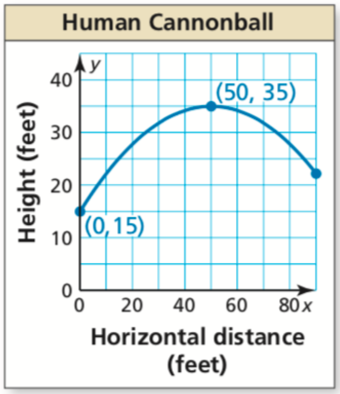
2.4 Modeling with Quadratic Functions

Objective: Write equations of quadratic functions using vertices, points, and x-intercepts

Example 1: Vertex and a Point
The graph shows the path of a performer who is shot out of a cannon, where y represents the height and x is the horizontal distance. Write an equation of the parabola. The performer lands in a net 95 feet from the cannon. What is the height of the net?



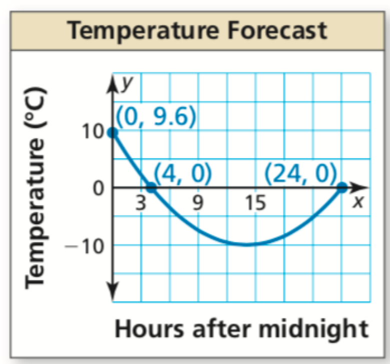
Try on your own:
Write an equation of a parabola that passes through the point (-1, 2) and has a vertex (4,-9)

Example 2: Point and X-Intercepts
A meteorologist creates a parabolic function that predicts the temperature tomorrow, where x is the number of hours after midnight and y is the temperature (in degrees Celsius).

a) Write a function f that models the temperature over time.

b) What is the coldest temperature?

c) What is the temperature at 17 hours?



Try on your own:
Write an equation of the parabola that passes through the point (2,5) and has x-intercepts -2 and 4.

Thinking back. How can you determine that the following is linear?

x	-2	-1	0	1	2	3
F(x)	-10	-5	0	5	10	15

Critical Thinking: What can you determine about the following?

X	-3	-2	-1	0	1	2	3
F(x)	9	4	1	0	1	4	9

Example 3: Three Points
NASA can create a weightless environment by flying a plan in parabolic paths. The table shows heights h (in feet) of the plane t seconds after starting the flight path. After about 20.8 seconds, passengers being to experience a weightless environment. Write and evaluate a function to approximate the height at which this occurs.

Time, *t*	Height, *h*
10	26,900
15	29,025
20	30,600
25	31,625
30	32,100
35	32,025
40	31,400

Try on your own: Write an equation of a parabola that passes through the points (-1,4), (0,1), and (2,7).

Example 4: Using Quadratic Regression

The table shows fuel efficiencies of a vehicle at difference speeds. Write a function that models the data. Use the model to approximate the optimal driving speed.

Miles per hour, x	Miles per gallon, y
20	14.5
24	17.5
30	21.2
36	23.7
40	25.2
45	25.8
50	25.8
56	25.1
60	24.0
70	19.5

Homework:

3, 5, 10, 13, 17, 19, 23, 27, 29, 30, 33, 38-41

2.4 Exercises

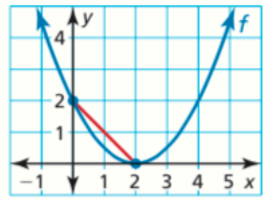
Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain when it is appropriate to use a quadratic model for a set of data.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find "both" answers.

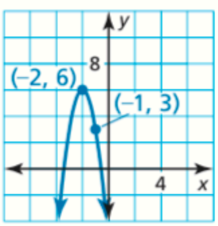
What is the average rate of change over $0 \le x \le 2$?	What is the distance from $f(0)$ to $f(2)$?
What is the slope of the line segment?	What is $\dfrac{f(2) - f(0)}{2 - 0}$?



Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, write an equation of the parabola in vertex form. *(See Example 1.)*

3.



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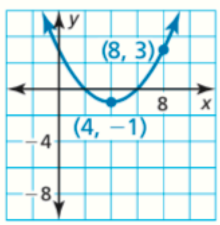


5. passes through (13, 8) and has vertex (3, 2)

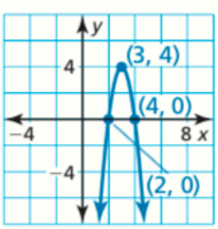
6. passes through (−7, −15) and has vertex (−5, 9)

7. passes through (0, −24) and has vertex (−6, −12)

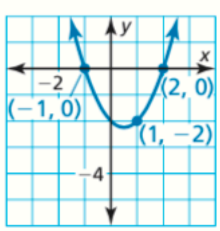
8. passes through (6, 35) and has vertex (−1, 14)

In Exercises 9–14, write an equation of the parabola in intercept form. *(See Example 2.)*

9.



