2.6 Solving Absolute Value Inequalities

Thinking Back: $|x| = 5$ means the distance between x and 0 is 5 units.

Core Concepts: pay close attention to your signs!

$|ax + b| < c$ means $ax + b < c$ **AND** $ax + b > -c$

$|ax + b| > c$ means $ax + b > c$ **OR** $ax + b < -c$

Example 1: Solving Absolute Value Inequalities
Solve each inequality. Graph each solution, if possible.

a) $|x + 7| < 2$

b) $|8x - 11| < 0$

Example 2: Solving Absolute Value Inequalities
Solve each inequality.

a) $|c - 1| \geq 5$

b) $|10 - m| \geq -2$

c) $4|2x - 5| + 1 > 21$

Concept Summary

Solving Inequalities

One-Step and Multi-Step Inequalities

- Follow the steps for solving an equation. Reverse the inequality symbol when multiplying or dividing by a negative number.

Compound Inequalities

- If necessary, write the inequality as two separate inequalities. Then solve each inequality separately. Include *and* or *or* in the solution.

Absolute Value Inequalities

- If necessary, isolate the absolute value expression on one side of the inequality. Write the absolute value inequality as a compound inequality. Then solve the compound inequality.

2.6 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **REASONING** Can you determine the solution of $|4x - 2| \geq -6$ without solving? Explain.
2. **WRITING** Describe how solving $|w - 9| \leq 2$ is different from solving $|w - 9| \geq 2$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–18, solve the inequality. Graph the solution, if possible. *(See Examples 1 and 2.)*

3. $|x| < 3$
4. $|y| \geq 4.5$
5. $|d + 9| > 3$
6. $|h - 5| \leq 10$
7. $|2s - 7| \geq -1$
8. $|4c + 5| > 7$
9. $|5p + 2| < -4$
10. $|9 - 4n| < 5$
11. $|6t - 7| - 8 \geq 3$
12. $|3j - 1| + 6 > 0$
13. $3|14 - m| > 18$
14. $-4|6b - 8| \leq 12$
15. $2|3w + 8| - 13 \leq -5$
16. $-3|2 - 4u| + 5 < -13$
17. $6|-f + 3| + 7 > 7$
18. $\frac{2}{3}|4v + 6| - 2 \leq 10$

19. **MODELING WITH MATHEMATICS** The rules for an essay contest say that entries can have 500 words with an absolute deviation of at most 30 words. Write and solve an absolute value inequality that represents the acceptable numbers of words. *(See Example 3.)*

20. **MODELING WITH MATHEMATICS** The normal body temperature of a camel is 37°C. This temperature varies by up to 3°C throughout the day. Write and solve an absolute value inequality that represents the range of normal body temperatures (in degrees Celsius) of a camel throughout the day.

ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in solving the absolute value inequality.**

21. ✗
$$|x - 5| < 20$$
$$x - 5 < 20$$
$$x < 25$$

22. ✗
$$|x + 4| > 13$$
$$x + 4 > -13 \quad \text{and} \quad x + 4 < 13$$
$$x > -17 \quad \text{and} \quad x < 9$$
$$-17 < x < 9$$

In Exercises 23–26, write the sentence as an absolute value inequality. Then solve the inequality.

23. A number is less than 6 units from 0.
24. A number is more than 9 units from 3.
25. Half of a number is at most 5 units from 14.
26. Twice a number is no less than 10 units from −1.

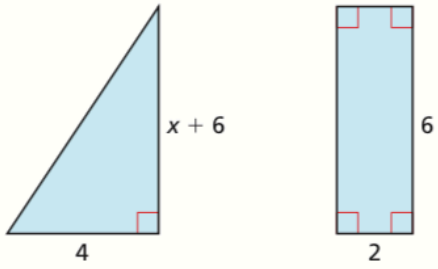
27. **PROBLEM SOLVING** An auto parts manufacturer throws out gaskets with weights that are not within 0.06 pound of the mean weight of the batch. The weights (in pounds) of the gaskets in a batch are 0.58, 0.63, 0.65, 0.53, and 0.61. Which gasket(s) should be thrown out?

28. **PROBLEM SOLVING** Six students measure the acceleration (in meters per second per second) of an object in free fall. The measured values are shown. The students want to state that the absolute deviation of each measured value x from the mean is at most d. Find the value of d.

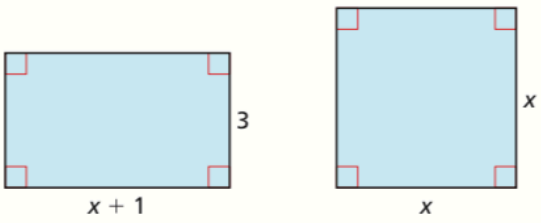
10.56, 9.52, 9.73, 9.80, 9.78, 10.91

MATHEMATICAL CONNECTIONS **In Exercises 29 and 30, write an absolute value inequality that represents the situation. Then solve the inequality.**

29. The difference between the areas of the figures is less than 2.



30. The difference between the perimeters of the figures is less than or equal to 3.



REASONING **In Exercises 31–34, tell whether the statement is true or false. If it is false, explain why.**

31. If a is a solution of $|x + 3| \leq 8$, then a is also a solution of $x + 3 \geq -8$.

32. If a is a solution of $|x + 3| > 8$, then a is also a solution of $x + 3 > 8$.

33. If a is a solution of $|x + 3| \geq 8$, then a is also a solution of $x + 3 \geq -8$.

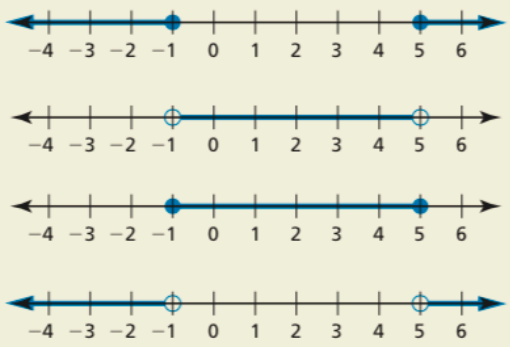
34. If a is a solution of $x + 3 \leq -8$, then a is also a solution of $|x + 3| \geq 8$.

35. MAKING AN ARGUMENT One of your classmates claims that the solution of $|n| > 0$ is all real numbers. Is your classmate correct? Explain your reasoning.

36. THOUGHT PROVOKING Draw and label a geometric figure so that the perimeter P of the figure is a solution of the inequality $|P - 60| \leq 12$.

37. REASONING What is the solution of the inequality $|ax + b| < c$, where $c < 0$? What is the solution of the inequality $|ax + b| > c$, where $c < 0$? Explain.

38. HOW DO YOU SEE IT? Write an absolute value inequality for each graph.



How did you decide which inequality symbol to use for each inequality?

39. WRITING Explain why the solution set of the inequality $|x| < 5$ is the *intersection* of two sets, while the solution set of the inequality $|x| > 5$ is the *union* of two sets.

40. PROBLEM SOLVING Solve the compound inequality below. Describe your steps.

$$|x - 3| < 4 \text{ and } |x + 2| > 8$$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Plot the ordered pair in a coordinate plane. Describe the location of the point. *(Skills Review Handbook)*

41. $A(1, 3)$ **42.** $B(0, -3)$ **43.** $C(-4, -2)$ **44.** $D(-1, 2)$

Copy and complete the table. *(Skills Review Handbook)*

45.

x	0	1	2	3	4
$5x + 1$					

46.

x	−2	−1	0	1	2
$-2x - 3$					