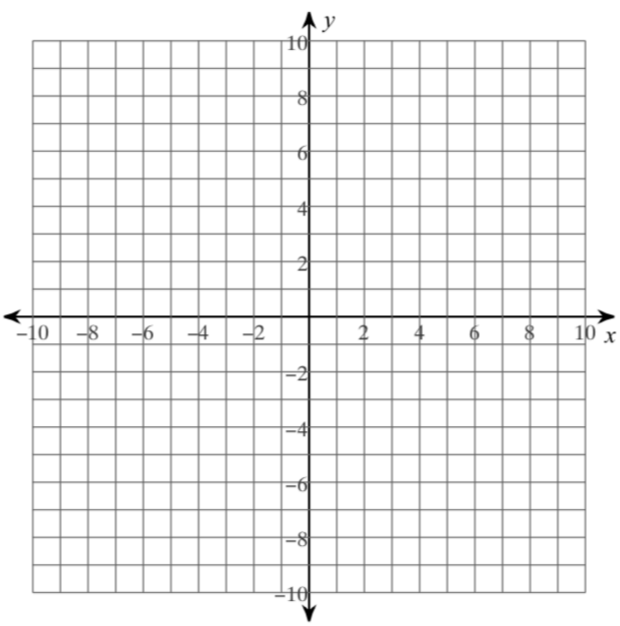
Example 1: Translations of a Quadratic Function
Describe the transformation of $f(x) = x^2$
represented by $g(x) = (x+4)^2 - 1$. Then graph each function.

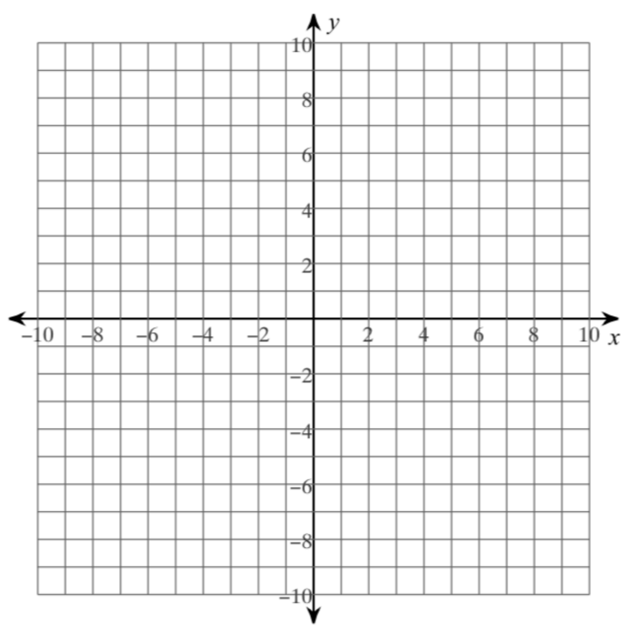


Example 2: Transformations of Quadratic Functions
Describe the transformation of $f(x) = x^2$ represented by g. Then graph each function.

a) $f(x) = -\frac{1}{2}x^2$

b) $f(x) = (2x)^2 + 1$





Try on your own: Describe the transformation of $f(x) = x^2$ represented by g

a) $g(x) = (\frac{1}{3}x)^2$

b) $g(x) = 3(x-1)^2$

c) $f(x) = -(x+3)^2 + 2$

Vertex Form: $f(x) = a(x-h)^2 + k$

Example 3: Writing a Transformed Quadratic Function
Let the graph of g be a vertical stretch by a factor of 2 and a reflection in the x-axis, followed by a translation 3 units down of the graph of $f(x) = x^2$. Write a rule for g and identify the vertex. Classify if it has a max or min.

Example 4: Writing a Transformed Quadratic Function
Let the graph of g be a translation 3 units right and 2 units up, followed by a reflection in the y-axis of the graph of $f(x) = x^2 - 5x$. Write a rule for g.

Example 5: Modeling with Mathematics
A firetruck arrives at a fire. Jacob is on the ladder spraying the fire hose. The water can be modeled by $f(x) = -0.03x^2 + x + 25$ where x is the horizontal distance (in feet) form the fire truck. The crew raises the ladder so that the water hits the ground 10ft further from the truck. Write a function that models the new path of the water.

Homework:
17-24(graph on calc), 31-34, 44, 45

2.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

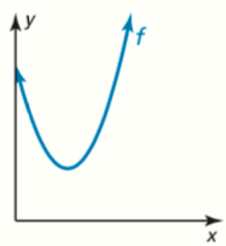
1. **COMPLETE THE SENTENCE** The graph of a quadratic function is called a(n) ________.
2. **VOCABULARY** Identify the vertex of the parabola given by $f(x) = (x + 2)^2 - 4$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–12, describe the transformation of $f(x) = x^2$ represented by g. Then graph each function. *(See Example 1.)*

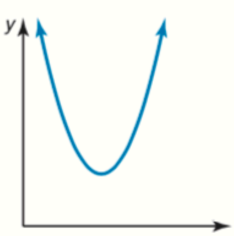
3. $g(x) = x^2 - 3$
4. $g(x) = x^2 + 1$
5. $g(x) = (x + 2)^2$
6. $g(x) = (x - 4)^2$
7. $g(x) = (x - 1)^2$
8. $g(x) = (x + 3)^2$
9. $g(x) = (x + 6)^2 - 2$
10. $g(x) = (x - 9)^2 + 5$
11. $g(x) = (x - 7)^2 + 1$
12. $g(x) = (x + 10)^2 - 3$

ANALYZING RELATIONSHIPS **In Exercises 13–16, match the function with the correct transformation of the graph of f. Explain your reasoning.**



13. $y = f(x - 1)$
14. $y = f(x) + 1$
15. $y = f(x - 1) + 1$
16. $y = f(x + 1) - 1$

A.