10.



11. x-intercepts of 12 and −6; passes through (14, 4)

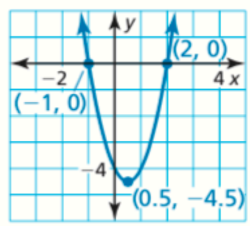
12. x-intercepts of 9 and 1; passes through (0, −18)

13. x-intercepts of −16 and −2; passes through (−18, 72)

14. x-intercepts of −7 and −3; passes through (−2, 0.05)

15. **WRITING** Explain when to use intercept form and when to use vertex form when writing an equation of a parabola.

16. **ANALYZING EQUATIONS** Which of the following equations represent the parabola?



Ⓐ $y = 2(x - 2)(x + 1)$

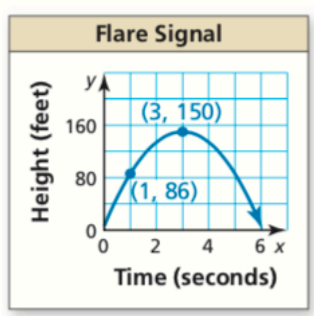
Ⓑ $y = 2(x + 0.5)^2 - 4.5$

Ⓒ $y = 2(x - 0.5)^2 - 4.5$

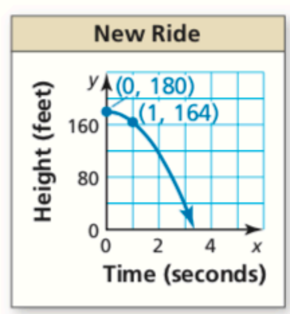
Ⓓ $y = 2(x + 2)(x - 1)$

In Exercises 17–20, write an equation of the parabola in vertex form or intercept form.

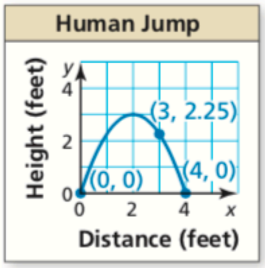
17.



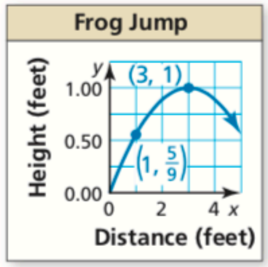
18.



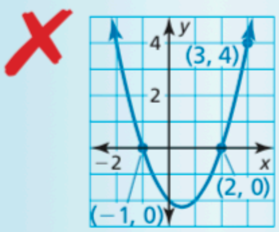
19.



20.



21. **ERROR ANALYSIS** Describe and correct the error in writing an equation of the parabola.



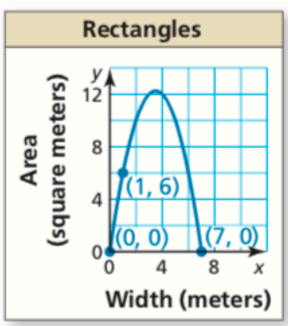
$y = a(x - p)(x - q)$

$4 = a(3 - 1)(3 + 2)$

$a = \frac{2}{5}$

$y = \frac{2}{5}(x - 1)(x + 2)$

22. **MATHEMATICAL CONNECTIONS** The area of a rectangle is modeled by the graph where y is the area (in square meters) and x is the width (in meters). Write an equation of the parabola. Find the dimensions and corresponding area of one possible rectangle. What dimensions result in the maximum area?



23. **MODELING WITH MATHEMATICS** Every rope has a safe working load. A rope should not be used to lift a weight greater than its safe working load. The table shows the safe working loads S (in pounds) for ropes with circumference C (in inches). Write an equation for the safe working load for a rope. Find the safe working load for a rope that has a circumference of 10 inches. *(See Example 3.)*

Circumference, C	0	1	2	3
Safe working load, S	0	180	720	1620

24. **MODELING WITH MATHEMATICS** A baseball is thrown up in the air. The table shows the heights y (in feet) of the baseball after x seconds. Write an equation for the path of the baseball. Find the height of the baseball after 5 seconds.

Time, x	0	2	4	6
Baseball height, y	6	22	22	6

25. **COMPARING METHODS** You use a system with three variables to find the equation of a parabola that passes through the points $(-8, 0)$, $(2, -20)$, and $(1, 0)$. Your friend uses intercept form to find the equation. Whose method is easier? Justify your answer.