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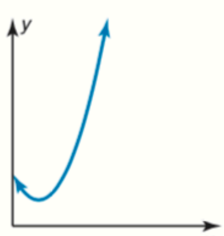

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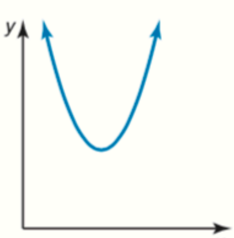

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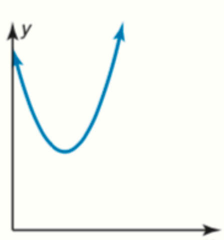


In Exercises 17–24, describe the transformation of $f(x) = x^2$ represented by g. Then graph each function. *(See Example 2.)*

17. $g(x) = -x^2$
18. $g(x) = (-x)^2$
19. $g(x) = 3x^2$
20. $g(x) = \frac{1}{3}x^2$
21. $g(x) = (2x)^2$
22. $g(x) = -(2x)^2$
23. $g(x) = \frac{1}{5}x^2 - 4$
24. $g(x) = \frac{1}{2}(x - 1)^2$

ERROR ANALYSIS **In Exercises 25 and 26, describe and correct the error in analyzing the graph of $f(x) = -6x^2 + 4$.**

25. ✗ The graph is a reflection in the y-axis and a vertical stretch by a factor of 6, followed by a translation 4 units up of the graph of the parent quadratic function.

26. ✗ The graph is a translation 4 units down, followed by a vertical stretch by a factor of 6 and a reflection in the x-axis of the graph of the parent quadratic function.

USING STRUCTURE **In Exercises 27–30, describe the transformation of the graph of the parent quadratic function. Then identify the vertex.**

27. $f(x) = 3(x + 2)^2 + 1$
28. $f(x) = -4(x + 1)^2 - 5$
29. $f(x) = -2x^2 + 5$
30. $f(x) = \frac{1}{2}(x - 1)^2$

In Exercises 31–34, write a rule for g described by the transformations of the graph of f. Then identify the vertex. *(See Examples 3 and 4.)*

31. $f(x) = x^2$; vertical stretch by a factor of 4 and a reflection in the x-axis, followed by a translation 2 units up

32. $f(x) = x^2$; vertical shrink by a factor of $\frac{1}{3}$ and a reflection in the y-axis, followed by a translation 3 units right

33. $f(x) = 8x^2 - 6$; horizontal stretch by a factor of 2 and a translation 2 units up, followed by a reflection in the y-axis

34. $f(x) = (x + 6)^2 + 3$; horizontal shrink by a factor of $\frac{1}{2}$ and a translation 1 unit down, followed by a reflection in the x-axis

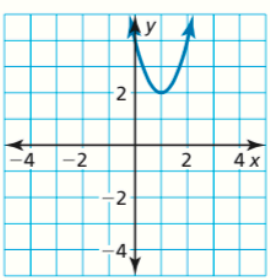
USING TOOLS In Exercises 35–40, match the function with its graph. Explain your reasoning.

35. $g(x) = 2(x - 1)^2 - 2$ **36.** $g(x) = \frac{1}{2}(x + 1)^2 - 2$

37. $g(x) = -2(x - 1)^2 + 2$

38. $g(x) = 2(x + 1)^2 + 2$ **39.** $g(x) = -2(x + 1)^2 - 2$

40. $g(x) = 2(x - 1)^2 + 2$

A.


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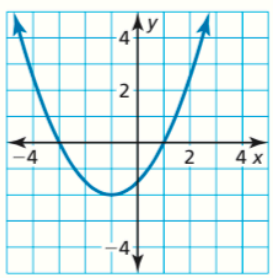

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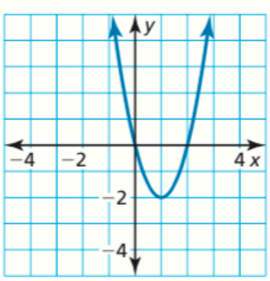

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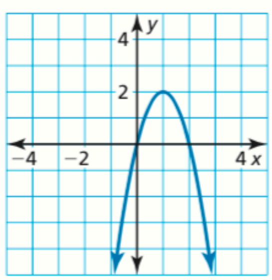

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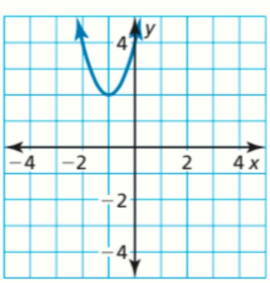

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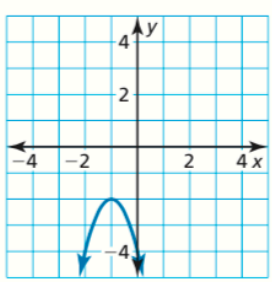


JUSTIFYING STEPS In Exercises 41 and 42, justify each step in writing a function g based on the transformations of $f(x) = 2x^2 + 6x$.

41. translation 6 units down followed by a reflection in the x-axis

$$\begin{aligned} h(x) &= f(x) - 6 \\ &= 2x^2 + 6x - 6 \\ g(x) &= -h(x) \\ &= -(2x^2 + 6x - 6) \\ &= -2x^2 - 6x + 6 \end{aligned}$$

42. reflection in the y-axis followed by a translation 4 units right

$$\begin{aligned} h(x) &= f(-x) \\ &= 2(-x)^2 + 6(-x) \\ &= 2x^2 - 6x \\ g(x) &= h(x - 4) \\ &= 2(x - 4)^2 - 6(x - 4) \\ &= 2x^2 - 22x + 56 \end{aligned}$$

43. MODELING WITH MATHEMATICS The function $h(x) = -0.03(x - 14)^2 + 6$ models the jump of a red kangaroo, where x is the horizontal distance traveled (in feet) and $h(x)$ is the height (in feet). When the kangaroo jumps from a higher location, it lands 5 feet farther away. Write a function that models the second jump. *(See Example 5.)*

44. MODELING WITH MATHEMATICS The function $f(t) = -16t^2 + 10$ models the height (in feet) of an object t seconds after it is dropped from a height of 10 feet on Earth. The same object dropped from the same height on the moon is modeled by $g(t) = -\frac{8}{3}t^2 + 10$. Describe the transformation of the graph of f to obtain g. From what height must the object be dropped on the moon so it hits the ground at the same time as on Earth?

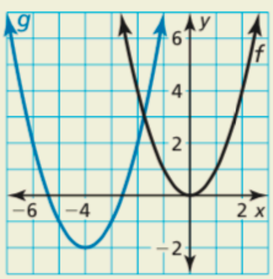
45. **MODELING WITH MATHEMATICS** Flying fish use their pectoral fins like airplane wings to glide through the air.

a. Write an equation of the form $y = a(x - h)^2 + k$ with vertex (33, 5) that models the flight path, assuming the fish leaves the water at (0, 0).

b. What are the domain and range of the function? What do they represent in this situation?

c. Does the value of a change when the flight path has vertex (30, 4)? Justify your answer.

46. **HOW DO YOU SEE IT?** Describe the graph of g as a transformation of the graph of $f(x) = x^2$.



47. **COMPARING METHODS** Let the graph of g be a translation 3 units up and 1 unit right followed by a vertical stretch by a factor of 2 of the graph of $f(x) = x^2$.

a. Identify the values of a, h, and k and use vertex form to write the transformed function.

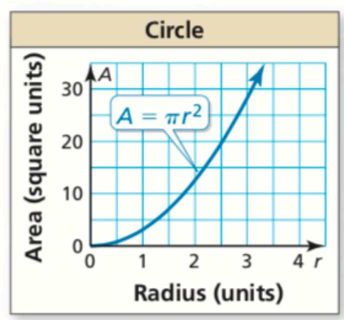
b. Use function notation to write the transformed function. Compare this function with your function in part (a).

c. Suppose the vertical stretch was performed first, followed by the translations. Repeat parts (a) and (b).

d. Which method do you prefer when writing a transformed function? Explain.

48. **THOUGHT PROVOKING** A jump on a pogo stick with a conventional spring can be modeled by $f(x) = -0.5(x - 6)^2 + 18$, where x is the horizontal distance (in inches) and $f(x)$ is the vertical distance (in inches). Write at least one transformation of the function and provide a possible reason for your transformation.

49. **MATHEMATICAL CONNECTIONS** The area of a circle depends on the radius, as shown in the graph. A circular earring with a radius of r millimeters has a circular hole with a radius of $\frac{3r}{4}$ millimeters. Describe a transformation of the graph below that models the area of the blue portion of the earring.

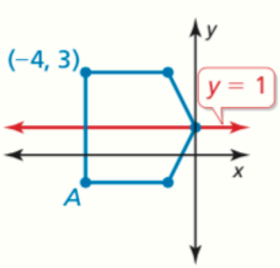


Maintaining Mathematical Proficiency

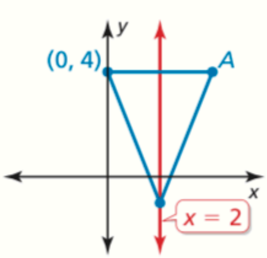
Reviewing what you learned in previous grades and lessons

A line of symmetry for the figure is shown in red. Find the coordinates of point A.
(Skills Review Handbook)

50.



51.



52.