26. **MODELING WITH MATHEMATICS** The table shows the distances y a motorcyclist is from home after x hours.

Time (hours), x	0	1	2	3
Distance (miles), y	0	45	90	135

a. Determine what type of function you can use to model the data. Explain your reasoning.

b. Write and evaluate a function to determine the distance the motorcyclist is from home after 6 hours.

27. **USING TOOLS** The table shows the heights h (in feet) of a sponge t seconds after it was dropped by a window cleaner on top of a skyscraper. *(See Example 4.)*

Time, t	0	1	1.5	2.5	3
Height, h	280	264	244	180	136

a. Use a graphing calculator to create a scatter plot. Which better represents the data, a line or a parabola? Explain.

b. Use the *regression* feature of your calculator to find the model that best fits the data.

c. Use the model in part (b) to predict when the sponge will hit the ground.

d. Identify and interpret the domain and range in this situation.

28. **MAKING AN ARGUMENT** Your friend states that quadratic functions with the same x-intercepts have the same equations, vertex, and axis of symmetry. Is your friend correct? Explain your reasoning.

In Exercises 29–32, analyze the differences in the outputs to determine whether the data are *linear*, *quadratic*, or *neither*. Explain. If linear or quadratic, write an equation that fits the data.

29.

Price decrease (dollars), x	0	5	10	15	20
Revenue ($1000s), y	470	630	690	650	510

30.

Time (hours), x	0	1	2	3	4
Height (feet), y	40	42	44	46	48

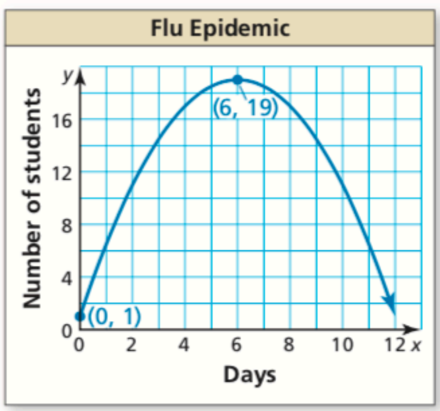
31.

Time (hours), x	1	2	3	4	5
Population (hundreds), y	2	4	8	16	32

32.

Time (days), x	0	1	2	3	4
Height (feet), y	320	303	254	173	60

33. PROBLEM SOLVING The graph shows the number y of students absent from school due to the flu each day x.



a. Interpret the meaning of the vertex in this situation.

b. Write an equation for the parabola to predict the number of students absent on day 10.

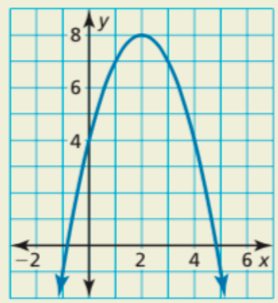
c. Compare the average rates of change in the students with the flu from 0 to 6 days and 6 to 11 days.

34. THOUGHT PROVOKING Describe a real-life situation that can be modeled by a quadratic equation. Justify your answer.

35. PROBLEM SOLVING The table shows the heights y of a competitive water-skier x seconds after jumping off a ramp. Write a function that models the height of the water-skier over time. When is the water-skier 5 feet above the water? How long is the skier in the air?

Time (seconds), x	0	0.25	0.75	1	1.1
Height (feet), y	22	22.5	17.5	12	9.24

36. HOW DO YOU SEE IT? Use the graph to determine whether the average rate of change over each interval is *positive*, *negative*, or *zero*.



a. $0 \le x \le 2$

b. $2 \le x \le 5$

c. $2 \le x \le 4$

d. $0 \le x \le 4$

37. REPEATED REASONING The table shows the number of tiles in each figure. Verify that the data show a quadratic relationship. Predict the number of tiles in the 12th figure.

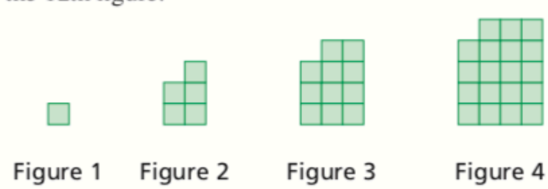


Figure	1	2	3	4
Number of Tiles	1	5	11	19

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Factor the trinomial. *(Skills Review Handbook)*

38. $x^2 + 4x + 3$

39. $x^2 - 3x + 2$

40. $3x^2 - 15x + 12$

41. $5x^2 + 5x - 30$